Performance Evaluation of Wireless Cellular Networks with Mixed Channel Holding Times

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***Abstract*—In most analytical models for wireless cellular networks, the channel holding times for both new and handoff calls are usually assumed to be independent and identically distributed. However, simulation study and field data show that this assumption is invalid. In this paper, we present a new general analytical model in wireless cellular networks where channel holding times for new and handoff calls are distinctly distributed with different average values. For our proposed model, we first derive the explicit matrix product-form solution of the stationary probability for number of new and handoff calls in the system. We then show that the expression of the stationary probability for total number of calls in the system possesses a scalar product-form solution if and only if the expected channel holding times for both new and handoff calls are the same. Moreover, we derive analytical results for the blocking probabilities of new and handoff calls. Finally, we compare our new theoretical results with the corresponding simulation results and two already existing approximations. Through this comparison study, we show that our analytical results are indeed the same as the simulation results and that there are certainly significant estimation errors for the existing approximations.**

***Index Terms*—Teletraffic, product-form solution, blocking probability, cellular networks.**

I. Introduction

IN wireless cellular networks, there are usually two classes of calls, viz., new calls and handoff calls. The new calls in a cell are the ones which are initiated in this cell and the handoff calls are the ones which are the ongoing calls handed off to this cell (see [2], [3], [10], [11], [12] and the references therein). In the existing literature [8] [10], to simplify the analysis, the handoff call holding time is always assumed to be the same as that of the new call holding time, and the handoff call cell residence time is always assumed to be the same as that of the new call cell residence time. However, recent studies in [2], [3] [4] and the references therein showed that the new call channel holding time and the handoff call channel holding time may have different distributions. Worse yet, they may have different average values. Simulation study and field data also confirmed that they are different random variables [4]. The four diagrams of Figure 1 in paper [5] showed the difference of the average channel holding times for new calls and handoff calls. In this paper, we present an analytical model to show how difficult it could be if the handoff call holding time is different from the new call holding time and the handoff call cell residence time is different from the new call cell residence time. We want to point out that although our model targets at the second generation wireless cellular systems, it can easily modified for future generation wireless cellular systems. Surprisingly, the problem itself is not addressed well for even the early wireless cellular systems.

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To simplify the analysis, we assume that the cellular system is homogeneous [16], i.e., the underlying traffic processes and the parameters for all cells within the cellular networks are statistically identical. The detailed assumptions and notation for this wireless mobile network are as follows:

1) Each cell consists of C channels. In addition, there is a buffer of size $K \geq 0$ for holding the waiting type-two calls (e.g., handoff calls) in each cell (base station site).
2) Type-*i* calls in the cell are generated as a Poisson process with rate λ_i (i=1,2). Here, type-1 calls are for new calls while type-2 calls are for handoff calls.
3) The requested call connection time of type-*i* calls at each cell, say, $\mathbf{H}_i$, is assumed to be exponentially distributed with mean $1/h_i$ (i=1,2).
4) The cell residence time of type-*i* calls in each cell , say, $\mathbf{R}_i$, the interval that a type-*i* call stays in this cell, is exponentially distributed with mean $1/r_i$.
5) When the number of the busy channels at base station (BS) is i ($i = 0, 1, ..., C-1$), a type-1 call is admitted with probability α_i ($0 \leq \alpha_i \leq 1$). Otherwise, a type-1 call will be blocked and then cleared from the system.
6) When the number of the busy channels at BS is i ($i = 0, 1, ..., C-1$), a type-2 call is *admitted* with probability β_i ($0 \leq \beta_i \leq 1$) and is blocked and cleared from the system with probability $1-\beta_i$; When the number of the type-2 calls at BS (including the calls in the buffer and the calls in communication) is i where $i = C, C+1, ..., C+K-1$, a type-2 call is *buffered* with probability β_i and is blocked and cleared from the system with probability $1-\beta_i$. If a call is put on hold in the buffer, it will stay in the buffer until either it gains a channel for service or until it departs from the current cell due to mobility; When the number of the type-2 calls at BS is $C+K$, a type-2 call will be blocked and cleared from the system. If the type-2 call moves out of the cell before it gains a channel, it will terminate at the current BS and cleared from the system. In all other cases, a

