A Return and Risk Model for Efficient Spectrum Sharing in Cognitive Radio Networks

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***Abstract*—Cognitive Radio technology releases the spectrum from shackles of authorized licenses and facilitates the trading of spectrum bands. In the spectrum market, primary users (PUs) set prices for their vacant bands and sell them for monetary gains, and secondary users (SUs) buy the bands and opportunistically use them to satisfy their service demands when the PUs are not active. However, when there are multiple bands available for the SUs to access, the SU confronts the challenges of how to choose bands and how to split his traffic over them considering both the contention from peer SUs as well as the unpredictable activities of the PUs. In this paper, we propose a return and risk model to represent these concerns of the SU, and help the SU to make appropriate decisions of traffic distribution over available spectrum bands, either the bands belonging to SU itself or the bands shared with PUs. The simulation and analysis show that our spectrum sharing scheme is efficient in terms of maximum return for given risk or minimum risk for given return, and is also effective in improving the spectrum utilization and SUs' satisfactory degrees.**

***Index Terms*—Cognitive Radio, Return, Risk, Mix of Bands**

I. INTRODUCTION

Recent years has witnessed the booming growth of wireless networks and flourish of various wireless services. In parallel with that, current static spectrum allocation policy of Federal Communications Commission (FCC) [1] results in the exhaustion of available spectrum, while many licensed spectrum bands are extremely under usage. As one of the most promising solutions to improve spectrum utilization, cognitive radio technology allows the SUs to opportunistically access to vacant bands belonging to PUs in either temporal or spatial domain [2], [3].

The idea of opening up the licensed spectrum bands in cognitive radio networks initiates the market of spectrum trading, and promotes a batch of interesting research on related topics. Specifically, in [4], Grandblaise has generally described the possible scenarios and introduced some microeconomics inspired mechanisms of dynamic spectrum access; from the PUs' aspect, Xing [5] and Niyato [6] have well investigated the spectrum pricing issues in the spectrum market where multiple PUs compete with each other to offer spectrum access to the SUs; from the SUs' aspect, people are interested in how the SU chooses to distribute his traffic over the spectrum bands when there is more than one unoccupied licensed band. Motamedi [7] has formulated this problem as a multiarmed bandit problem and proposed a reinforce learning algorithm to consistently track the best band in terms of band condition.

However, if the SU jointly considers the contention among the SUs and the activities of PUs in cognitive radio networks, the concept of the best band tends to be ambiguous since he can neither identify the band with least competition among SUs as the best band, nor regard the one with most possibility of PU's absence as the best band.

In this paper, we propose to leverage SU's expected return to model the contention among SUs and the variance of return to represent the potential risk of PUs' coming back. Instead of swarming all the traffic over the best band, we make the SU split his traffic on a diversified mix of bands. In particular, we illustrate how the SU efficiently distributes his traffic over the risk free band of his own and risky bands shared with PUs, and how the SU efficiently selects the mix of risky bands belonging to PUs.[1] Simulation results show that the proposed spectrum sharing scheme is efficient in the sense that the SU is able to obtain maximum return for given risk or carry on minimum risk for given return, except for it is effective in improving the spectrum utilization and SUs' satisfaction.

II. SYSTEM MODEL

A. Overview

We consider a wireless system with multiple PUs operating on different spectrum bands and multiple SUs who can take opportunistic use of these frequencies. That is to say, considering the constraints of geographical location, SUs are able to access to different spectrum bands belonging to different PUs when the PUs' services are not on; SUs must disconnect and move out of PU's spectrum bands immediately when PUs become active. For example, as shown in Fig.1(a), the leftmost SU can opportunistically access to the spectrum bands owned by $\{PU_1, PU_2, \cdots, PU_i\}$ since he is in the coverage of these PUs (The grey part of PU's spectrum represents the currently vacant spectrum bands which the SUs can access to. A PU may own more than one available spectrum band at a given time point.). Similarly, the rightmost SU has to opportunistically

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[1] The process of selecting a mix of bands may be divided into two stages. The first stage starts with sampling and observation, and ends with statistical values about the future performances of available spectrum bands [8]; the second stage starts with the relevant values about future performances, and ends with the selection of the band mix. In this paper, we are only concerned with the second stage.

