Teletraffic Analysis and Mobility Modeling of PCS Networks

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Abstract—Channel holding time is of primary importance in teletraffic analysis of PCS networks. This quantity depends on user's mobility which can be characterized by the cell residence time. In this paper we show that when the cell residence time is not exponentially distributed, the channel holding time is not exponentially distributed either, a fact also confirmed by available field data. In order to capture the essence of PCS network behavior, including the characterization of channel holding time, a correct mobility model is therefore necessary. The new model must be good enough to fit field data, while at the same time resulting in a *tractable* queueing system. In this paper we propose a new mobility model, called the *hyper-Erlang distribution model*, which is consistent with these requirements. Under the new realistic operational assumption of this model, in which the cell residence time is generally distributed, we derive analytical results for the channel holding time distribution, which are readily applicable to the hyper-Erlang distribution models. Using the derived analytical results we demonstrate how the distribution of cell residence time affects the channel holding time distribution. The results presented in this paper can provide guidelines for field data processing in PCS network design and performance evaluation.

Index Terms—Call holding time, cell residence time, channel holding time, mobility, mobility model, PCS.

I. INTRODUCTION

CHANNEL holding (occupancy) time is an important quantity in teletraffic analysis of PCS networks. This quantity is needed to derive key network design parameters such as the new call blocking probability and the handoff call blocking probability [17]. Bolotin [1] studied common-channel signaling (CCS) systems and found that channel throughput drops significantly more under an exponential call holding time distribution model than under the actual measured call holding time distribution. This observation is expected to be true for PCS networks. Thus, it is important to realistically characterize channel holding time in PCS networks and investigate how its distribution affects PCS network traffic. In

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order to accomplish this, we need to have an appropriate traffic model to reflect the actual traffic situation and the user mobility patterns.

For the sake of convenience and tractability, most previous traffic analysis made the assumption that the channel holding time is distributed exponentially [9], [10], [17], [28], [37], [38]. However, this assumption is not valid for PCS networks. For these networks, Guerin [16] demonstrated that when the rate of direction change is "low," the channel holding time is no longer exponentially distributed. Bolotin [1] showed that the channel holding time for CCS (common channel signaling) networks is no longer exponentially distributed either. Lin et al. [28] gave a condition under which the channel holding time is exponentially distributed, that is, the cell residence time needs to be exponentially distributed. Recent experiments with operational systems and field data showed that channel holding times for cellular systems and mobile radio systems are not exponentially distributed. Using field data Jedrzycki and Leung [19] observed that channel holding time distribution for cellular telephony systems is not exponential, and statistically showed that the lognormal distribution provided a better fit for the field data for channel holding time. Orlik and Rappaport [32] observed that the data profile used in [19] can also be approximately modeled by the SOHYP (the Sum of Hyperexponential) distribution. Barcelo et al. [3], [4], [20], [21] conducted a series of experiments for mobile radio and cellular systems and concluded that channel holding times and related time variables are not exponentially distributed. They further showed that the lognormal distribution and the mixture of Erlang distributions provided better statistical fitting to the experimental data. In summary, the above research and experiments demonstrate that the exponential assumption for channel holding time is not appropriate. Although it is a wellknown fact in queueing theory [25] that blocking probability in M/G/m/m is insensitive to service time distribution (corresponding, in our case, to channel holding time), this may not be true for G/G/m/m systems. Chlebus and Ludwin [5] and Rajartnam and Takawira [35] showed that the handoff traffic in cellular and PCS networks is not Poissonian. Rajartnam and Takawira [35] also showed that the handoff call blocking probability under realistic mobility modeling is significantly different from that under the assumption that the channel holding time is exponentially distributed. Thus, for cellular or PCS networks, the queueing system for each cell is a G/G/m/m system and the blocking probability is indeed sensitive to the service time (i.e., the channel holding time) distribution. This phenomena is another important reason why

it is necessary to study channel holding time for PCS network performance evaluation under more realistic situations.

In this paper we deal with channel holding time (i.e., the time a call spends in a cell) under more general assumptions. We observe that channel holding time depends on the users' mobility, which in turn can be characterized by the cell residence time (the time a mobile user stays, or dwells, in a cell). Thus, in order to appropriately characterize the channel holding time, it is necessary to have a good mobility model for the cell residence time. One approach to modeling the cell residence time can be had by assuming that a cell has specific (hexagonal or circular) shape. When combined with specific distributions of speed and movement direction of a mobile user it then becomes possible to determine the probability distribution of cell residence time [10], [17]. However, in practical systems cell shapes are irregular, and the speed and direction of mobile users may be hard to characterize. It is therefore more appropriate to directly model the cell residence time as a random variable with an appropriate probability distribution to capture the overall effects of the cellular shape and the users' mobility patterns. This approach has been adopted in the past by a few researchers. Zonoozi and Dassanayake [39] used the generalized Gamma distribution to model the cell residence time. Unfortunately, the generalized Gamma distribution leads to the loss of Markovian property in the resulting queueing model of the cellular network, which makes the resulting queueing system intractable. Orlik and Rappaport [31], [32] modeled the cell residence time as a SOHYP random variable. The advantage of using the SOHYP distribution is the preservation of the Markovian property in the queueing network model. It was shown in [33] that SOHYP models can be tuned to have the coefficient of variation of the cell residence time less than, equal to, and greater than unity, while the exponential (even Erlang) distribution model for cell residence time only applies to cases where the coefficients of variation are less than unity. However, it is not known whether the SOHYP models have the capability of universal approximations. Moreover, the Laplace transform of the SOHYP distribution remains a complex rational function.