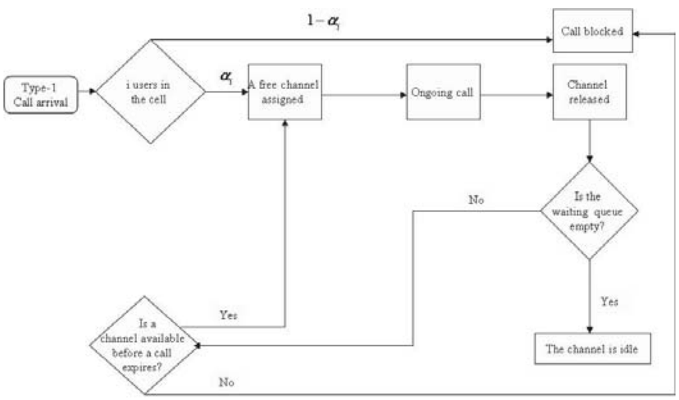

Fig. 1. Flowchart of type 1 call for the proposed thinning scheme.

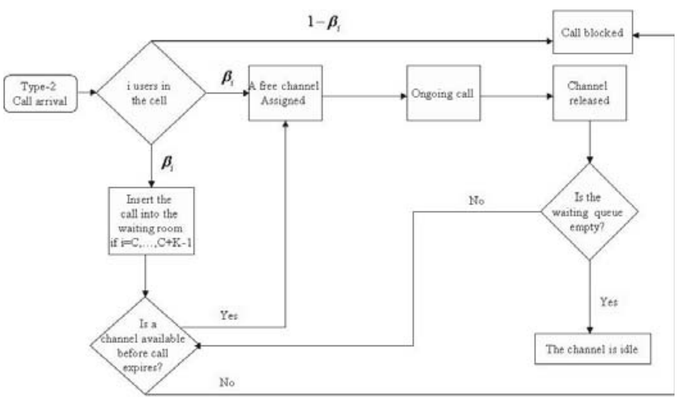

Fig. 2. Flowchart of type 2 call for the proposed thinning scheme.

type-2 call is blocked and cleared from the system.

The diagrams for the type-1 call and type-2 call control process in our proposed scheme are given in Figure 1 and Figure 2, respectively.

From the above description, we observe the following:

- The major differences in our model from the existing wireless cellular models are that (1) different types of calls possess different call holding time and different cell residence time distributions and (2) type-2 calls have higher priority in terms of buffer usage.
- If there are no priority on either reservation or buffer for the type-2 calls, the model can be reduced to the standard two dimensional traffic model by specifying the admission probabilities (α_i and β_j). In this case, product form results can be derived in terms of reversible Markov process, from which a generalized two-dimensional Erlang's loss formula can be then obtained [17]. If the admitted rates are all the same for new calls and handoff calls, and the call holding time and cell residence time for two types of calls have the same rates, our model is the same as the thinning scheme I in [1] and [5]. If two types of calls have the same call holding time and the same cell residence time, the model is reduced to the well known cutoff priority model ([9], [11]).
- We also remark that the proposed schemes can be generalized to handle the call admission control problem in wireless multimedia networks with different prioritized services. For example, we can classify multimedia services into different priority levels according to the QoS requirements, then apply multiple thresholds for call admission control or choose different admission probabilities for different priority levels to reflect the QoS. We will present such studies in a separate paper.
- The proposed call admission schemes are ones in which an arrived call is admitted with certain probability. The idea behind these schemes is to smoothly throttle the both new call and handoff call stream as the network traffic is building up. Thus, when the network is approaching the congestion, the admitted call stream becomes thinner. Due to the flexible choice of the call admission probabilities, these schemes can be made very general. For example, in paper [5], the authors studied several specific thinning schemes, called "new call thinning scheme" and "fractional guard channel scheme" etc.