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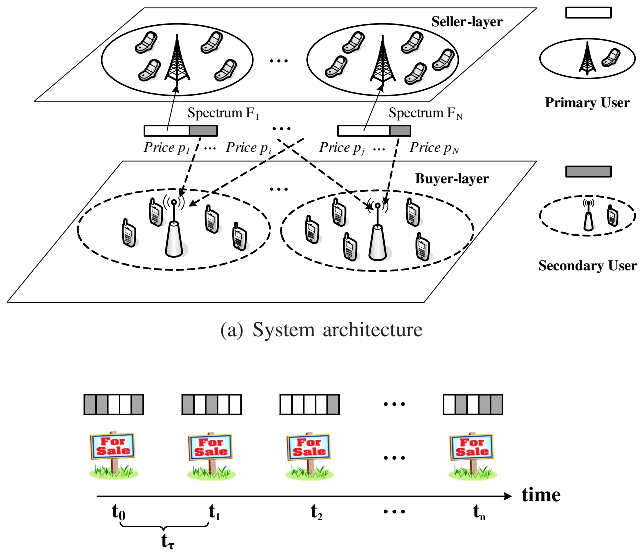


(a) System architecture

(b) Periodical spectrum selling

Fig. 1. System model for spectrum sharing

use the spectrum bands of $\{PU_j, PU_{j+1}, \cdots, PU_n\}$ because of his location.

From the view of PUs, they allow unlicensed SUs to opportunistically use their bands for monetary purposes. In this case, a PU will set reasonable prices for his unoccupied bands considering the band condition as well as competition among the PUs in the spectrum market [5], [6], and sell those bands periodically to enlarge his revenue as shown in Fig.1(b).

On the other hand, the SUs need to decide how to access to, or say how to buy, those vacant spectrum bands with different characteristics. Intuitively, the decision-making process of the SU must take the contention from the other SUs (i.e., the competition for the transmission opportunities among the SUs, just the same as in the pure Ad-hoc networks) and price of the available spectrum bands into consideration. Besides, the SUs are also concerned with quality of the bands, and worried about the unpredictable coming back of PUs' traffic.

B. Quality of the Spectrum Bands

As for a SU, the characteristics of the available bands shared by the PUs are not identical. The quality of those bands may be evaluated quite differently by different SUs regarding the location of the SUs, the frequency of the unoccupied bands, the contiguity of bands provided, etc.; Table.I quoted from [5] roughly shows the change in terms of received power when the transmission is switched from one frequency to another. When doing the computation, we suppose the antenna gain (both G_t and G_r) to be 1, and the distance d to be the same. As we can see, for instance, when the transmission frequency is switched from 3000 MHz to 300 MHz, for the same transmission power, the received power will increase by 20 dB[2].

To model these phenomena, spectral efficiency q of transmission of the SUs can be summarized as [5], [6],

$$q(W_i, d_i) = W_i \log_2 \Big(1 + \frac{y_b}{N_0}(d_i)^{-2}\Big). \tag{1}$$

[2]Here, we ignore the effect of interference and multi-path effects.

TABLE I
GAIN WITH DIFFERENT FREQUENCIES

Initial Frequency (MHz)	Target Frequency (MHz)	Ratio	Gain (dB)
3000	3000	1.00	0
3000	2700	1.11	0.9152
3000	2400	1.25	1.9382
3000	2100	1.43	3.0980
3000	1800	1.67	4.4370
3000	1500	2.00	6.0206
3000	1200	2.50	7.9588
3000	900	3.33	10.4576
3000	600	5.00	13.9794
3000	300	10.00	20.0000

where y_b is signal power, W_i is the bandwidth of the spectrum band i, and d_i is the distance between the PU and the SU. With the help of spectrum efficiency q, different spectrum bands can be ranked according to the SU's preferences.

III. Return and Risk

But, in cognitive radio networks, quality is not the only criterion for the SUs to evaluate the condition of spectrum bands, especially the ones bought from the PUs. Since the PUs can become active at any time, the SU must carry on the risk of sharing spectrum bands with PUs if we assume the payment for the spectrum is non-refundable. It is necessary for a SU to take the PUs' activities into account when he makes choice of the spectrum bands.