A good mobility model must satisfy at least the following two conditions: 1) it must be simple but good enough to fit field data, and 2) the resulting queueing system model must still be tractable. The classical Cox model [7], [15], [26], used in queueing systems in the past is known to provide a good approximation for general distribution. In particular, the exponential distribution, the Erlang distribution, the hyperexponential distribution, and the SOHYP distribution are all special cases of the Cox distribution models. However, the Cox models (including SOHYP models) contain too many parameters to be identified, hence the statistical fitting using a Cox model is very complicated.

In this paper, we propose a new mobility model, called the *hyper-Erlang distribution model*, which satisfies the above two conditions 1) and 2). We find that the hyper-Erlang distribution preserves the Markovian property of the resulting queueing network models and has universal approximation capability, which will be demonstrated in this paper. We observe that the above two approximation models (hyper-Erlang or Cox)

share the same attractive property: they all have rational Laplace transforms and preserve the Markovian property of the resulting queueing networks, hence the multidimensional Markov chain theory can be applied to find the required teletraffic parameters such as the call blocking probabilities.

In this paper we first discuss the hyper-Erlang distribution model and its universal approximation capability, then we derive formulae for the (conditional) distribution of channel holding time with general cell residence time distributions. We then provide an easy-to-compute procedure when the cell residence time has rational Laplace transform, in particular for the hyper-Erlang models. Using our results, we show analytically that when the cell residence time is not exponentially distributed and the channel holding time is indeed not exponentially distributed. Surprisingly, a counterintuitive result is observed: the low variance of the cell residence time leads to the invalidity of the exponential assumption for channel holding time. We observe that while for cellular networks the cell size is large, and hence the variance of the cell residence time is high, the exponential assumption for channel holding time may be appropriate for some cellular networks. For the PCS networks, the cell size becomes much smaller and the variance of the cell residence time will be lower. Hence, the exponential assumption is not valid anymore. Therefore, if field data for cell residence time shows a low variation, the exponential assumption for the channel holding time in the teletraffic analysis for PCS networks cannot be used. In this instance, our analytical results can be conveniently used to characterize the channel holding time. With the mobility model for the cell residence time and the analytical results for the channel holding time we present in this paper, we can study the resulting queueing systems for the PCS network using the multidimensional Markov chain models as illustrated in [26]. In this way we can investigate the validity of several classical analytical results in traffic theory for PCS systems. This paper takes the first step toward this goal.

II. HYPER-ERLANG DISTRIBUTION MODEL

As mentioned in the previous section, the cell residence time can be used to characterize the users' mobility. We observe that the cell residence time can be treated as a nonnegative random variable. Hence, a good distribution model for the random variable will be sufficient for characterizing the users' mobility. In this section we discuss such a model, the *hyper-Erlang distribution model*.

The *hyper-Erlang* distribution has the following probability density function and Laplace transform:

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$$f_{he}(t) = \sum_{i=1}^{M} \alpha_i \frac{(m_i \eta_i)^{m_i} t^{m_i - 1}}{(m_i - 1)!} e^{-m_i \eta_i t}, \qquad t \ge 0$$
$$f_{he}^*(s) = \sum_{i=1}^{M} \alpha_i \left(\frac{m_i \eta_i}{s + m_i \eta_i}\right)^{m_i} \tag{1}$$

where

$$\alpha_i \ge 0, \qquad \sum_{i=1}^M \alpha_i = 1$$

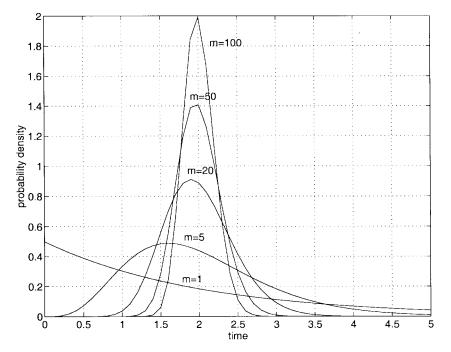


Fig. 1. Probability density function for Erlang distribution: m = 1, 5, 20, 50, 100.

and M, m_1, m_2, \dots, m_M are nonnegative integers, $\eta_1, \eta_2, \dots, \eta_M$ are positive numbers.

We next show that these distribution functions provide sufficiently general models, i.e., hyper-Erlang distributions are universal approximators. This can be accomplished by the following result (in what follows we will use star * to denote the Laplace transformation):

Lemma [24]: Let G(t) be the cumulative distribution function of a nonnegative random variable. Then it is possible to choose a sequence of distribution functions $G_m(t)$, each of which corresponds to a mixture of Erlang distributions, so that

$$\lim_{m \to \infty} G_m(t) = G(t)$$

for all t at which G(t) is continuous. In particular, $G_m(t)$ can be chosen as

$$G_m(t) = \sum_{k=1}^{\infty} \left[G\left(\frac{k}{m}\right) - G\left(\frac{k-1}{m}\right) \right] G_m^k(t), \qquad t \ge 0$$

where $G_m^k(t)$ is the distribution function of an Erlang distribution with mean k/m and variance k/m^2 (i.e., the distribution of the sum of k exponential random variables each with mean 1/m).