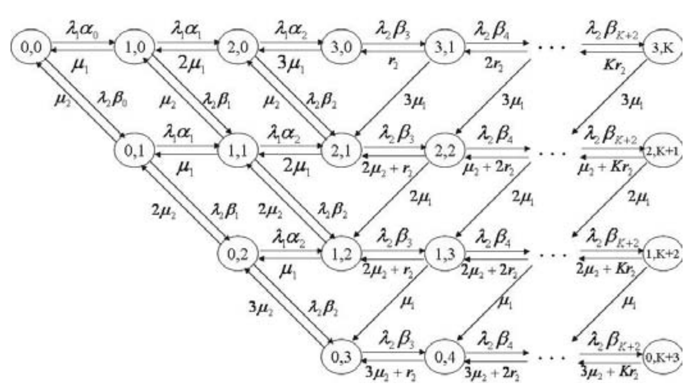

Fig. 3. Transition rate diagram when $C = 3$.

II. Analytical Results

The purpose of this section is to find the stationary probability of the system when there are i type-1 calls and j type-2 calls in the cell, where $i = 0, 1, ..., C; j = 0, 1, ..., C + K$ and $i + j \leq C + K$. Let $X(t)$ be the number of type-1 calls in the cell at time t, and $Y(t)$ be the number of the type-2 calls (including those in the buffer) in a cell at time t. It is easy to show that $\{(X(t), Y(t))\}$ forms a two-dimensional Markov process with the state space

$$\Omega = \bigcup_{i=0}^{C-1} \{(i,0),(i-1,1),(i-2,2),...,(1,i-1),(0,i)\} \bigcup_{j=0}^{K} \{(C,j),(C-1,j+1),...,(1,j+C-1),(0,j+C)\},$$

where, without loss of generality, we order the states lexicographically.

The transition rate diagram of this Markov process, for the special case when $C = 3$, is given by Figure 3. The transition rate diagram for general C can be similarly obtained. Consequently, the infinitesimal generator of above constructed general two-dimensional Markov Process $\{(X(t), Y(t))\}$ can be derived as follows:

$$Q=\begin{bmatrix} E_0 & A_0 & 0 & \cdots & 0 & 0 & 0 \\ B_1 & E_1 & A_1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & B_{N-1} & E_{N-1} & A_{N-1} \\ 0 & 0 & 0 & \cdots & 0 & B_N & E_N \end{bmatrix},$$

where $N = C + K$ and the matrices A_i, B_i and E_i are given as follows.

- Matrix A_i $(i = 1, 2, ..., C + K - 1)$ refers to an arrival of a call to the cell in which there are currently i calls, and
  - If $i = 0, 1, ..., C-1$, A_i is an $(i+1)\times(i+2)$ matrix given by

$$A_i=\begin{bmatrix} \lambda_1\alpha_i & \lambda_2\beta_i & \cdots & 0 & 0 \\ 0 & \lambda_1\alpha_i & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_2\beta_i & 0 \\ 0 & 0 & \cdots & \lambda_1\alpha_i & \lambda_2\beta_i \end{bmatrix}. \quad (1)$$

  - If $i = C, C+1, ..., C+K-1$, A_i is a $(C+1)\times(C+1)$ square matrix given by

$$A_i = \lambda_2\beta_i I_{(C+1)\times(C+1)}. \quad (2)$$

- Matrix B_i $(i = 1, 2, ..., C + K)$ refers to a departure of a call from the cell in which there are currently i calls, and
  - If $i = 1, 2, ..., C$, B_i is an $(i + 1) \times i$ matrix given by

$$B_i=\begin{bmatrix} i\mu_1 & 0 & \cdots & 0 & 0 \\ \mu_2 & (i-1)\mu_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 2\mu_1 & 0 \\ 0 & 0 & \cdots & (i-1)\mu_2 & \mu_1 \\ 0 & 0 & \cdots & 0 & i\mu_2 \end{bmatrix}, \quad (3)$$

  where $\mu_2 = h_2 + r_2$ *and* $\mu_1 = h_1 + r_1$.
  - If $i = C + 1, ..., , N$, B_i is a $(C + 1) \times (C + 1)$ matrix given by

$$B_i=\begin{bmatrix} d_{i,0} & C\mu_1 & \cdots & 0 & 0 \\ 0 & d_{i,1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & d_{i,C-1} & \mu_1 \\ 0 & 0 & \cdots & 0 & d_{i,C} \end{bmatrix}, \quad (4)$$