To formulate all SU's concerns, we let a random variable r_i $(0 \leq r_i \leq 1)$ represent the contention situation for a SU in band i, i.e., r_i denotes the time that the SU can use for his transmission during a unit selling/buying time period considering the competition from the peer SUs. Thus, when the SU is using band i bought from PUs, his monetary revenue for one period is

$$\mathscr{R}_i = r_i \cdot q_i \cdot v - p_i, \tag{2}$$

where q_i is the spectral efficiency of band i as described in Section II-B, v is the revenue the SU attains for satisfying the traffic demands per Mbps, p_i is the unit price for sharing band i with PUs, and $\mathscr{R}_i$ is the monetary gains the SU obtains by using band i. Given a specific band i, $\mathscr{R}_i$ is a random variable depending on r_i since v, q_i and p_i can be regarded as constants. Without loss of generality, we use the expected value[3], $E(\mathscr{R}_i)$, to represent the expected return of the SU, and the variance, $\sigma_{\mathscr{R}_i}$ $(\sigma_{\mathscr{R}_i} > 0)$, to represent the risk of return, i.e., the SU's potential worry about PUs' traffic coming back on band i.

Similarly, if the SU is using his own bands, his return can be expressed as,

$$\mathscr{R}_s = r_s \cdot q_s \cdot v - c_s, \tag{3}$$

where $\mathscr{R}_s$ is the risk-free return $(\sigma_{\mathscr{R}_s} = 0)$ per Mbps, r_s is a random variable reflecting the contention status, q_s is the

[3]These values can be abstracted from the history record of the previous m or directly from database of "white space" [8].

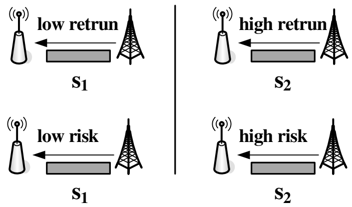


Fig. 2. Return vs Risk.

spectrum efficiency coefficient, and c_s is the inherent cost of the SU's own band.

We assume there are n different bands (denoted by $N = \{1, 2, \cdots, n\}$) owned by PUs, and can be opportunistically accessed by the SUs. Then, there will be an interesting question for the SU, i.e., how does a SU choose his way to share the bands with PUs, or say, how to distribute his traffic over all the bands including both risk-free band of his own and risky bands from PUs. The problem can be mathematically expressed as the follows,

$$E\big(\mathscr{R}\big) = (1-\eta)E\big(\mathscr{R}_s\big) + \eta E\big(\mathscr{R}_p\big), \tag{4}$$

and

$$E\big(\mathscr{R}_p\big) = \sum_{i=1}^{n} \omega_i E\big(\mathscr{R}_i\big), \tag{5}$$

where $\mathscr{R}$ is the overall monetary return per Mbps, $\mathscr{R}_s$ is the risk-free return per Mbps from distributing traffic on the SU's own band, and $\mathscr{R}_p$ is the risky return per Mbps from distributing traffic on the PUs' bands. The value of $0 \leq \eta < 1$ indicates that the SU takes opportunistic use of PUs' vacant bands, while $(1-\eta)\mathscr{R}_s$ is the return from the SU's own band. $\mathscr{R}_p$ is a weighted average of $\mathscr{R}_i$ with ω_i as non-negative weights, where $\mathscr{R}_i$ is independent of ω_i.

Intuitively, we find that the SU is able to maximize his expected return by pouring all his traffic over the band k with the maximal expected return, i.e., $k = \underset{i \in N}{\operatorname{argmax}}\, E\big(\mathscr{R}_i\big)$. However, the risk of using the chosen band k may be quite high, which means the maximum return is not guaranteed with regard to the PU's activities, e.g., for the SU, the return $E(\mathscr{R}_2)$ is larger than $E(\mathscr{R}_1)$, but the risk $\sigma_{\mathscr{R}_2}$ is also greater than $\sigma_{\mathscr{R}_1}$, as shown in Fig.2. Consequently, a rational or risk-averse SU is not likely to gamble all his traffic on this band since the return may be extraordinarily low.