Let $g_m(t)$ and $g_m^*(s)$ denote the density function and Laplace transform of $G_m(t)$, and $g_m^k(t)$ denote the density function of $G_m^k(t)$. Then we have

$$g_m(t) = \sum_{k=1}^{\infty} \left[G\left(\frac{k}{m}\right) - G\left(\frac{k-1}{m}\right) \right] g_m^k(t)$$
$$g_m^*(s) = \sum_{k=1}^{\infty} \left[G\left(\frac{k}{m}\right) - G\left(\frac{k-1}{m}\right) \right] \left(\frac{k/m}{s+k/m}\right)^k.$$
(2)

The resulting distribution is called the mixed Erlang distribution. The advantage of using this distribution is that the coefficients can be determined from the experimental data. We can use a finite number of terms to approximate the distribution function. In this case, the resulting distribution approximates the hyper-Erlang distribution.

In fact, we can intuitively illustrate from the Sampling Theorem [34] why the distribution $G_m(t)$ provides the universal approximation to general distribution models. Fig. 1 shows the density function by varying the shape parameter m [see (26)]. We observe that as the shape parameter m becomes sufficiently large, the density function approaches the Dirac δ function. This can also be shown analytically. Consider the Erlang distribution with the following density function

$$f_e(t) = \frac{(m\eta)^m t^{m-1}}{(m-1)!} e^{-m\eta t}, \qquad t \ge 0.$$

This density function attains the maximum value

$$f_{\max} = \frac{m\eta(m-1)^{m-1}e^{-(m-1)}}{(m-1)!}$$

at the point $t_{\rm max} = (m-1)/(m\eta)$. Using the Stirling's approximation [24]

$$n! \sim \sqrt{2\pi} n^{n+1/2} e^{-n}$$

(where \sim indicates that the ratio of the two sides tends to unity as $n \to \infty$), we obtain

$$f_{\max} \sim \frac{\eta}{\sqrt{2\pi}} \sqrt{m}$$

which implies that $f_{\text{max}} \to \infty$ as $m \to \infty$. Hence, $f_e(t)$ approaches the δ function as m sufficiently large. From signal processing theory [22] we know that the δ function can be used to sample a function and reconstruct the function from the sampled data (the Sampling Theorem). We can replace the δ function by the Erlang density function with sufficiently large

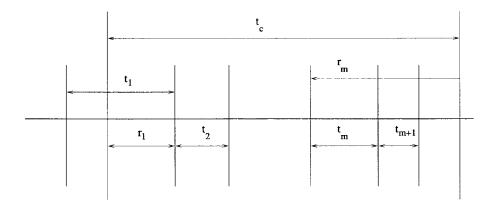


Fig. 2. The time diagram for call holding time and cell residence time.

shape parameter, and the resulting approximation is exactly in the form of the hyper-Erlang distribution.

We remark that the hyper-Erlang distribution model is much easier to use than the Cox models. Let t be the generic form for the cell residence time t_i . If t is modeled by the hyper-Erlang distribution as in (1), we can easily find its kth moment given below

$$E[t^k] = (-1)^k f_{he}^{*(k)}(0) = \sum_{i=1}^M \alpha_i \, \frac{(m_i + k - 1)!}{(m_i - 1)!} \, (m_i \eta_i)^{-k}.$$

The parameters α_i , m_i , and η_i $(i = 1, 2, \dots, M)$ can be found by fitting a number of moments from field data. Moreover, if the number of moments exceeds the number of variables, then the least-square method can be used to find the best fit to minimize the least-square error.

We also point out that hyper-Erlang distribution can be used to model other time variables such as channel holding time, interarrival time of calls and call holding time. In this research we only focus on cell residence time modeling. Specifically, we model cell residence time instead of channel holding time directly, since channel holding time can be derived from the cell residence time distribution and the cell residence time distribution can also be obtained directly from any other specific mobility models.

III. CHANNEL HOLDING TIME

The channel holding time distribution depends on the mobility of users, which can be characterized by the cell residence time. As the assumption of exponentially distributed cell residence time is too restrictive for real world systems, we wish to relax this assumption. In this section, we study the channel holding time under the condition that the cell residence time is generally distributed, in particular, hyper-Erlang distributed.

Let the call holding time t_c (i.e., the unencumbered call holding time of requested connection to a PCS network for a new call, as in wireline telephony) be exponentially distributed with parameter μ . Let t_m be the cell residence time, r_1 be the time between the instant a new call is initiated and the instant the new call moves out of the cell if the new call is not completed, and let r_m (m > 1) be the residual life time distribution of call holding time when the call finishes mth handoff successfully. Let t_{nh} and t_{hh} denote the channel holding times for a new call and a handoff call, respectively. Then, from Fig. 2, the channel holding time for a new call will be

$$t_{nh} = \min\{t_c, r_1\}\tag{3}$$

and the channel holding time for a handoff call is

$$t_{hh} = \min\{r_m, t_m\}.\tag{4}$$

Let t_c , t_m , r_1 , t_{hh} , and t_{nh} have density functions $f_c(t)$, f(t), $f_r(t)$, $f_{hh}(t)$, and $f_{nh}(t)$ with their corresponding Laplace transforms $f_c^*(s)$, $f^*(s)$, $f_r^*(s)$, $f_{hh}^*(s)$, and $f_{nh}^*(s)$, respectively, and with cumulative distribution functions $F_c(t)$, F(t), $F_r(t)$, $F_{hh}(t)$, and $F_{nh}(t)$, respectively.

