Channel Occupancy Times and Handoff Rate for Mobile Computing and PCS Networks

Yuguang Fang, *Member, IEEE*, Imrich Chlamtac, *Fellow, IEEE*, and Yi-Bing Lin, *Senior Member, IEEE*

Abstract—This paper presents a study of channel occupancy times and handoff rate for mobile computing in MC (Mobile Computing) and PCS (Personal Communications Services) networks, using general operational assumptions. It is shown that, for exponentially distributed call holding times, a distribution more appropriate for conventional voice telephony, the channel occupancy times are exponentially distributed if and only if the cell residence times are exponentially distributed. It is further shown that the merged traffic from new calls and handoff calls is Poisson if and only if the cell residence times are exponentially distributed, too. When cell residence times follow a general distribution, a more appropriate way to model mobile computing sessions, new formulae for channel occupancy time distributions are obtained. Moreover, when the call holding times and the cell residence times have general (nonlattice) distributions, general formulae for computing the handoff rate during a call connection and handoff call arrival rate to a cell are given. Our analysis illustrates why the exponential assumption for call holding time results in the underestimation of handoff rate, which then leads to the actual blocking probabilities being higher than the blocking probabilities for MC/PCS networks designed using the exponential distribution approximation for call holding time. The analytical results presented in this paper can be expected to play a significant role in teletraffic analysis and system design for MC/PCS networks.

Index Terms—PCS, mobile computing, call holding time, cell residence times, call blocking, handoff rate, channel occupancy times.

1 Introduction

A convergence of Mobile Computing and Personal Communications Services (MC/PCS) being witnessed today leads to the emergence of networks poised to provide integrated services, such as voice and data, to mobile users anywhere, anytime ([19], [4]) in an uninterrupted and seamless way, using advanced microcellular and handoff concepts ([15]). Due to the growing importance of these emerging networks, it is necessary to study their behavior for performance evaluation, optimization, management, as well as billing, under realistic conditions. We observe that the channel occupancy time corresponds to the service time, while the handoff call traffic to a cell forms a major part of the cell traffic, the resulting queuing network is used to find the very important design parameters, such as blocking probability. The handoff rate is used to characterize the handoff call traffic to a cell. Therefore, the channel occupancy time and the handoff rate are two important parameters for the MC/PCS networks.

The channel occupancy time distribution has been studied quite extensively in the past for classical cellular systems. A common assumption in these studies has been that the call holding time (the time requested for a call connection) is exponentially distributed. The first traffic model for cellular mobile radio telephone systems was proposed by Hong and Rappaport ([11]), who analyzed the performance and showed that the channel occupancy time distribution could be approximated by the exponential distribution when the call holding times are exponentially distributed. Using a simulation model, Guerin ([10]) showed that, for the mobile users with "low" change rate of direction of the movement, the channel occupancy time distribution displays a rather poor agreement with the exponential distribution fitting. In ([22]), Lin and Chlamtac obtained the expected channel holding times for both new calls and handoff calls. Nanda ([27]) studied the handoff rate under the assumption that the call holding times are exponentially distributed and there are no blocking and forced terminations (i.e., corresponding to the ideal assumption that there is an infinite number of channels available in each cell). In other words, most analytical studies in the literature assume that the channel occupancy times are exponentially distributed. However, recent field studies ([1], [12], [14]) showed that the channel occupancy times are not exponentially distributed for cellular systems. Therefore, further investigation on channel occupancy times is needed.

To model the MC/PCS networks in a realistic way, several observations are in order. First, due to the wide spectrum of the integrated communications services (such as phone calls, information retrievals, Internet surfing, etc.) carried jointly over an MC/PCS network, the assumption of call holding times being exponentially distributed, made in the past for evaluating the behavior of classical wireless or wireline telephone networks, may no longer be valid. This is confirmed via field studies, as we mentioned earlier. Second, due to user mobility (portables or mobile computers)

- Y. Fang and I. Chlamtac are with the Erik Jonsson School of Engineering and Computer Science, The University of Texas at Dallas, P.O. Box 830688, EC 33, Richardson, TX 75083. E-mail: {yfang, chlamtac}@utdallas.edu.
- Y.-B. Lin is with the Department of Computer Sciences and Information Engineering, National Chiao Tung University, Hsinchu, Taiwan, Republic of China. E-mail: liny@csie.nctu.edu.tw.

Manuscript received 14 Oct. 1996.
For information on obtaining reprints of this article, please send e-mail to: tc@computer.org, and reference IEEECS Log Number 102134.

0018-9340/98/$10.00 © 1998 IEEE

and the irregular geographical cell shapes, the cell residence times (the time a user spends in a cell) will also typically have a general distribution. Third, the channel occupancy time in a cell, i.e., the time the channel is occupied by a call in a cell (a new call, a handoff call, regardless of the call being completed in the cell or moving out of the cell) is also not necessarily distributed exponentially, as generally assumed in the past ([23], [27], [33], [34]). Last, during a communication session in an MC/PCS network, a user may traverse several cells and the call may consequently be handed off many times before it completes.

Recently, we ([7], [8]) have developed some analytical tools for analyzing the call completion probability, effective call holding times, and billing rate planning. In this paper, we present a systematic study of channel occupancy times and handoff rate in MC/PCS systems under general systems assumptions, leading to a number of new results. Under the assumption that the call holding time is exponentially distributed, we present a necessary and sufficient condition for the new call channel occupancy time (the handoff call channel occupancy time or the channel occupancy time) to be exponentially distributed. Specifically, we show that, in an MC/PCS system, the channel occupancy time is exponentially distributed if and only if cell residence times are exponentially distributed. When cell residence times are not exponentially distributed, we derive formulae to compute the distribution of channel occupancy times. In order to apply Erlang-B formula, the cell arrival traffic, which consists of merged traffic of new calls and handoff calls, has been assumed to be Poisson, a common assumption for the computation of blocking probability in cellular systems ([5], [9], [23], [31], [33], [34]). However, for cell arrival traffic in MC/PCS networks, we show that the cell arrival traffic is Poisson if and only if the cell residence times are exponentially distributed.