On the other hand, if a SU diversifies his traffic over a mix of different bands, he may obtain given return with the corresponding minimal risk or maximum return with the given risk. So, the spectrum sharing is considered to be *efficient* if the SU chooses a mix of PUs' spectrum bands with maximum $E(\mathscr{R})$ for given $\sigma_{\mathscr{R}}$ or less and minimum $\sigma_{\mathscr{R}}$ for given $E(\mathscr{R})$ or more, which can be abbreviated as $E-\sigma$ criterion.

As for the SU, the efficient spectrum sharing involves two issues: (1). How to split the traffic over the risk-free band belonging to the SU and the risky bands belonging to PUs; (2). How to choose the mix of the PUs' bands.

IV. The Efficient Traffic Distribution over Risky and Risk-free Bands

A. Traffic Splitting over the Mix of Bands

In last section, we establish the relation between the SU's traffic distribution over arbitrary mix of spectrum bands and his total expected return $E(\mathscr{R})$ as illustrated in equation 4. With simple derivation, we rewrite $E\big(\mathscr{R}\big)$ and $\sigma_{\mathscr{R}}$ of the return of the band mix as,

$$\begin{cases} E\big(\mathscr{R}\big) = E\big(\mathscr{R}_s\big) + \eta\Big(E\big(\mathscr{R}_p\big) - E\big(\mathscr{R}_s\big)\Big) \\ \sigma_{\mathscr{R}}^2 = \eta^2\sigma_p^2. \end{cases} \tag{6}$$

After eliminating η between these two equations above, we find that the direct relation between the expected value of the SU's return of the traffic per Mbps and the risk parameters of his traffic splitting is:

$$\begin{cases} E\big(\mathscr{R}\big) = E\big(\mathscr{R}_s\big) + \theta\sigma_{\mathscr{R}} \\ \theta = \Big(E\big(\mathscr{R}_p\big) - E\big(\mathscr{R}_s\big)\Big)/\sigma_p. \end{cases} \tag{7}$$

In terms of any arbitrarily selected mix of spectrum bands, therefore, the SU's expected return on his traffic distribution over these bands is linearly related to the risk of return, as measured by the standard deviation of his return. Given any selected mix, the intercept of this linear function is the risk-free return $\mathscr{R}_s$ and its slope is given by θ, which is determined by the parameters $\mathscr{R}_p$ and σ_p of the particular band mix of PUs being considered.

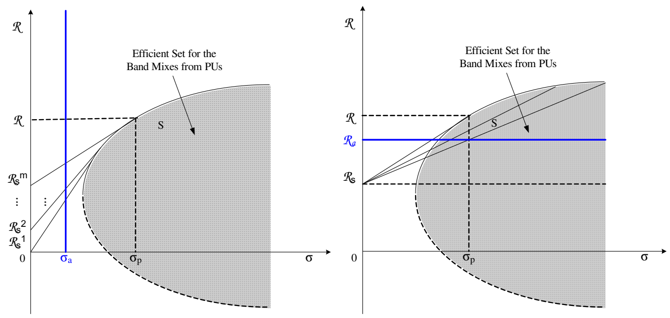
Moreover, let us consider all possible mixes of spectrum bands. Those mixes with the same θ value lie on the same "Opportunistic Spectrum Utilization Line" (OSUL), but those have different θ values will produce different OSULs between return and risk for the SUs as shown in Fig.3. The SU's problem is to decide which OSUL to choose (the proper value of η), and how intensively to use it (the proper value of ω_i).

A rational SU adhering to the proposed efficient sharing criterion will minimize the variance of his overall return $\sigma_{\mathscr{R}}^2$ associated with any expected return $E(\mathscr{R})$ he may choose by confining all his selection in bands to the mix with the largest θ value. After choosing the band mix by maximizing θ, the SU can complete his efficient sharing choice and determine a unique best value of η, i.e., the ratio of the SU's traffic distribution on the risky bands and the amount of traffic on his own risk-free band.

B. The Geometry of the Efficient Traffic Splitting

The algebraic analysis given above can be represented graphically as in Fig.3. Any available mix of spectrum bands from PUs is characterized by a pair of values $(\sigma_p, \mathscr{R}_p)$ which can be represented as a point in a plane with σ as the horizonal axis and $\mathscr{R}$ as the vertical axis. Our assumptions insure that the points representing all available band mixes lie in a finite region, all parts of which lie to the right of the line $\sigma = 0$, and that this region is bounded by a closed curve.