From (4) we obtain the probability

$$Pr(t_{hh} \le t)$$

$$= Pr(r_m \le t \text{ or } t_m \le t)$$

$$= Pr(r_m \le t) + Pr(t_m \le t) - Pr(r_m \le t, t_m \le t)$$

$$= Pr(r_m \le t) + Pr(t_m \le t) - Pr(r_m \le t) Pr(t_m \le t)$$

$$= Pr(t_c \le t) + Pr(t_m \le t) - Pr(t_c \le t) Pr(t_m \le t)$$
(5)

where we have used the independency of r_m and t_m , and the memoryless property of the exponential distribution from which we have that the distribution of r_m has the same distribution as t_c . Differentiating (5), we obtain

$$f_{hh}(t) = f_c(t) + f(t) - f_c(t) \operatorname{Pr}(t_m \le t) - \operatorname{Pr}(t_c \le t) f(t)$$
$$= f_c(t) \int_t^\infty f(\tau) \, d\tau + f(t) \int_t^\infty f_c(\tau) \, d\tau.$$
(6)

From (6), applying Laplace transform, we obtain

$$\begin{aligned} f_{hh}^*(s) &= f^*(s) + f_c^*(s) - \int_0^\infty e^{-st} f(t) \int_0^t f_c(\tau) \, d\tau \, dt \\ &- \int_0^\infty e^{-st} f_c(t) \int_0^t f(\tau) \, d\tau \, dt \\ &= f^*(s) + f_c^*(s) - \mu \int_0^\infty e^{-(s+\mu)t} \int_0^t f(\tau) \, d\tau \, dt \\ &- \int_0^\infty e^{-st} (1 - e^{-\mu t}) f(t) \, dt \\ &= \frac{\mu}{s+\mu} + \frac{s}{s+\mu} \, f^*(s+\mu). \end{aligned}$$

From the above equations, we obtain the expected handoff call channel holding time we will use $h^{(i)}(x)$ to denote the *i*th derivative of any function h(x) at point x in the subsequent development

$$E[t_{hh}] = -f_{hh}^{*(1)}(0) = \frac{1}{\mu} (1 - f^{*}(\mu)).$$

From (3) and a similar argument, we obtain

$$f_{nh}(t) = f_c(t) \int_t^\infty f_r(\tau) \, d\tau + f_r(t) \int_t^\infty f_c(\tau) \, d\tau.$$

Applying Laplace transform, we have

$$f_{nh}^{*}(s) = \frac{\mu}{s+\mu} + \frac{s}{s+\mu} f_{r}^{*}(s+\mu)$$
(7)

and the expected new call channel holding time is

$$E[t_{nh}] = -f_{nh}^{*(1)}(0) = \frac{1}{\mu} \left[1 - f_r^*(\mu)\right].$$

The preceding discussion differentiates between new calls and handoff calls when considering the channel holding times. If such distinction is not made, we need to consider the channel holding time distribution for any call (either new call or handoff call), i.e., the channel holding time for the merged traffic of new calls and handoff calls, as used in current literature. We will simply call this the channel holding time, using no modifiers such as new call or handoff call. Let t_h denote the channel holding time and λ_h the handoff call arrival rate, and let λ denote the new call arrival rate. Then, it is easy to show that $t_h = t_{nh}$ with probability $\lambda/(\lambda + \lambda_h)$ and $t_h = t_{hh}$ with probability $\lambda_h/(\lambda + \lambda_h)$. Let $f_h(t)$ and $f_h^*(s)$ be its density function and the corresponding Laplace transform. It is easy to obtain

$$f_{h}(t) = \frac{\lambda}{\lambda + \lambda_{h}} f_{nh}(t) + \frac{\lambda_{h}}{\lambda + \lambda_{h}} f_{hh}(t)$$
$$f_{h}^{*}(s) = \frac{\lambda}{\lambda + \lambda_{h}} f_{nh}^{*}(s) + \frac{\lambda_{h}}{\lambda + \lambda_{h}} f_{hh}^{*}(s).$$
(8)

Summarizing the above we arrive at:

Theorem 1: For a PCS network with exponential call holding time and Poisson new call arrivals with arrival rate λ , we have the following.