  where $d_{i,j} = j\mu_2 + (i - C)r_2$, for $j = 0, 1, ..., C$.
- Matrix E_i $(i = 0, 1, 2, ..., C + K)$ refers to no change in the total number of calls in the cell in which there are currently i calls, and
  - $E_0 = -(\lambda_1\alpha_0 + \lambda_2\beta_0)$;
  - If $i = 1, 2, ..., C - 1$, E_i is an $(i + 1) \times (i + 1)$ diagonal matrix given by

$$E_i = diag\left\{a_{(i,j)}\right\},$$

  where $a_{(i,j)} = -[\lambda_1\alpha_i + \lambda_2\beta_i + (i-j)\mu_1 + j\mu_2]$ for $j = 0, 1, ..., i$, $\mu_1 = h_1 + r_1$ and $\mu_2 = h_2 + r_2$.
  - If $i = C, C+1, ..., N-1$, E_i is a $(C+1)\times(C+1)$ diagonal matrix given by

$$E_i = diag\left\{a_{(i,j)}\right\},$$

  where $a_{(i,j)} = -[\lambda_2\beta_i + (C-j)\mu_1 + j\mu_2 + (i-C)r_2]$ for $j = 0, 1, ..., C$.
  - If $i = N$, E_i is a $(C+1)\times(C+1)$ diagonal matrix given by

$$E_i = diag\left\{a_{(i,j)}\right\},$$

  where $a_{(i,j)} = -[(C - j)\mu_1 + j\mu_2 + (i - C)r_2]$ for $j = 0, 1, ..., C$.

Let $\pi_{i,j}$ denote the steady-state probability when there are i type-1 calls and j type-2 calls (including those in the buffer) in the cell,

$$\boldsymbol{\pi}_n = (\pi_{n,0}, \pi_{n-1,1}, \cdots, \pi_{1,n-1}, \pi_{0,n}),$$

for $0 \le n \le C - 1$, and

$$\boldsymbol{\pi}_n = (\pi_{C,n-C}, \pi_{C-1,n-C+1}, \cdots, \pi_{1,n-1}, \pi_{0,n}),$$

for $C \le n \le N$.

By using Lemma 3 in [6], if we denote $\prod_{i=1}^{n} b_i = b_1 b_2 \cdots b_n$ for any matrices $b_1, \cdots, b_n$, then the steady-state probability can be determined by

$$\boldsymbol{\pi}_n = \boldsymbol{\pi}_0 \prod_{i=1}^{n}\left[A_{i-1}(-D_i)^{-1}\right], \quad (5)$$

where D_i $(i = 0, 1, \cdots, N)$ are recursively calculated by $D_N = E_N$ and

$$D_n = E_n + A_n(-D_{n+1}^{-1})B_{n+1}, \quad (6)$$

for $n = 0, 1, \cdots, N - 1$, and

$$\boldsymbol{\pi}_0 = \left[1 + \sum_{n=1}^{N}\prod_{i=1}^{n}\left[A_{i-1}(-D_i)^{-1}\right]\mathbf{e}\right]^{-1}, \quad (7)$$

where, and also in the rest of this paper, **e** is defined as a column vector with all its components equal to one.

Remarks 1: In the analytical models of the existing literature such as [5][8][10], the handoff call holding time is always assumed to be the same as that of the new call holding time and that the handoff call cell residence time is assumed to be the same as that of the new call cell residence time. Under those assumptions, it is easily to have $h_1 = h_2 = h$ and $r_1 = r_2 = r$, and therefore $h_1 + r_1 = h_2 + r_2$. In the following analysis, however, we do not need to have the conditions: $h_1 = h_2$ and $r_1 = r_2$ but only need to assume $\mu = \mu_1 = \mu_2$. The information behind this condition is that the expected call holding time for new call and handoff call may not be the same, and the expected cell residence time for new call and handoff call may not be the same either.