All the $(\sigma, \mathscr{R})$ pairs with any particular mix lie on a series of line segments from the point $(0, \mathscr{R}_f)$, where $\eta = 0$, to the points on the boundary of attainable set, where $\eta = 1$. Each possible band mix thus determines a unique OSUL. Thus,



(a) Given σ_a (b) Given $\mathscr{R}_a$

Fig. 3. Geometry of traffic splitting between risk free and risky bands

the optimal band mix is determined by two factors: the SU's selection of $\mathscr{R}_s$ and his selection of θ.

According to the $\mathscr{R}$-$\sigma_{\mathscr{R}}$ rule, the optimal $\mathscr{R}_s$ selection strategy for the SU is to make full use of his own band. As shown in Fig.3(a), for given risk σ_a, the total return of the SU with the maximum $\mathscr{R}_s$ (i.e., $\mathscr{R}_s^m$ in Fig.3(a)) is greater than that with less risk-free return (i.e., $\mathscr{R}_s^1$, $\mathscr{R}_s^2$, ..., $\mathscr{R}_s^{m-1}$).

Then, the second factor is θ, the slope of OSUL. Note that shifts from one possible mix to another, rotating the associated OSUL counter clockwise, move the SU to preferred positions. The limit of the favorable rotation is OSUL with the maximum attainable θ, which identifies the efficient mix of spectrum bands.

V. The Efficient Mix of PUs' Spectrum Bands

Now, the problem left is how to select the efficient mix of PUs' spectrum bands, i.e., how to determine the proper value of ω_i. We see that if ω_i is assigned to the i^{th} band for service providing, the return of the i^{th} band will be $\omega_i\mathscr{R}_i$, which can also be written in the form

$$\omega_i\mathscr{R}_i = \omega_i(\mathscr{R}_i - \mathscr{R}_s) + \omega_i\mathscr{R}_s. \tag{8}$$

By using the constraint $\sum_i \omega_i = 1$, the expected return per Mbps for any mix of PUs' bands can be expressed as

$$\begin{aligned} E(\mathscr{R}_p) &= \sum_i \left[\omega_i\big(E(\mathscr{R}_i) - E(\mathscr{R}_s)\big) + \omega_i E(\mathscr{R}_s)\right] \\ &= E(\mathscr{R}_s) + \sum_i \omega_i\big(E(\mathscr{R}_i) - E(\mathscr{R}_s)\big). \end{aligned} \tag{9}$$

Thus, the expectation and variance of the return on any mix of PUs' bands is

$$\begin{cases} E(\mathscr{R}_p) = E(\mathscr{R}_s) + \sum_i \omega_i\big(E(\mathscr{R}_i) - E(\mathscr{R}_s)\big) \\ \sigma_p = \sum_j \sum_i \omega_i\omega_j\sigma_{\mathscr{R}_{ij}}. \end{cases} \tag{10}$$

where $\sigma_{\mathscr{R}_{ij}}$ represents the variance $\sigma_{\mathscr{R}_i}$ when $i = j$, and covariances when $i \neq j$. By defining $\tilde{x}_i = \mathscr{R}_i - \mathscr{R}_s$, the notations in the right-hand expressions are able to be further simplified :

$$\begin{cases} E(\mathscr{R}_p) = E(\mathscr{R}_s) + \sum_i \omega_i\tilde{x}_i \\ \sigma_p = \sum_i \sum_j \omega_i\omega_j\sigma_{\tilde{x}_{ij}}. \end{cases} \tag{11}$$

By substituting equation 11 into equation 7, θ defined in Section IV-A can be written as

$$\theta = \frac{E(\mathscr{R}_p) - E(\mathscr{R}_s)}{(\sigma_p)^{1/2}} = \frac{\sum_i \omega_i\tilde{x}_i}{(\sum_i\sum_j \omega_i\omega_j\sigma_{\tilde{x}_{ij}})^{1/2}}. \tag{12}$$