 The Laplace transform of the density function of the new call channel holding time is given by

$$f_{nh}^{*}(s) = \frac{\mu}{s+\mu} + \frac{s}{s+\mu} f_{r}^{*}(s+\mu)$$
(9)

and the expected new call channel holding time is

$$E[t_{nh}] = -f_{nh}^{*(1)}(0) = \frac{1}{\mu} [1 - f_r^*(\mu)].$$
(10)

2) The Laplace transform of the density function of the handoff call channel holding time is given by

$$f_{hh}^*(s) = \frac{\mu}{s+\mu} + \frac{s}{s+\mu} f^*(s+\mu)$$
(11)

and the expected handoff call channel holding time is

$$E[t_{hh}] = \frac{1}{\mu} \left(1 - f^*(\mu) \right). \tag{12}$$

3) Let λ_h denote the handoff call arrival rate, then the Laplace transform of the density function of channel holding time is given by

$$f_h^*(s) = \frac{\lambda}{\lambda + \lambda_h} f_{nh}^*(s) + \frac{\lambda_h}{\lambda + \lambda_h} f_{hh}^*(s).$$
(13)

and the expected channel holding time is given by

$$E[t_h] = \frac{\lambda}{\mu(\lambda + \lambda_h)} \left(1 - f_r^*(\mu)\right) + \frac{\lambda_h}{\mu(\lambda + \lambda_h)} \left(1 - f^*(\mu)\right).$$
(14)

When the residual life time r_1 of t_1 is exponentially distributed with parameter μ_r , then its Laplace transform $f_r^*(s)$ is $\mu_r/(s + \mu_r)$. Taking this into (9), we obtain

$$f_{nh}^*(s) = \frac{\mu}{s+\mu} + \frac{\mu_r s}{(s+\mu)(s+\mu+\mu_r)} = \frac{\mu+\mu_r}{s+\mu+\mu_r}$$

which implies that the new call channel holding time is exponentially distributed with parameter $\mu + \mu_r$. Similarly, if the cell residence time t_i is exponentially distributed with parameter η , then the handoff call channel holding time is also exponentially distributed with parameter $\mu + \eta$. In this case, the channel holding time is hyperexponentially distributed. If $\mu_r = \eta$, then the channel holding time [see (13)] is exponentially distributed with parameter $\mu + \eta$. In fact, since r_1 is the residual life of t_1 , from the Residual Life Theorem [26], we have

$$f_r^*(s) = \frac{\eta [1 - f^*(s)]}{s} = \frac{\eta}{s + \eta} = f^*(s).$$

Hence, the channel holding time is exponentially distributed with parameter $\mu + \eta$ when the cell residence time is exponentially distributed.

Based on the field data collected for channel holding time, Jedrzycki and Leung [19] demonstrated that the channel holding time is not exponentially distributed and that the lognormal distribution provides a satisfactory approximation after the spikes of data are removed (the spikes correspond to the handoff calls). Orlik and Rappaport [31], [32] interpreted the distribution reported in [19] as the *conditional distribution given that the call completes in its current cell*, and derived the results for conditional distributions for the channel holding time when the cell residence time is SOHYP distributed. We adopt a different approach and use a more general distribution model for cell residence time. Simple results for the conditional distribution for channel holding time when the cell residence time is generally distributed are presented next.

Let $f_{cnh}(t)$, $f_{chh}(t)$, and $f_{ch}(t)$ denote the conditional density functions for new call channel holding time, the handoff call channel holding time, and the channel holding time, respectively, with Laplace transforms $f_{cnh}^*(s)$, $f_{chh}^*(s)$, and $f_{ch}^*(s)$, and with cumulative distribution functions $F_{cnh}(t)$, $F_{chh}(t)$, and $F_{ch}(t)$. We first study the conditional distribution for the handoff be c call channel holding time. We have

$$F_{chh}(h) = \Pr(t_{hh} \le h | r_m \le t_m)$$

= $\frac{\Pr(r_m \le h, r_m \le t_m)}{\Pr(r_m \le t_m)}$
= $\frac{\int_0^h f_c(t) \int_t^\infty f(\tau) d\tau dt}{\Pr(r_m \le t_m)}$
= $\frac{\int_0^h f_c(t) [1 - F(t)] dt}{\Pr(r_m \le t_m)}$.

Differentiating both sides, we obtain the conditional density function

$$f_{chh}(h) = \frac{f_c(h)[1 - F(h)]}{\Pr(r_m \le t_m)}.$$
 (15)

We observe that

$$\Pr(r_m \le t_m) = \int_0^\infty \int_0^t f(t) f_c(\tau) \, d\tau \, dt$$

= $\int_0^\infty f(t) [1 - e^{\mu t}] \, dt$
= $1 - \int_0^\infty f(t) e^{-\mu t} \, dt = 1 - f^*(\mu).$

Taking this into (15), we obtain

$$f_{chh}(h) = \frac{[1 - F(h)]\mu e^{-\mu h}}{1 - f^*(\mu)}.$$

Hence,

$$f_{chh}^{*}(s) = \frac{\mu \int_{0}^{\infty} e^{-(s+\mu)h} [1-F(h)] dh}{1-f^{*}(\mu)}$$
$$= \frac{\mu}{s+\mu} \cdot \frac{1-f^{*}(s+\mu)}{1-f^{*}(\mu)}.$$
 (16)

In a similar fashion, we obtain the following result for the new call channel holding time:

$$f_{cnh}(h) = \frac{[1 - F_r(h)]\mu e^{-\mu h}}{1 - f_r^*(\mu)}$$
$$f_{cnh}^*(s) = \frac{\mu}{s + \mu} \cdot \frac{1 - f_r^*(s + \mu)}{1 - f_r^*(\mu)}.$$

The conditional channel holding time distribution $f_{ch}(t)$ $[f_{ch}^*(s)]$ is the average of the conditional new call channel holding time distribution and handoff call channel holding time distribution. In summary, we therefore have