When the expected channel holding time is the same for new calls and handoff calls, by noting $D_N = E_N$, we can easily get $D_N^{-1}\mathbf{e} = -\frac{1}{C\mu+(N-C)r_2}\mathbf{e}$. Recursively from equation (6), we have that

$$D_n^{-1}\mathbf{e} = -\frac{1}{n\mu}\mathbf{e}, \quad for\ \ n = 1, 2, ..., C - 1,$$

and

$$D_n^{-1}\mathbf{e} = -\frac{1}{C\mu + (n - C)r_2}\mathbf{e}, \quad for\ \ n = C, C+1, ..., C+K.$$

Therefore, from equation (5) and the expression of A matrix, we finally have

- for $n = 0, 1, ..., C$,

$$\begin{aligned}\boldsymbol{\pi}_n \mathbf{e} &= \boldsymbol{\pi}_0 \prod_{i=1}^{n} \left[A_{i-1}(-D_i)^{-1}\right] \mathbf{e} = ... \\ &= \boldsymbol{\pi}_0 \prod_{i=1}^{n} \left[\frac{\lambda_1 \alpha_{i-1} + \lambda_2 \beta_{i-1}}{i\mu}\right]; \end{aligned} \tag{8}$$

- for $n = C+1, ..., N$,

$$\begin{aligned}\boldsymbol{\pi}_n \mathbf{e} &= \boldsymbol{\pi}_0 \prod_{i=1}^{n} \left[A_{i-1}(-D_i)^{-1}\right] \mathbf{e} = ... \\ &= \boldsymbol{\pi}_0 \prod_{i=1}^{C} \left[\frac{\lambda_1 \alpha_{i-1} + \lambda_2 \beta_{i-1}}{i\mu}\right] \\ &\quad \prod_{j=C+1}^{n} \left[\frac{\lambda_2 \beta_{j-1}}{C\mu + (j-C) r_2}\right], \end{aligned} \tag{9}$$

where $\boldsymbol{\pi}_0$ is calculated by equation (7) as follows

$$\begin{aligned}\boldsymbol{\pi}_0^{-1} = 1 &+ \sum_{n=1}^{C} \prod_{i=1}^{n} \left[\frac{\lambda_1 \alpha_{i-1} + \lambda_2 \beta_{i-1}}{i\mu}\right] \\ &+ \sum_{n=C+1}^{C+K} \prod_{i=1}^{C} \left[\frac{\lambda_1 \alpha_{i-1} + \lambda_2 \beta_{i-1}}{i\mu}\right] \\ &\quad \prod_{j=C+1}^{n} \left[\frac{\lambda_2 \beta_{j-1}}{C\mu + (j-C) r_2}\right]. \end{aligned} \tag{10}$$

The stationary probability of this special case is consistent with the result in [5] without buffer when $K = 0$ and with a similar result in [10] when all α_i and β_j are all ones.

Remark 2: In equation (5), (6) and (7), we obtained the matrix product-form solution for the stationary probability of the system in general. As the problem is very interesting and fundamentally important from both the theory and application, many researchers attempt to find a simple product-form approximation even for some special cases of our model. For example, for a special case of our above model when the expected channel holding times for both type-1 calls and type-2 calls are different and there are no buffering mechanism, Fang and Zhang [5] proposed an approximate result

$$p_j^a = \frac{\prod_{i=0}^{j-1}(\beta_i \rho + \rho_h)}{j!} p_0.$$

Recently, the authors in paper [13] also proposed an approximation result for the stationary probability. Summarizing their results and denoting by p_j the stationary probability when j channels are being connected, we know that they in fact proposed a scalar product form formula $p_j = p_{j-1} K_j$ for all j in their model when the buffer size is zero ($K = 0$). Unfortunately, how good this approximation is to the real value when the holding times for new calls and handoff calls are different is not investigated. By using our matrix product solution for the stationary probability in equation (5), (6) and (7), we can obtain the following result.

Theorem 1: Denote by p_n ($n = 0, 1, 2, \cdots, C+K$) the stationary probability when there are totally n calls (including both new calls and handoff calls) in the system, the sufficient and necessary condition to have $p_n = p_{n-1} K_n$ for a constant K_n is

$$h_1 + r_1 = h_2 + r_2$$

and

$$K_n = \begin{cases} \frac{\lambda_1 \alpha_{n-1} + \lambda_2 \beta_{n-1}}{n\mu}, & \text{if } n = 1, 2, \cdots, C, \\ \frac{\lambda_2 \beta_n}{C\mu + (n-C) r_2}, & \text{if } n = C+1, \cdots, C+K. \end{cases}$$