The handoff rate, which is defined as the average number of handoffs during a call connection, is the second focus of this paper. This quantity is not only used in obtaining the handoff call arrival rate to a cell, a quantity used in characterizing the channel occupancy time distribution, it can also be tied to the service quality and cost parameters. In this paper, we, therefore, derive a general formula for the computation of the handoff rate under general conditions, i.e., when the call holding times and cell residence times are generally distributed with nonlattice distribution functions.

By studying the CCS network, Bolotin ([2]) observed that the SS7 channel throughput under the actual call holding time distribution is greater than the theoretical channel throughput under exponential distribution approximation for call holding time, and showed that the call holding time cannot simply be modeled by exponential distribution. In our study of MC/PCS networks, we address a similar problem. We find that, for high mobility users, due to the variation of the call holding time, the handoff rate in a call and, hence, the handoff call arrival rate are significantly different from the case when the call holding time is assumed to be exponentially distributed. The exponential distribution model for call holding time underestimates the handoff call arrival rate, hence the overall cell traffic rate, thus the blocking probability under the actual call holding time, are higher than those under exponential approximation for channel holding time. This is due to the fact that the exponential distribution only captures the effects of the mean value. Our study shows that the effects of distribution models on the system performance parameters must be accounted for in teletraffic analysis in order to design a practical MC/PCS network.

Last, the technique proposed in this paper also leads to a practical model for deriving performance evaluation of GSM based mobile computing systems. GSM provides data capabilities to support mobile computing applications. The data capabilities include GSM phase 2 bearer data service, phase 2+ high speed circuit switched data (HSCSD), and general packet radio service (GPRS) ([6], [25]). These data services are either circuit switched or packet switched at the transport layer (i.e., at the wireline backbone network), but are all circuit switched at the current radio interface. In other words, a mobile computing user will occupy a traffic channel similarly to a typical voice user, hence, when a mobile user gets a channel in a cell, he/she will use the channel during the cell residence of the mobile user. Thus, the modeling of radio resource allocation for GSM-supported mobile computing system is similar to the radio channel allocation for PCS, except that the computing session times in mobile computing are unlikely to be exponentially distributed as in traditional PCS systems. As a result, previously proposed approaches ([5], [21], [23], [31], [33], [34]) are not appropriate for modeling these mobile computing systems (due to the exponential call holding time assumption). By allowing the use of general call holding time distribution, our model can be used to study GSM-based mobile computing systems with arbitrary computing session periods.

This paper is organized as follows: In the next section, we discuss the properties of channel occupancy times and the merged cell traffic. In the third section, we present analytical formulae for the computation of channel holding time distributions for general cell residence times. In the fourth section, we present results on handoff rate calculation. Performance study is provided in Section 5, with the last section concluding the paper.

2 Comments on Classical Assumptions Regarding Handoff Traffic and Channel Occupancy Times

In order to find the blocking probability in MC/PCS networks, a commonly used assumption ([5], [9], [31], [34]) is that the channel occupancy times are exponentially distributed. While this is a reasonable assumption for wired telephone traffic and most currently used cellular networks, we show that this assumption holds only for the case when cell residence times are exponentially distributed, a property which does not hold for emerging MC/PCS networks. The second commonly used assumption is that the merged traffic from the new call traffic and handoff call traffic in a cell is a Poisson process ([5], [9], [23], [31], [33], [34]). This assumption is needed to apply the well-known Erlang-B formula (or its extended version) using the $M/M/c/c$ (or $M/G/c/c$) queuing model to compute the

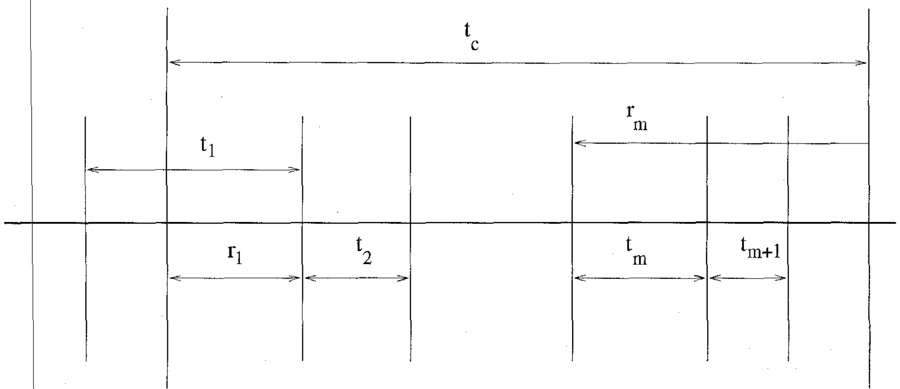


Fig. 1. The time diagram for call holding time and cell residence time.

blocking probability in a cell. Although this assumption regarding merged traffic properties may be a good approximation for some cases, it cannot be expected to be appropriate for most MC/PCS applications. In fact, the fit of the exponential distribution for these applications has not been quantified in the literature. We show that this assumption is valid only for the case when cell residence times are exponentially distributed.