Theorem 2: For a PCS network with exponential call holding time and Poisson new call arrivals, the conditional distributions for the new call channel holding time, the handoff call channel holding time, and the channel holding time can be characterized by their Laplace transforms as follows:

$$f_{cnh}^{*}(s) = \frac{\mu}{s+\mu} \cdot \frac{1 - f_{r}^{*}(s+\mu)}{1 - f_{r}^{*}(\mu)}$$
(17)

$$f_{chh}^{*}(s) = \frac{\mu}{s+\mu} \cdot \frac{1 - f^{*}(s+\mu)}{1 - f^{*}(\mu)}$$
(18)

$$f_{ch}^{*}(s) = \frac{\mu}{s+\mu} \left(\frac{\lambda}{\lambda+\lambda_{h}} \cdot \frac{1-f_{r}^{*}(s+\mu)}{1-f_{r}^{*}(\mu)} + \frac{\lambda_{h}}{\lambda+\lambda_{h}} \cdot \frac{1-f^{*}(s+\mu)}{1-f^{*}(\mu)} \right).$$
(19)

Let T_{cnh} , T_{chh} , and T_{ch} denote the expected conditional new call channel holding time, the expected conditional handoff call channel holding time, and the expected conditional channel holding time, respectively, then we have

$$T_{cnh} = \frac{1}{\mu} + \frac{f_r^{*(1)}(\mu)}{1 - f_r^*(\mu)}$$
(20)

$$T_{chh} = \frac{1}{\mu} + \frac{f^{*(1)}(\mu)}{1 - f^{*}(\mu)}$$
(21)

$$T_{ch} = \frac{1}{\mu} + \frac{\lambda}{\lambda + \lambda_h} \cdot \frac{f_r^{*(1)}(\mu)}{1 - f_r^{*}(\mu)} + \frac{\lambda_h}{\lambda + \lambda_h} \frac{f^{*(1)}(\mu)}{1 - f^{*}(\mu)}.$$
 (22)

When the residual life r_1 is exponentially distributed with parameter μ_r , from (17) we obtain $f_{cnh}^*(s) = (\mu + \mu_r)/(s + \mu + \mu_r)$. Hence, the conditional new call channel holding time is also exponentially distributed. Moreover, this holding time has the same distribution as the unconditional new call channel holding time due to the memoryless property of the exponential distribution. Similarly, the handoff call channel holding time is also exponentially distributed if the cell residence time is exponentially distributed.

In order to apply Theorems 1 and 2, we need to determine the handoff call arrival rate λ_h . This parameter depends on the new call arrival rate, the new call blocking probability, and the handoff call blocking probability. Let p_o and p_f denote the new call and handoff call blocking probabilities, respectively. Let H be the number of handoffs for a call. Its expectation E[H] is also called handoff rate. Using a procedure similar to the one in [11] and [14], we can obtain the following:

$$E[H] = -(1 - p_o) \sum_{p \in \sigma_c} \operatorname{Res}_{s=p} \frac{f_r^*(s)}{s[1 - (1 - p_f)f^*(s)]} f_c^*(-s)$$

where σ_c denotes the set of poles of $f_c^*(-s)$ in the right half of the complex plane and $\operatorname{Res}_{s=p}$ denotes the residue at the pole s = p. Since t_c is exponentially distributed with parameter μ , hence $f_c^*(s) = \mu/(s + \mu)$, from the above we obtain

$$E[H] = \frac{(1 - p_o)f_r^*(\mu)}{1 - (1 - p_f)f^*(\mu)}.$$
(23)

Since each unblocked call initiates E[H] handoff calls on the

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average, the handoff call arrival rate can be obtained

$$\lambda_h = \lambda E[H] = \frac{(1 - p_o)\lambda f_r^*(\mu)}{1 - (1 - p_f)f^*(\mu)}.$$
 (24)

From the discussion above we can observe that as long as $f_r^*(s)$ and $f^*(s)$ are proper rational functions, then the Laplace transforms of distribution functions of all channel holding times (either conditional or unconditional) are all rational functions (see Theorems 1 and 2). To find the corresponding density functions, we only need to find the inverse Laplace transforms. This can be accomplished by using the partial fractional expansion [22]. To illustrate the idea, we present the following procedure. Suppose that g(s) is a proper rational function with poles p_1, p_2, \dots, p_k with multiplicities n_1, n_2, \dots, n_k . Then g(s) can be expanded as

$$g(s) = \sum_{i=1}^{k} \sum_{j=0}^{n_i} A_{ij} \frac{s^j}{(s+p_i)^{n_i}}$$
(25)

where the constants A_{ij} can be found easily by the formula

$$A_{ij} = \frac{d^{j}}{ds^{j}} \left[(s+p_{i})^{n_{i}} g(s) \right]_{s=0},$$

$$j = 0, 1, \dots, i, \quad i = 1, 2, \dots, k.$$

Notice the relationship $(\mathcal{L}^{-1}$ denotes the inverse Laplace transform operator)

$$\mathcal{L}^{-1}[s^{j}f(s)] = \frac{d^{j}}{dt^{j}} \{\mathcal{L}^{-1}[f(s)]\},\$$
$$\mathcal{L}^{-1}[1/(s+\beta)^{h}] = \frac{t^{h}}{h!} e^{-\beta t}.$$