In fact, sufficient condition is already obtained as in the Remark 1 (see Equations 8 and 9). We now only focus on the necessary condition. If equation $p_n = p_{n-1} K_n$ holds for all n, by noting equation (5), we have both $\pi_n \mathbf{e} = \pi_{n-1} \mathbf{e} K_n$ and

$$\pi_n \mathbf{e} = \boldsymbol{\pi}_0 \prod_{i=1}^{n-1} \left[A_{i-1}(-D_i)^{-1}\right] \left[A_{n-1}(-D_n)^{-1}\right] \mathbf{e}.$$

This means that we should have $\left[A_{n-1}(-D_n)^{-1}\right] \mathbf{e} = K_n e$. By noting the special structure of matrix A_{n-1}, we should then have

$$(-D_n)^{-1} \mathbf{e} = \begin{cases} \frac{K_n}{\lambda_1 \alpha_{n-1} + \lambda_2 \beta_{n-1}} \mathbf{e}, & \text{if } n = 1, 2, \cdots, C, \\ \frac{K_n}{\lambda_2 \beta_{n-1}} \mathbf{e}, & \text{if } n = C+1, \cdots, C+K. \end{cases}$$

That is equivalent to

$$\mathbf{e} = -\frac{K_n}{\lambda_1 \alpha_{n-1} + \lambda_2 \beta_{n-1}} D_n \mathbf{e} = \frac{K_n}{\lambda_1 \alpha_{n-1} + \lambda_2 \beta_{n-1}} B_n \mathbf{e}, \tag{11}$$

for $n = 1, 2, ..., C$, and

$$\mathbf{e} = -\frac{K_n}{\lambda_2 \beta_{n-1}} D_n \mathbf{e} = \frac{K_n}{\lambda_2 \beta_{n-1}} B_n \mathbf{e}, \tag{12}$$

for $n = C+1, ..., C+K$.

From the equations (3) and (4), we have

$$B_n e = \begin{bmatrix} n \\ n-1 \\ \vdots \\ 1 \\ 0 \end{bmatrix} \mu_1 + \begin{bmatrix} 0 \\ 1 \\ \vdots \\ n-1 \\ n \end{bmatrix} \mu_2 \tag{13}$$

for $n = 1, 2, ..., C$, and

$$\begin{aligned} B_n e &= \begin{bmatrix} (n-C) r_2 + C\mu_1 \\ (n-C) r_2 + (C-1)\mu_1 + \mu_2 \\ \vdots \\ (n-C) r_2 + \mu_1 + (C-1)\mu_2 \\ (n-C) r_2 + C\mu_2 \end{bmatrix} \\ &= (n-C) r_2 e + \begin{bmatrix} C \\ C-1 \\ \vdots \\ 1 \\ 0 \end{bmatrix} \mu_1 + \begin{bmatrix} 0 \\ 1 \\ \vdots \\ C-1 \\ C \end{bmatrix} \mu_2, \end{aligned} \tag{14}$$

for $n = C+1, ..., C+K$. Therefore, from equation (11) to (14), we conclude if equation $p_n = p_{n-1} K_n$ holds for all n, we should have $h_1 + r_1 = h_2 + r_2$. Furthermore, in this case, by taking the results in equations (13) and (14) into equations (11) and (12), we can easily obtain

$$K_n = \begin{cases} \frac{\lambda_1 \alpha_{n-1} + \lambda_2 \beta_{n-1}}{n\mu}, & \text{if } n = 1, 2, \cdots, C, \\ \frac{\lambda_2 \beta_{n-1}}{C\mu + (n-C) r_2}, & \text{if } n = C+1, \cdots, C+N. \end{cases}$$

It is worth noting that the key condition of Theorem 1 is $h_1 + r_1 = h_2 + r_2$, which is equivalent to that the expected channel holding time of new call is the same as the expected channel holding time of handoff calls. Therefore, we may still