We first consider the first assumption using the time diagram in Fig. 1. Assume that the call holding times (the times of requested connections to a MC/PCS network for new calls) are exponentially distributed with parameter μ. Let t_c be the call holding time for a typical new call, t_m be the cell residence time, r_1 be the time between the instant the new call is initiated at and the instant the new call moves out the cell if the new call is not completed, r_m $(m > 1)$ be the residual call holding time when the call finishes mth handoff successfully. Let t_{no} and t_{ho} denote the channel occupancy times for a new call and a handoff call, respectively. Then, from Fig. 1, the new call channel occupancy time is

$$t_{no} = \min\{t_c, r_1\}, \tag{1}$$

and the handoff call channel occupancy time is

$$t_{ho} = \min\{r_m, t_m\}. \tag{2}$$

Let t_c, t_m, r_1, t_{ho}, and t_{no} have density functions $f_c(t)$, $f(t)$, $f_r(t)$, $f_{ho}(t)$, and $f_{no}(t)$ with their corresponding Laplace transforms $f_c^*(s)$, $f^*(s)$, $f_r^*(s)$, $f_{ho}^*(s)$, and $f_{no}^*(s)$, respectively.

We next show that the handoff call channel occupancy time t_{ho} is exponentially distributed if and only if the cell residence time t_m is exponentially distributed. From (2), we obtain the probability

$$\begin{aligned} Pr(t_{ho} \le t) &= Pr(r_m \le t \text{ or } t_m \le t) \\ &= Pr(r_m \le t) + Pr(t_m \le t) - Pr(r_m \le t, t_m \le t) \\ &= Pr(r_m \le t) + Pr(t_m \le t) - Pr(r_m \le t)Pr(t_m \le t) \\ &= Pr(t_c \le t) + Pr(t_m \le t) - Pr(t_c \le t)Pr(t_m \le t), \end{aligned} \tag{3}$$

where we have used $Pr(r_m \le t, t_m \le t) = Pr(r_m \le t)Pr(t_m \le t)$ from the independency of r_m and t_m, and $Pr(r_m \le t) = Pr(t_c \le t)$ from the memoryless property of the exponential distribution, where r_m is just the residual life of t_c (either from the Residual Life Theorem or the argument in [23]). Differentiating (3), we obtain

$$\begin{aligned} f_{ho}(t) &= f_c(t) + f(t) - f_c(t)Pr(t_m \le t) - Pr(t_c \le t)f(t) \\ &= f_c(t)\int_t^{\infty} f(\tau)d\tau + f(t) + f(t)\int_t^{\infty} f_c(\tau)d\tau. \end{aligned} \tag{4}$$

Suppose that the cell residence times are exponentially distributed with parameter η, then, from (4), we obtain

$$f_{ho}(t) = \mu e^{-\mu t} \times e^{-\eta t} + \eta e^{-\eta t} \times e^{-\mu t} = (\mu + \eta)e^{-(\mu+\eta)t},$$

which is an exponential distribution. Conversely, suppose that the handoff call channel occupancy time has exponential distribution with parameter γ, let $Y(t) = \int_t^{\infty} f(\tau)d\tau$, then $\dot{Y}(t) = -f(t)$ (overdot $\cdot$ denotes the differentiation symbol). From (4), we obtain

$$\mu e^{-\mu t}Y(t) + e^{-\mu t}f(t) = \gamma e^{-\gamma t},$$

i.e.,

$$\dot{Y}(t) = \mu Y(t) - \gamma e^{-(\gamma-\mu)t},$$

from which we obtain

$$\begin{aligned} Y(t) &= e^{\mu t}Y(0) + \int_0^t e^{\mu(t-\tau)}\left[-\gamma e^{-(\gamma-\mu)\tau}\right]d\tau \\ &= e^{\mu t}\left\{Y(o) - \gamma\int_0^t e^{-\gamma\tau}d\tau\right\} = e^{-(\gamma-\mu)t}. \end{aligned}$$

Thus, $f(t) = -\dot{Y}(t) = (\gamma - \mu)e^{-(\gamma-\mu)t}$, i.e., the cell residence time must be exponentially distributed.

We now consider the new call channel occupancy time distribution case. From (1) and a similar argument, we obtain

$$f_{no}(t) = f_c(t)\int_t^{\infty} f_r(\tau)d\tau + f_t(t)\int_t^{\infty} f_c(\tau)d\tau.$$

Suppose that the new call channel occupancy time is exponentially distributed with parameter, say, μ_1, from this identity and a similar argument as for the handoff call channel occupancy time case, we can deduce that

$$f_r(t) = (\mu_1 - \mu)e^{-(\mu_1-\mu)t},$$

which is also an exponential distribution. Let $F(t)$ denote the distribution function of the cell residence time with mean $1/\eta$, from the Residual Life Theorem ([18], [26]), we have

$$f_r(t) = \eta(1 - F(t)),$$

from which we obtain

$$F(t) = 1 - \frac{\mu_1 - \mu}{\eta} e^{-(\mu_1 - \mu)t}.$$

From this and $F(0) = 0$, we obtain $\mu_1 - \mu = \eta$, so $F(t) = 1 - e^{-\eta t}$, and we conclude that the cell residence times are also exponentially distributed. This shows that, for an MC/PCS network with exponential call holding times, the new call channel occupancy time is exponentially distributed if and only if the cell residence times are exponentially distributed.