Taking this into (25), we obtain the inverse Laplace transform

$$\mathcal{L}^{-1}[g(s)] = \sum_{i=1}^{k} \sum_{j=0}^{m_i} A_{ij} \frac{d^j}{dt^j} \left(\frac{t^j}{j!} e^{-p_i t}\right).$$

We also notice that the inverse Laplace transform of a rational function is in fact the impulse response of a linear system in which the rational function is the system transfer function of the resulting linear system [22] and the cumulative distribution function is the step response of the linear system. By studying the impulse response and step response of the channel holding time. Several ready-to-use software packages for the study of the impulse response and step response in signals and systems [22] are available. In the well-known software package Matlab, the commands *impulse* and *step* can be used to find the density function and the distribution function. Therefore, Theorems 1 and 2 can be readily applicable to the case where the cell residence time distribution is Coxian [26] or SOHYP [31].

When applying the hyper-Erlang distribution model for cell residence time, we can in fact reduce the computation further. As an example, we use Theorem 1 (2) to illustrate this point. If we substitute $f^*(s)$ with $f^*_{he}(s)$, we obtain

$$f_{hh}^*(s) = \sum_{i=1}^M \alpha_i \left[\frac{\mu}{s+\mu} + \frac{s}{s+\mu} \left(\frac{m_i \eta_i}{s+m_i \eta_i} \right)^{m_i} \right]$$
$$\equiv \sum_{i=1}^M \alpha_i f_e^*(s; m_i, \eta_i)$$

where $f_e^*(s; m_i, \eta_i)$ corresponds to the handoff call channel holding time when the cell residence time is Erlang distributed with parameters (m_i, η_i) . Thus, we can concentrate on finding the algorithm for computing the channel holding time for the case when the cell residence time is Erlang distributed.

As a final remark in this section, we illustrate the relationship between $f_r^*(s)$ and $f^*(s)$. If we are interested in all calls for a long run (i.e., we have large samples for cell residence time), the residual life time r_1 can be regarded as the residual life of the cell residence time, as it is the time that a mobile user spends in the initiating cell (where the call is made). From the Residual Life Theorem [26], we obtain

$$f_r^*(s) = \frac{\eta [1 - f^*(s)]}{s}$$

where $1/\eta$ is the average cell residence time. If we only have small number of samples for cell residence time, then the Residual Life Theorem may not be appropriate [26] and we can only use the best distribution fit for r_1 from the available samples, in which case we can regard the cell residence time sequence r_1, t_2, t_3, \cdots as a renewal process [8].

IV. EFFECT OF DISTRIBUTION OF THE CELL RESIDENCE TIME ON THE CHANNEL HOLDING TIME DISTRIBUTION

It is well known that the exponential distribution is completely determined by a single parameter, i.e., the average value. Thus, if we use exponential distribution to model the cell residence time for the field trials, the fitted distribution is completely determined by the average value of the field data. Therefore, this model hardly captures the variation of the cell residence time for a mobile user. In this case, the resulting channel holding time, which is also exponentially distributed, is also completely determined by the average channel holding time (or the average cell residence time). In a real situation, however, a mobile user's cell residence time significantly deviates from the average value from time to time and from cell to cell. It is important to understand how the distribution of the cell residence time affects the channel holding time distribution. One statistical quantity to characterize the deviation of the field data from the average value is the variance. In fact, the variance of the cell residence time is one of the reasons why the channel holding time is not exponentially distributed when the cell residence time is not exponentially distributed.

In this section, we present a case study to show the applications of our analytical results and analytically show how variance of cell residence time affects the channel holding time distribution. We show that when the cell residence time is not exponentially distributed, the channel holding time is not

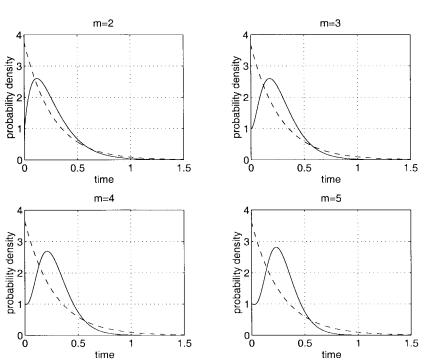


Fig. 3. Probability density function of handoff call channel holding time (solid line) and its exponential fitting (dashed line) when cell residence time is Erlang distributed with parameter (m, η) .

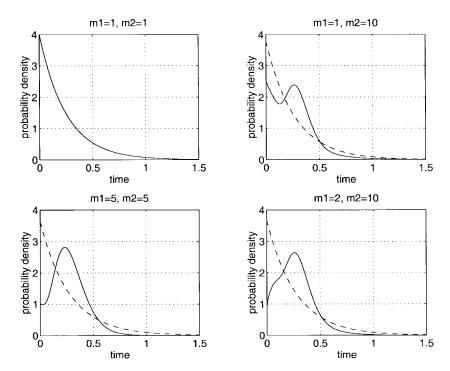


Fig. 4. Probability density function of handoff call channel holding time (solid line) and its exponential fitting (dashed line) when cell residence time is hyper-Erlang distributed with parameter (m_1, m_2, η) .

exponentially distributed either. In fact, for some cases (where the variance is small), the approximation using the exponential distribution is severely inappropriate. This suggests that a careful study is needed for the channel holding time in teletraffic analysis.