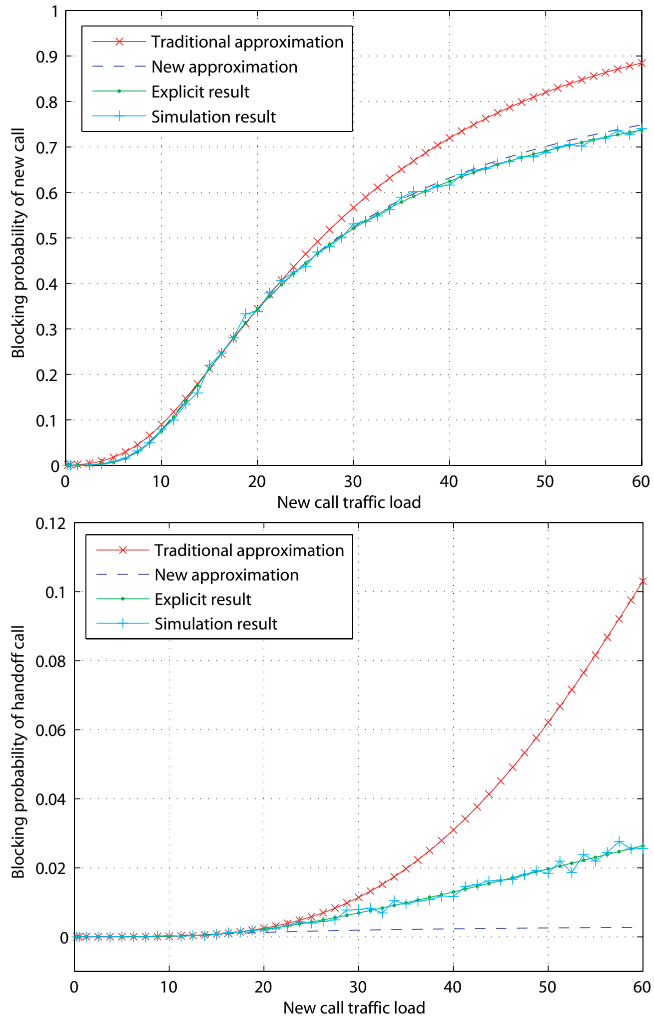

Fig. 4. Blocking probability vs. new call traffic load.

have the scalar product form probability even if the expected call holding time of new call is different from that of the handoff calls, or the expected cell residence time of new call is different from that of the handoff calls.

Once we obtain the stationary probability distribution of the system as in equation (5), we can derive many performance metrics of interest. Here, for illustration purpose, we only show the results of the blocking probability for each type of calls as follows.

Theorem 2: Define p_i $(i = 1, 2)$ the blocking probability for a type-i call, i.e., the probability that a type-i call is blocked when it arrives, we have

$$p_1 = 1 - \sum_{n=0}^{C-1} \alpha_n \boldsymbol{\pi}_0 \prod_{i=1}^{n} \left[A_{i-1}(-D_i)^{-1}\right] \mathbf{e}, \tag{15}$$

and

$$p_2 = 1 - \sum_{n=0}^{C+K-1} \beta_n \boldsymbol{\pi}_0 \prod_{i=1}^{n} \left[A_{i-1}(-D_i)^{-1}\right] \mathbf{e}. \tag{16}$$

The verification of these results are straightforward, if we notice the description of the $\boldsymbol{\pi}_n$ in equation (5), and

$$\begin{aligned} p_1 &= \sum_{i+j<C} (1-\alpha_{i+j})\pi_{i,j} + \sum_{i+j\geq C} \pi_{i,j} \\ &= 1 - \sum_{n=0}^{C-1} \alpha_n \boldsymbol{\pi}_n \mathbf{e}, \end{aligned}$$

as well as

$$\begin{aligned} p_2 &= \sum_{i+j<C+K-1} (1-\beta_{i+j})\pi_{i,j} + \sum_{i+j=C+K} \pi_{i,j} \\ &= 1 - \sum_{n=0}^{C+K-1} \beta_n \boldsymbol{\pi}_n \mathbf{e}. \end{aligned}$$

III. Comparison Study

Now, we present our analytical results and the simulation results for the new call blocking probability and the handoff termination probability. We also compare with two well-known approximate results: the traditional approximation [7] and the new approximation [5]. Here, for our comparison study, we assume $C = 30$ and $K = 0$, $\alpha_i = 1$ for $i = 0, 1, ..., 24$, $\alpha_i = 0$ for $i = 25, ..., 29$, and $\beta_i = 1$ for $i = 1, ..., 29$.