In the preceding discussion, we separated calls into new calls and handoff calls when considering the channel occupancy times. If such distinction is not made, then we need to consider the channel occupancy time distribution for any call (either new call or handoff call), i.e., the channel occupancy time for the merged traffic of new calls and handoff calls, as used in current literature. We will simply call this the channel occupancy time, without any modifiers such as new call or handoff call. Let t_{co} denote the channel occupancy time and λ_h the handoff call arrival rate (which will be discussed in a later section). Then, it is easy to show that $t_{co} = t_{no}$ with probability $\lambda/(\lambda + \lambda_h) \triangleq p$ and $t_{co} = t_{ho}$ with probability $\lambda_h/(\lambda + \lambda_h) \triangleq q$. Let $f_{co}(t)$ and $f^*_{co}(s)$ be its density function and the corresponding Laplace transform. It is easy to obtain

$$\begin{aligned} f_{co}(t) &= pf_{no}(t) + qf_{ho}(t) \\ &= pf_c(t)\int_t^\infty f_r(\tau)d\tau + pf_r(t)\int_t^\infty f_c(\tau)d\tau + qf_c(t) \\ &\int_t^\infty f(\tau)d\tau + qf(t)\int_t^\infty f_c(\tau)d\tau \\ &= e^{-\mu t}\left\{p\mu\eta\int_t^\infty \left[1 - F(\tau)\right]d\tau + (p\eta + q\mu)\left[1 - F(t)\right] + qf(t)\right\}. \end{aligned} \quad (5)$$

It is straightforward to show that when the cell residence times are exponentially distributed, then the channel occupancy time is also exponentially distributed. Conversely, suppose that the channel occupancy time is exponentially distributed with, say, parameter γ, i.e., $f_{co}(t) = \gamma e^{-\gamma t}$, from (5), we obtain

$$p\mu\eta\int_t^\infty \left[1 - F(\tau)d\tau\right] + (p\eta + q\mu)\left[1 - F(t)\right] + qf(t) = \gamma e^{-(\gamma-\mu)t}. \quad (6)$$

From the left hand side of (6), with the properties of the distribution function, we can deduce that $\gamma - \mu \geq 0$. Let $Y(t) = \int_t^\infty [1 - F(\tau)]d\tau$, then $F(t) = 1 + \dot{Y}(t)$, $\ddot{Y}(t) = f(t)$, taking these into (6), we obtain

$$\ddot{Y}(t) - \left(\frac{p}{q}\eta + \mu\right)\dot{Y}(t) + \frac{p}{q}\eta\mu Y(t) = \frac{\gamma}{q}e^{-(\gamma-\mu)t}. \quad (7)$$

One particular solution of (7) is in the form

$$Y_1(t) = Be^{-(\gamma-\mu)t},$$

where

$$B = \frac{\gamma}{q\gamma^2 + \gamma(p\eta + q\mu) + p\mu\eta} > 0.$$

Noticing that the characteristic equation of (7) is

$$s^2 - \left(\frac{p}{q}\eta + \mu\right)s + \frac{p}{q}\eta\mu = (s - \mu)\left(s - \frac{p}{q}\eta\right) = 0,$$

which has roots $s = \mu > 0$ and $s = (p/q)\eta > 0$. If these two roots are equal, then all solutions of (7) are given by

$$Y(t) = C_1e^{\mu t} + C_2te^{\mu t} + Be^{-(\gamma-\mu)t},$$

where C_1 and C_2 are constants. Since $\lim_{t\to\infty}Y(t) = 0$, we must have $C1 = C_2 = 0$, so $Y(t) = Be^{-(\gamma-\mu)t}$. Similarly, if the two roots are not equal, then all solutions of (7) are

$$Y(t) = C_1e^{\mu t} + C_2e^{-[(p/q)\eta]t} + Be^{-(\gamma-\mu)t},$$

so $C_1 = C_2 = 0$ from the definition of $Y(t)$. In any case, $Y(t)$ must be in the form $Y(t) = Be^{-(\gamma-\mu)t}$. So, we have

$$F(t) = 1 + \dot{Y}(t) = 1 - B(\gamma - \mu)e^{-(\gamma-\mu)t}.$$

From $F(0) = 0$, we obtain $B(\gamma - \mu) = 1$, hence $F(t) = 1 - e^{-(\gamma-\mu)t}$, which implies that the cell residence times are exponentially distributed. In summary, we have shown that the channel occupancy time is exponentially distributed if and only if the cell residence times are exponentially distributed.

Next, we discuss the second commonly used assumption, i.e., the merged traffic of new calls and handoff calls in a cell is Poissonian. Assume that, in a typical cell of an MC/PCS network, the new call arrivals are Poisson, then the handoff call arrivals to the cell are independent of the new call arrivals. Let $N_n(t)$ and $N_h(t)$ be the numbers of new calls and handoff calls, respectively, up to time t. Let $N(t)$ be the number of calls from the merged traffic of the new call traffic and the handoff call traffic. Then, we have

$$N(t) = N_n(t) + N_h(t), \quad (8)$$

We use the Z–transform theory and the following result ([18]): For a traffic with counting process $N(t)$, $N(t)$ is a Poisson process if and only if its Z–transform $E[z^{N(t)}]$ is equal to $\exp(-\lambda t(1 - z))$. If $N_h(t)$ is a Poisson process, then, obviously, $N(t)$ is a Poisson process, i.e., the merged traffic is a Poisson process. Suppose that $N(t)$ is a Poisson process with parameter λ_m and $N_n(t)$ is a Poisson process with parameter λ, then, from (8), we have

$$\begin{aligned} E\left[z^{N(t)}\right] &= e^{-\lambda_m t(1-z)} = E\left[z^{N_n(t)+N_h(t)}\right] \\ &= E\left[z^{N_n(t)}\right]E\left[z^{N_h(t)}\right] = e^{-\lambda t(1-z)}E\left[z^{N_h(t)}\right]. \end{aligned}$$

From this, we obtain

$$E\left[z^{N_h(t)}\right] = e^{-(\lambda_m - \lambda)t(1-z)}.$$

Thus, the handoff call traffic $N_h(t)$ is also a Poisson process. It is well known ([3], [18], [26]) that, for the $M/G/c$ queuing system with first-come-first-serve (FCFS) strategy, the departure process is a Poisson process if and only if the service time is exponentially distributed. We next observe that the handoff call traffic is the departure process of the queuing system with the Poisson arrivals (the merged traffic) and with two virtual "servers": one "server" for the calls which complete the connection successfully in the cell (the service time distribution is exponentially distributed due to the memoryless property of exponential distribution), the other "server" for calls which need handoffs (which forms the handoff call traffic). The departure process from the first server is a Poisson process, since the departure process from the $M/M/1$ queue is a Poisson process ([3]). As the handoff call traffic is Poisson from the above discussion, from the earlier referenced Burke's result, the channel occupancy time must be exponentially distributed, hence, the cell residence times must be exponentially distributed, too.