As for the SU, the efficient mix is the one which maximizes θ as we discussed in the Section IV. Therefore, we examine the partial derivatives of θ with respect to the ω_i and find,

$$\frac{\partial\theta}{\partial\omega_i} = (\sigma_p)^{-1}[\tilde{x}_i - \lambda(\omega_i\sigma_{\tilde{x}_{ii}} + \sum_j \omega_j\sigma_{\tilde{x}_{ij}})], \tag{13}$$

where

$$\lambda = \frac{\tilde{x}}{\sigma_p^2} = \frac{\sum_i \omega_i\tilde{x}}{\sum_i\sum_j \omega_i\omega_j\sigma_{\tilde{x}_{ij}}}. \tag{14}$$

The necessary and sufficient conditions on the relative values of the ω_i for a stationary and the unique maximum are obtained by setting the derivatives above equal to zero, which give the set of equations

$$\zeta_i\sigma_{\tilde{x}_{ii}} + \sum_j \zeta_j\sigma_{\tilde{x}_{ij}} = \tilde{x}, \qquad \forall i \in N \tag{15}$$

where $\zeta_i = \lambda\omega_i$. Note that the set of equations are linear in terms of the own-variances, $\sigma_{\tilde{x}_{ii}}$, pooled covariances, $\sigma_{\tilde{x}_{ij}}$, and excess returns, $\tilde{x}$, of the respective spectrum bands from PUs. Since the covariance matrix is positive definite and hence non-singular, there is a unique solution to this system of equations,

$$\zeta_i^* = \sum_j (\sigma_{\tilde{x}^{ij}})^{-1}\tilde{x}_j, \tag{16}$$

where $(\sigma_{\tilde{x}^{ij}})^{-1}$ represents the ij^{th} element of the inverse of the covariance matrix. This solution may also be expressed in terms of the primary variables as

$$\omega_i^* = (\lambda^*)^{-1}\sum_j (\sigma_{\tilde{x}_{ij}})^{-1}(\mathscr{R}_j - \mathscr{R}_s), \quad \forall i \in N. \tag{17}$$

Moreover, $\zeta_i = \lambda\omega_i$ implies

$$\sum_i \zeta_i = \lambda\sum_i \omega_i. \tag{18}$$

When we leverage the constraint $\sum_i \omega_i = 1$, λ^* is readily evaluated as

$$\sum_i \zeta_i^* = \lambda^*\sum_i \omega_i^* = \lambda^*. \tag{19}$$

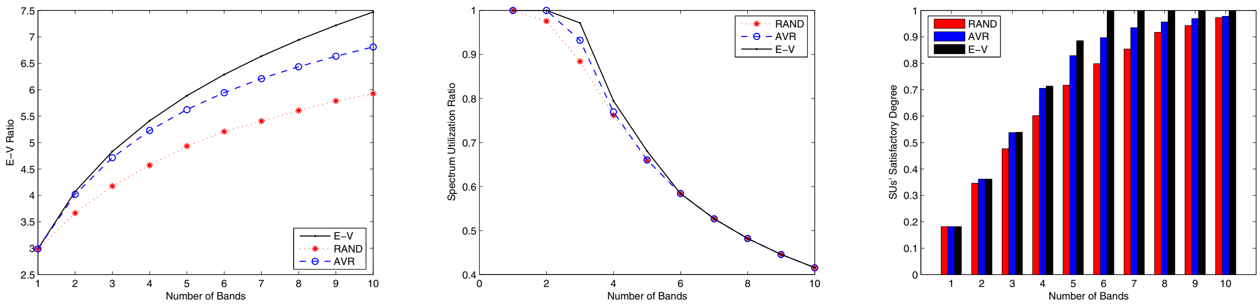
The optimal ζ_i^* can consequently be scaled to the efficient traffic splitting over the band mix ω_i^*, which is

$$\sum_i \zeta_i^* = \lambda^* = \frac{\tilde{x}^*}{\sigma_{\tilde{x}^*}^2}. \tag{20}$$

It is also interesting to note that if we represent the ratio of the expected excess return to its variance for the i^{th} band at the efficient sharing, we have

$$\omega_i^* = \frac{\lambda_i}{\lambda^*} - \sum_{i\neq j}\omega_j^*\frac{\bar{x}_{ij}}{\sigma_{\tilde{x}_{ii}}}, \quad \text{where } \lambda_i = \frac{\bar{x}_i}{\sigma_{\tilde{x}_{ii}}}. \tag{21}$$