We first study the channel holding time for the case when the cell residence time is Erlang distributed. The Erlang distribution is characterized by its density function and Laplace transform as follows:

$$f(t) = \frac{\beta^m t^{m-1}}{(m-1)!} e^{-\beta t}, \qquad f^*(s) = \left(\frac{\beta}{s+\beta}\right)^m \tag{26}$$

where $\beta = m\eta$ is called the scale parameter and m is called the shape parameter. The mean of this Erlang distribution is η and its variance is $1/(m\eta^2)$. When the mean η is fixed, varying the value m is equivalent to varying the variance and

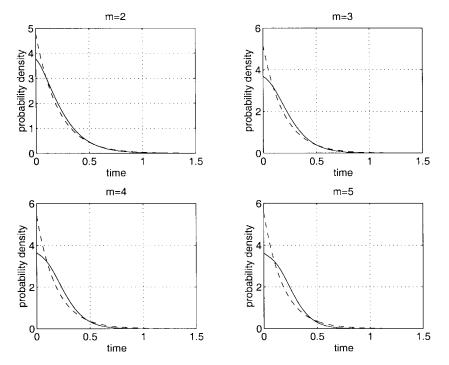


Fig. 5. Conditional probability density function of handoff call channel holding time (solid line) and its exponential fitting (dashed line) when cell residence time is Erlang distributed with parameter (m, η) .

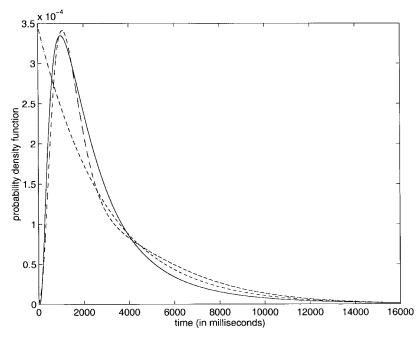


Fig. 6. Probability density function of a lognormal distribution (solid line), its hyper-Erlang approximation (dash line) and its exponential fitting (dashdot line).

larger m means smaller variance and lesser spread of the cell residence time.

Due to the similarity of the formulae for new call and handoff call, we only study the handoff call channel holding time. Fig. 3 shows the handoff call channel holding time probability density functions with different variance of cell residence time distributed according to Erlang distribution with the same mean. It can be observed that when the cell residence time become less spread, the handoff call channel holding time shows severe mismatch to the exponential distribution. This implies that we can not simply apply the exponential distributions for handoff call channel holding time during the network study of PCS networks where mobility is a major issue.

Next we study the case when the cell residence time is hyper-Erlang distributed with two terms as follows:

$$f^*(s) = \alpha_1 \left(\frac{m_1 \eta}{s + m_1 \eta}\right)^{m_1} + \alpha_2 \left(\frac{m_2 \eta}{s + m_2 \eta}\right)^{m_2}.$$

The Laplace transform of the handoff call channel holding

time can be written as

$$f_{hh}^*(s) = \alpha_1 \left\{ \frac{\mu}{s+\mu} + \frac{s}{s+\mu} \cdot \left(\frac{m_1 \eta}{s+m_1 \eta} \right)^{m_1} \right\} \\ + \alpha_2 \left\{ \frac{\mu}{s+\mu} + \frac{s}{s+\mu} \cdot \left(\frac{m_2 \eta}{s+m_2 \eta} \right)^{m_2} \right\}.$$

This representation illustrates that the distribution of the handoff call channel holding time is in fact an average of two distributions of handoff call channel holding times, each of which is obtained from Erlang distributed cell residence time cases. The mean value of the cell residence time for this case is still η , which is the same as in the Erlang case. Different values of m_1 and m_2 signify the different variances. Fig. 4 shows the distribution plotting. When m_1 and m_2 choose different values, the variances of cell residence time are different and the handoff call channel holding time is no longer exponentially distributed.

Fig. 5 shows the conditional probability density function for the handoff call channel holding time when the cell residence time is Erlang distributed. From this example, we observe the conditional distribution for handoff call channel holding time is a better match to the exponential distribution when the variance of cell residence time is large. However, when the variance becomes small, i.e., the cell residence time is less spread, this match disappears.

From the scenario of the field data [2], [4], [19]–[21], we observe that the channel holding time probability density function have one or multiple peaks, which shows why the channel holding time is not exponentially distributed, while the hyper-Erlang distribution is a better model. Lognormal distribution has been used [4], [19] for statistical fitting, we demonstrate that hyper-Erlang distribution can be used to approximate lognormal distribution. Fig. 6 shows the lognormal distribution from field data [21] for channel holding time and its exponential and hyper-Erlang fits. Hence with the field data, we can observe that the hyper-Erlang distribution indeed provides a very good fit to the lognormal distribution.

V. SUMMARY

This paper proposed a new mobility model and analytically characterized the distribution of the channel holding time under a realistic assumption, whereby the cell residence time is generally distributed. Our modeling effort focused on the characterization of the channel holding time under the assumption that the distribution of the cell residence time has rational Laplace transform. Hence, our analytical results are readily applicable to the hyper-Erlang distribution models for the cell residence time. The analytical results presented in this work provide a general framework for further study of teletraffic aspects in PCS networks in which classical assumptions do not hold.

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