Summarizing the preceding discussion, we finally obtain:

THEOREM 1. *For an MC/PCS network with exponential call holding times and Poisson new call arrivals, we can state:*

1) *the new call channel occupancy time is exponentially distributed if and only if the cell residence times are exponentially distributed,*
2) *the handoff call channel occupancy time is exponentially distributed if and only if the cell residence times are exponentially distributed,*
3) *the channel occupancy time is exponentially distributed if and only if the cell residence times are exponentially distributed, and*
4) *the merged traffic from the new call traffic and handoff call traffic is still Poissonian if and only if the cell residence times are exponentially distributed.*

3 Channel Occupancy Times

In the preceding section, we concluded that, for channel occupancy time to be exponentially distributed, the cell residence times had to be exponentially distributed. However, the assumption of exponential cell residence times is too restrictive, since it is important to know the distribution of channel occupancy time for generally distributed cell residence times. It is furthermore important to find out how "close" is the exponential distribution to the distribution of channel occupancy times. We address these issues next.

From (4), applying Laplace transform, we obtain

$$\begin{aligned} f_{ho}^*(s) &= f^*(s) + f_c^*(s) - \int_0^\infty e^{-st} f(t) \int_0^t f_c(\tau) d\tau\, dt - \\ &\quad \int_0^\infty e^{-st} f_c(t) \int_0^t f(\tau) d\tau\, dt \\ &= f^*(s) + f_c^*(s) - \mu \int_0^\infty e^{-(s+\mu)t} \int_0^t f(\tau) d\tau\, dt - \\ &\quad \int_0^\infty e^{-st} \left(1 - e^{-\mu t}\right) f(t) dt \\ &= \frac{\mu}{s+\mu} + \frac{s}{s+\mu} f^*(s+\mu). \end{aligned}$$

From this, it is easy to obtain that the expected handoff call channel occupancy time is given by (we will use $h^{(i)}(x)$ to denote the ith derivative of any function $h(x)$ at point x in the subsequent development)

$$E[t_{ho}] = -f_{ho}^{*(1)}(0) = \frac{1}{\mu}\left(1 - f^*(\mu)\right).$$

Similarly, from (1), we obtain

$$f_{no}^*(s) = \frac{\mu}{s+\mu} + \frac{s}{s+\mu} f_r^*(s+\mu). \tag{9}$$

Since r_1 is the residual life of the cell residence time, from the Residual Life Theorem ([18], [26]), we have

$$f_r^*(s) = \frac{\eta}{s}\left[1 - f^*(s)\right],$$

where $\eta = 1/E[t_m]$, i.e., the cell residence rate. Taking this into (9), we obtain

$$f_{no}^*(s) = \frac{\mu}{s+\mu} + \frac{\eta s}{(s+\mu)^2}\left[1 - f^*(s+\mu)\right],$$

from which we also obtain the expected new call channel occupancy time

$$E[t_{no}] = -f_{no}^{*(1)}(0) = \frac{1}{\mu} - \frac{\eta}{\mu^2}\left[1 - f^*(\mu)\right].$$

Similarly, we can obtain formulae for channel occupancy time.

It is commonly assumed that the new call channel occupancy time and the handoff call channel occupancy time have the same distribution. However, we claim that this is true only when the cell residence times are exponentially distributed. In fact, suppose that channel occupancy times for both new call and handoff call have the same distribution, then their Laplace transforms are equal, $f_{no}^*(s) = f_{ho}^*(s)$, we obtain that $f^*(s+\mu) = \eta/(s+\mu+\eta)$, hence, $f^*(s) = \eta/(s+\eta)$; this implies that the cell residence times must be exponentially distributed.

Summarizing the above discussions, we arrive at:

THEOREM 2. *For an MC/PCS network with exponential call holding times and Poisson new call arrivals with arrival rate λ, we have:*

1) *The Laplace transform of the density function of the new call channel occupancy time is given by*

$$f_{no}^*(s) = \frac{\mu}{s+\mu} + \frac{\eta s}{(s+\mu)^2}\left[1 - f^*(s+\mu)\right], \tag{10}$$

and the expected new call channel occupancy time is

$$E[t_{no}] = \frac{1}{\mu} - \frac{\eta}{\mu^2}\left[1 - f^*(\mu)\right]; \tag{11}$$

2) *The Laplace transform of the density function of the handoff call channel occupancy time is given by*

$$f_{ho}^*(s) = \frac{\mu}{s+\mu} + \frac{s}{s+\mu} f^*(s+\mu), \tag{12}$$

and the expected handoff call channel occupancy time is

$$E[t_{ho}] = \frac{1}{\mu}\left(1 - f^*(\mu)\right). \tag{13}$$

3) *Let λ_h denote the handoff call arrival rate to a cell, then the Laplace transform of the density function of channel occupancy time is given by*