Figure 4 shows how the blocking probabilities depend on the new call traffic load when $\lambda_1 = 1/20$, $\lambda_2 = 1/30$ and $\mu_2 = 1/300$ with μ_1 changing. From this figure, we observe that when the new calls and handoff calls have significant different average values of the channel holding time, our analytical results and the simulation results still match well (insignificant deviation is because the simulation time is not long enough), and the traditional approximation shows significant inaccuracy while the new approximation shows a better fit although significant error can be still observed, particularly when new call traffic load is high for the blocking probability of handoff calls.

Figure 5 shows how the blocking probability depends on the handoff call traffic load when $\lambda_1 = 1/20$, $\mu_1 = 1/300$ and $\lambda_2 = 1/40$ with μ_2 changing. From this figure, we observe the same: our analytical results and the simulation results still match well, and the traditional approximation shows significant inaccuracy while the new approximation shows a better fit.

From this simple comparison study, we observe that our analytical results for both new call blocking probability and handoff call blocking probability are verified by our simulation results. We also observe that both the traditional approximation and the new approximation provide good approximation only in certain parameter range, say, when traffic load is low. We confirm the conclusion in [5] that the new approximation indeed performs better than the traditional approximation. More importantly, this paper has completely addressed the problem, an open remark, in [5] in which it stated that "This paper calls again for the necessity of reexamining the classical analytical results in traffic theory, which are used for the analysis and design of wireless mobile networks."

IV. Conclusion

In this paper, we presented a new general analytical model for a thinning scheme in wireless cellular networks where

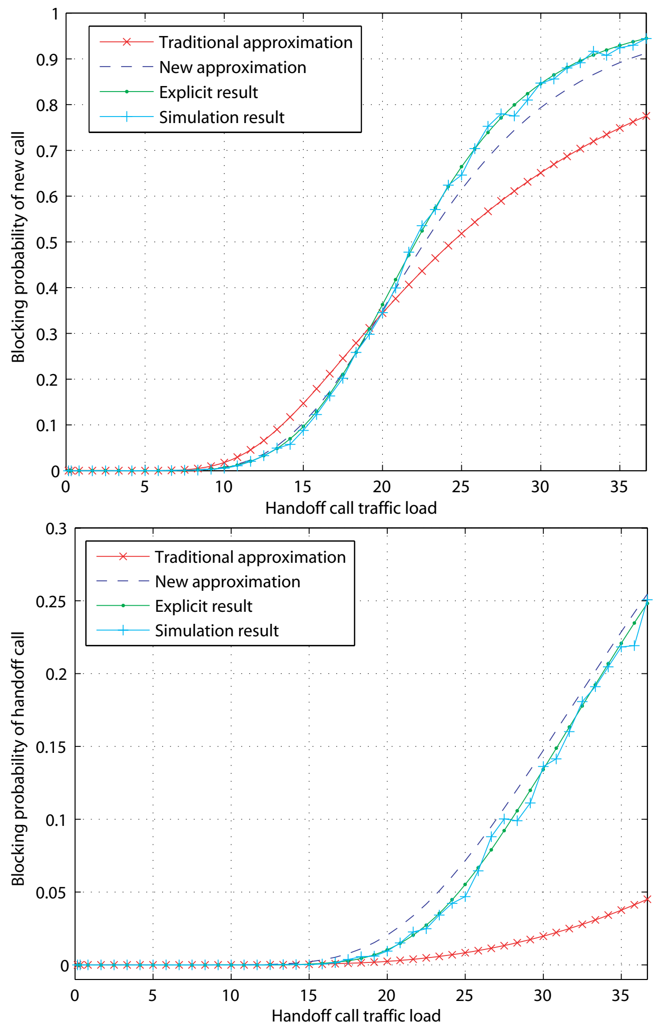

Fig. 5. Blocking probability vs. handoff call traffic load.

channel holding times for new calls and handoff calls are distinctly distributed with different average values. For the proposed model, the closed matrix product-form solution for the stationary probability were derived. Moreover, we verified that a sufficient and necessary condition for the stationary probability of the number of calls in the system to be a product-form is that the expected channel holding times for both new calls and handoff calls are equal. We also obtained the expressions for blocking probabilities of new calls and handoff calls. Finally, we verified our analytical results by our simulation study and understood better about the accuracy of the two existing approximate results. We can show that the complexity of the current algorithms in finding the solution of the stationary probability distribution is $O\left((N+1)^5 \ln(N+1)\right)$. Efficient computational algorithms are still under investigation.

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