The fraction of traffic distributed in each band in the efficient band mix of PUs is equal to the ratio of its λ_i to λ^* of the entire mix minus the ratio of its pooled covariance with other bands to its own variance. As a result, if the SU acts on the assumption that all the covariances were zero, he is able to pick his optimal band mix of PUs simply by determining the λ_i ratio of the expected excess return $\tilde{x}_i = \mathscr{R}_i - \mathscr{R}_s$ of each



(a) The ratio of return and risk

(b) Spectrum utilization

(c) SUs' satisfactory degrees

Fig. 4. Performance comparison of different traffic splitting schemes over spectrum bands

spectrum band to its variance $\sigma_{\tilde{x}_{ii}} = \sigma_{\tilde{\mathscr{R}}_i}$, and setting each $\omega_i = \lambda_i / \sum_i \lambda_i$. Consider the case of no covariances, we find $\sum_i \lambda_i = \lambda^* = \tilde{x}^* / \sigma^2_{x^*}$. With this simplified assumption, the λ_i ratios of each band is enough to determine the efficient mix[4].

VI. Simulation and Analysis

In this section, we compare the proposed $E-\sigma$ based traffic splitting with random splitting (RAND) and averaged splitting (AVR) over the bands to demonstrate its efficiency in spectrum sharing in terms of $E-\sigma$ ratio, spectrum utilization and SUs' satisfactory degrees.

We set up the simulation with a similar setting to that shown in Fig.1, modeling a SU to choose his traffic distribution over a mix of his own band and bands of PUs. For illustration purpose, we suppose that the risk free return $\mathscr{R}_f$ is equal to 5 and the number of PUs' bands n is 10. Without loss of generality, we assume the return of the band belonging to PUs $\mathscr{R}_i$ ($\mathscr{R}_i > 5$) monotonically increases with the risk of that band $\sigma_{\mathscr{R}_i}$ ($\sigma_{\mathscr{R}_i} > 0$).

As depicted in Fig.4(a), when the number of spectrum bands increases, the $E-\sigma$ ratio increases in all these three schemes. But the proposed spectrum sharing is more efficient than RAND and AVR because it distributes traffic to the i^{th} band with $\omega_i^* = \frac{\lambda_i}{\lambda^*}$, which makes it achieve the maximal θ and guarantee the maximum $\mathscr{R}$ with given $\sigma_{\mathscr{R}}$ or minimum $\sigma_{\mathscr{R}}$ with given $\mathscr{R}$.

In Fig.4(b), we evaluate the performance of spectrum utilization. Note that no matter the band is used by the PU or used by the SU, the spectrum band itself should be considered as utilized one. The spectrum utilization should be calculated as the ratio of the number of utilized bands to the number of total bands. According to this definition, we find that the $E-\sigma$ based traffic splitting is still slightly better than the other two schemes. The reason is that the SU will distribute less traffic on the more risky band if he shares the spectrum according to $E-\sigma$ rule. In this way, although the SU cannot relieve his services from impact of the PUs' coming back to 100%, he is able to take more opportunistic use of the licensed bands of PUs. By contrast, the spectrum utilization in both RAND and AVR are more likely affected by the PUs' activities.

Similar analysis also applies to the SUs' satisfactory degrees, which can be defined as the ratio of satisfied demands to overall demands of the SU. Compared with RAND and AVR, the satisfactory degrees of $E-\sigma$ based spectrum sharing is higher when the vacant spectrum of PUs is not abundant.

[4]In the more general case with non-zero covariances, a single set of linear equations must be solved in the usual way, but no (linear or non-linear) programming is required.

VII. Conclusion

In this paper, we propose a return and risk model for spectrum sharing in cognitive radio networks. Considering the risk of gathering all the traffic on one band, we elaborate on how to choose a efficient band mix consisting of risky and risk free bands, and guide the SUs to distribute his traffic over them. By numerical simulations, we show that our spectrum sharing scheme is efficient in terms of $E-\sigma$ criterion, and it improves the spectrum utilization and SUs' satisfactory degrees as well.

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