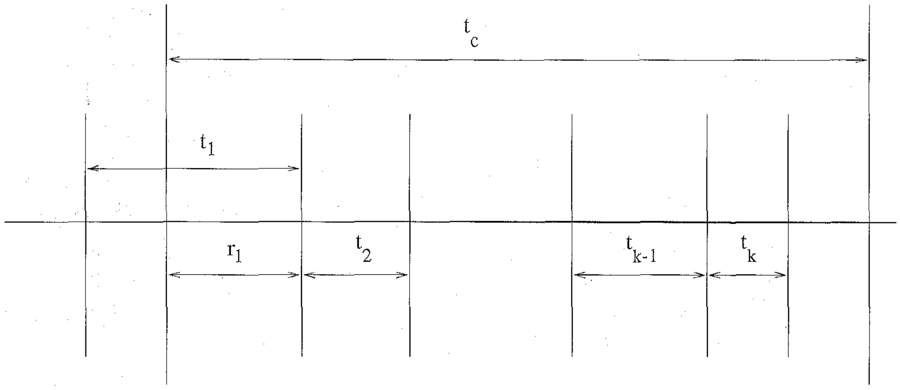


Fig. 2. The time diagram for k handoffs.

$$f_{co}^*(s) = \frac{\lambda}{\lambda + \lambda_h} f_{no}^*(s) + \frac{\lambda_h}{\lambda + \lambda_h} f_{ho}^*(s), \tag{14}$$

and the expected channel occupancy time is given by

$$E[t_{co}] = \frac{1}{\mu} - \frac{\lambda\eta}{(\lambda + \lambda_h)\mu^2}\left[1 - \left(1 - \frac{\lambda_h \mu}{\lambda\eta}\right) f^*(\mu)\right]. \tag{15}$$

4) *The new call channel occupancy time and the handoff call channel occupancy time have the same distribution if and only if the cell residence times are exponentially distributed.*

REMARK. *The expected channel occupancy times $E[t_{no}]$ and $E[t_{ho}]$ given in* (11) *and* (13) *have also been obtained in* [22] *from a direct approach.*

Using Theorem 2, we can obtain the channel occupancy time density functions by the inverse Laplace transform, from which the distribution functions can be obtained. In order to observe how "close" the exponential approximations can be, we have to determine which exponential distribution functions should be chosen. It is known that an exponential distribution is uniquely determined by its expected value. It is reasonable to use the exponential distribution whose expected value is equal to the real expected value of channel holding time (either (11), (13), or (15)). There are many criteria to evaluate this approximation, known in statistics as the "goodness of fit." A good choice will be the distance between the distribution functions of the real data and the exponential data. Hong and Rappaport ([11]) proposed the following measure for the "goodness of fit":

$$G = \frac{\int_0^\infty \left|F(t) - \left(1 - e^{-at}\right)\right| dt}{2\int_0^\infty (1 - F(t))dt},$$

where $F(t)$ is the distribution function of real data and a is the expected value of the exponential distribution used for the approximation. This measure is the normalized accumulated difference of distribution functions. Comparisons can also be done graphically by drawing distribution functions, which will be used in the performance analysis in fifth section.

4 Handoff Rate for General Call Holding Times and Cell Residence Times

As pointed out in the introduction, it is necessary to study the handoff rate (i.e., the expected number of handoffs occurring in a call) if one wishes to evaluate a MC/PCS network. For certain special cases, results for handoff rate exist in the current literature. In [27], Nanda presented an analytic result of handoff rate for the case when the call holding time is exponentially distributed and no handoff failure occurs (which is equivalent to the case where each cell has an infinite number of channels available). This is, of course, the ideal case. Lin et al. ([23]) considered the more practical case where handoff failures are taken into consideration and presented a formula for the case when call holding times are distributed exponentially. In this section, we present a formula for the general case where the call holding times and cell residence times are generally distributed, and handoff failures are accounted for.

Consider a typical call. Let t_1, t_2, $\cdots$ denote the cell residence times with expected value $1/\eta$ and r_1 denote the residual life of the new call (i.e., the time interval between the call arrival and the exit of the cell of the portable). Let t_c denote the call holding time for the typical call with expected value $1/\mu$. We use Fig. 2 to indicate the time diagram for k handoffs. Let H be the number of handoffs of a typical nonblocking call (either completed or forced to terminate) during the call connection. We can now study the property of the number H of handoffs under a general call holding time and cell residence time distributions.

Let $f^*(s)$ and $f_c^*(s)$ be defined as in the last section. We will assume that t_1, t_2, ... have independently and identically distributed (iid) nonlattice distributions and that the call holding time has generally distributed nonlattice distribution ([26]). The following is apparent: $H = 0$ if and only if the call is not blocked and the call holding time t_c is shorter than the residual life r_1, i.e., the call completes before the portable moves out of the cell; $H = 1$ if and only if the call is not blocked initially, then it either makes a successful handoff and completes the call successfully in the new cell or is forced to terminate because of the first handoff failure, and so on. If the blocking probability for a new call is p_o and the probability for a handoff call to be forced to terminate is p_f, then we have

$$
\begin{aligned}
Pr(H=0) &= (1-p_o)Pr(r_1 \ge t_c)\\
Pr(H=1) &= (1-p_o)Pr(r_1 < t_c \le r_1+t_2)(1-p_f) + (1-p_o)Pr(t_c > r_1)p_f\\
&\vdots\\
Pr(H=k) &= (1-p_o)Pr(r_1+t_2+\cdots+t_k < t_c \le r_1+t_2+\cdots+t_{k+1})(1-p_f)^k\\
&\quad + (1-p_o)Pr(t_c > r_1+t_2+\cdots+t_{k+1})(1-p_f)^{k-1}p_f\\
&\vdots
\end{aligned} \tag{16}
$$

We first calculate $Pr(H = 0)$. Since the Laplace transform of $\int_t^\infty f_r(\tau)d\tau$ is $(1-f_r^*(s))/s$ and the independency of t_c and r_1, from the first equation in (16) and the inverse Laplace transform, we have

$$
\begin{aligned}
Pr(H=0) &= (1-p_o)\int_0^\infty Pr(r_1 \ge t)f_c(t)dt\\
&= (1-p_o)\int_0^\infty \int_t^\infty f_r(\tau)d\tau\, f_c(t)dt\\
&= (1-p_o)\int_0^\infty \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{1-f_r^*(s)}{s}e^{st}ds f_c(t)dt\\
&= \frac{1-p_o}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{1-f_r^*(s)}{s}\int_0^\infty f_c(t)e^{st}dtds\\
&= \frac{1-p_o}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{1-f_r^*(s)}{s}f_c^*(-s)ds,
\end{aligned} \tag{17}
$$

where σ is a sufficiently small positive number which is chosen for the Laplace inverse transform.

Next, we compute $Pr(H = k)$ for $k > 0$. Before we do that, we need to compute $Pr(r_1 + t_2 + \cdots + t_k \le t_c)$. Let $\xi = r_1 + t_2 + \cdots + t_k$. Let $f_\xi(t)$ and $f_\xi^*(s)$ be the density function and the Laplace transform of ξ. From the independency of $r_1, t_2, t_3, \cdots$, we have

$$
f_\xi^*(s) = E\left[e^{-s\xi}\right] = E\left[e^{-sr_1}\right]\prod_{i=2}^{k}E\left[e^{-st_i}\right] = f_r^*(s)\left(f^*(s)\right)^{k-1}.
$$

So, the density function is given by

$$
f_\xi(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} f_r^*(s)\left(f^*(s)\right)^{k-1}e^{st}ds.
$$

Also, the Laplace transform of $Pr(\xi \le t)$ (the distribution function) is $f_\xi^*(s)/s$. Thus, we have

$$
\begin{aligned}
Pr(r_1+t_2+\cdots+t_k \le t_c) &= \int_0^\infty Pr(\xi \le t)f_c(t)dt\\
&= \int_0^\infty \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{f_r^*(s)\left[f^*(s)\right]^{k-1}}{s}e^{st}dsf_c(t)dt\\
&= \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{f_r^*(s)\left[f^*(s)\right]^{k-1}}{s}f_c^*(-s)ds.
\end{aligned}
$$

Taking this into (16), we obtain

$$
\begin{aligned}
Pr(H=k) &= (1-p_o)\left[Pr(t_c \ge r_1+t_2+\cdots+t_k) - Pr(t_c > r_1+t_2+\cdots+t_{k+1})\right](1-p_f)^k\\
&\quad + (1-p_0)Pr(t_c \ge r_1+t_2+\cdots+t_k)(1-p_f)^{k-1}p_f\\
&= \frac{(1-p_o)(1-p_f)^k}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{f_r^*(s)\left[f^*(s)\right]^{k-1}\left[1-f^*(s)\right]}{s}f_c^*(-s)ds\\
&\quad + \frac{(1-p_o)(1-p_f)^{k-1}p_f}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{f_r^*(s)\left[f^*(s)\right]^{k-1}}{s}f_c^*(-s)ds\\
&= \frac{(1-p_o)(1-p_f)^{k-1}}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{f_r^*(s)\left[1-(1-p_f)f^*(s)\right]\left[f^*(s)\right]^{k-1}}{s}f_c^*(-s)ds.
\end{aligned} \tag{18}
$$

Now, we find the Z–transform (moment generating function) for the number of handoffs. Let $H(z)$ be the moment generating function, from (17) and (18), we have

$$
\begin{aligned}
H(z) &= E\left[z^H\right] = \sum_{k=0}^{\infty} z^k Pr(H=k)\\
&= Pr(H=0) + \sum_{k=1}^{\infty} z^k Pr(H=k)\\
&= Pr(H=0) + \frac{(1-p_o)z}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{f_r^*(s)\left[1-(1-p_f)f^*(s)\right]}{s}\left(\sum_{k=1}^{\infty}\left[(1-p_f)zf^*(s)\right]^{k-1}\right)f_c^*(-s)ds\\
&= \frac{(1-p_o)}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{1-f_r^*(s)}{s}f_c^*(-s)ds + \frac{(1-p_o)z}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{f_r^*(s)\left[1-(1-p_f)f^*(s)\right]}{s\left[1-(1-p_f)zf^*(s)\right]}f_c^*(-s)ds\\
&= \frac{1-p_o}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{1-f_r^*(s)+z\left[f_r^*(s)-(1-p_f)f^*(s)\right]}{s\left[1-(1-p_f)zf^*(s)\right]}f_c^*(-s)ds\\
&= \frac{1-p_o}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{s-\eta\left[1-f^*(s)\right]+z\left[\eta(1-f^*(s))-(1-p_f)sf^*(s)\right]}{s^2\left[1-z(1-p_f)f^*(s)\right]}f_c^*(-s)ds,
\end{aligned} \tag{19}
$$

where $f_r^*(s) = \eta(1-f^*(s))/s$. It is obvious that when $\|z\| \le 1$, the integrand without term $f_c^*(-s)$ is analytic on the right half open complex plane. If $f_c^*(-s)$ has no branch point and has only finite possible isolated singular points in the right half plane (which is equivalent to saying that $f_c^*(s)$ has only finite possible isolated singular point in the left half plane), then the Residue Theorem can be applied to (19) using a semicircular contour in the right half plane. Indeed, if we use σ_c denote the singular points of $f_c^*(-s)$ in the right half complex plane, then, from (19) and the Residue Theorem ([20]), we obtain

THEOREM 3. *If the density function of the iid calling holding times has only finite possible isolated singular points, then the moment generating function for the number of handoffs is given by*

$$H(z) = -(1-p_o)\sum_{p\in\sigma_c}\operatorname*{Res}_{s=p} \frac{s-\eta\left[1-f^*(s)\right]+z\left[\eta\left(1-f^*(s)\right)-\left(1-p_f\right)sf^*(s)\right]}{s^2\left[1-z\left(1-p_f\right)f^*(s)\right]} f_c^*(-s), \quad (20)$$

where $\operatorname*{Res}_{s=p}$ *denotes the residue at singular point* $s = p$.

PROOF. Choosing such σ that all singular points of $f_c^*(-s)$ in the right half plane are on the right of vertical line $s = \sigma$, and choosing the contour enclosed by the semicircle at center $s = \sigma + j0$ and with radius sufficiently large, then we can apply the Residue Theorem to complete the proof. □

Theorem 3 can be used to obtain the moments of the number of handoffs or we can apply (19) to easily find the handoff rate $E[H]$. Differentiating $H(z)$ at $z = 1$, we obtain

$$E[H] = \frac{1-p_o}{2\pi j}\int_{\delta-j\infty}^{\sigma+j\infty} \frac{\eta\left[1-f^*(s)\right]}{s^2\left[1-\left(1-p_f\right)f^*(s)\right]} f_c^*(-s)ds. \quad (21)$$

Applying the Residue Theorem, we obtain

THEOREM 4. *If the density function of the iid calling holding times has only finite possible isolated singular points, then the handoff rate for a nonblocking call is given by*

$$E[H] = -\eta(1-p_o)\sum_{p\in\sigma_c}\operatorname*{Res}_{s=p} \frac{1-f^*(s)}{s^2\left[1-\left(1-p_f\right)f^*(s)\right]} f_c^*(-s). \quad (22)$$

If the call holding times are exponentially distributed with parameter μ, then $f_c^*(-s) = \mu/(-s+\mu)$, which has a unique singular point, and $\sigma_c = \{\mu\}$. From Theorem 4, we obtain

$$E[H] = (1-p_o)\frac{\eta\left(1-f^*(\mu)\right)}{\mu\left[1-\left(1-p_f\right)f^*(\mu)\right]}. \quad (23)$$

This has been obtained in [22] and [23] using different approaches.

If there is no blocking and no forced termination (the ideal case when there are infinitely many number of channels available in each cell), then $p_o = p_f = 0$ and, from (23), $E[H] = \eta/\mu$, which is also obtained in [27]. If there are handoff failures, the handoff rate is intuitively smaller. Indeed, for any handoff schemes, since $1 - f^*(\mu) \le 1 - (1-p_f)f^*(\mu)$ for any p_f ($0 \le p_f \le 1$), we have, from (23),

$$E[H] \le (1-p_o)\frac{\eta}{\mu} \le \frac{\eta}{\mu}.$$

Assume that the call holding times are iid with Erlang distribution

$$f_c(t) = \frac{\alpha^m t^{m-1}}{(m-1)!}e^{-\alpha t}, \quad f_c^*(s) = \left(\frac{\alpha}{s+\alpha}\right)^m, \quad (24)$$

where $\alpha = m\eta$ is the scale parameter and m is a positive integer. When $m = 1$, it gives the exponential distribution. Let

$$g(s) = \frac{1-f^*(s)}{s^2\left[1-\left(1-p_f\right)f^*(s)\right]}. \quad (25)$$

Then, $f_c^*(-s) = \left[\alpha/(-s+\alpha)\right]^m$ has a unique singular point $\sigma_c = \{\alpha\}$, from Theorem 4, we obtain

$$E[H] = \eta(-1)^{m+1}\alpha^m(1-p_o)\frac{g^{(m-1)}(\alpha)}{(m-1)!}, \quad \alpha = m\mu. \quad (26)$$

For this case, if there is no handoff failure, i.e., $p_o = p_f = 0$, from (25), we have $g(s) = 1/s^2$ and $g^{(m-1)})(\alpha) = (-1)^{m-1}m!\alpha^{-(m+1)}$. From (26), we have

$$E[H] = \frac{m\eta}{\alpha} = \frac{\eta}{\mu}.$$

In fact, it can be shown that, for the ideal case ($p_o = p_f = 0$), the handoff rate for any cell residence time distribution and any call holding time distribution is $E[H] = \eta/\mu$. Indeed, from (21), if $p_f = 0$, we obtain

$$\begin{aligned} E[H] &= \frac{(1-p_o)\eta}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{1}{s^2}f_c^*(-s)ds \\ &= \eta(1-p_o)\int_0^\infty f_c(t)\left[\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\frac{1}{s^2}e^{st}ds\right]dt \\ &= \eta(1-p_o)\int_0^\infty tf_c(t)dt = (1-p_o)\frac{\eta}{\mu} = \frac{\eta}{\mu} \quad (\text{if } p_o = 0). \end{aligned}$$

Next, we show how to find the handoff call arrival rate to a cell from the handoff rate in a call. It is easy to observe that, for each nonblocking new call, there will be, on the average, $E[H]$ number of handoff calls induced, so the handoff call traffic will have arrival rate $\lambda_h = \lambda E[H]$. From Theorem 4, we obtain

THEOREM 5. *If the density function of the iid calling holding times has only finite possible isolated singular points, then the handoff call traffic has the arrival rate given by*

$$\lambda_h = -\eta(1-p_o)\lambda\sum_{p\in\sigma_c}\operatorname*{Res}_{s=p} \frac{1-f^*(s)}{s^2\left[1-\left(1-p_f\right)f^*(s)\right]} f_c^*(-s). \quad (27)$$

The traffic intensity in a cell is given by

$$\rho = \lambda E[t_{no}] + \lambda_h E[t_{ho}], \quad (28)$$

where $E[t_{no}]$ *and* $E[t_{ho}]$ *are the channel occupancy times for new calls and handoff calls, respectively.*

PROOF. We only need to prove the formula for the cell traffic intensity. The overall traffic arrival rate is $\lambda + \lambda_h$ and the expected channel occupancy time is

$$\frac{\lambda}{\lambda+\lambda_h}E\left[t_{no}\right] + \frac{\lambda_h}{\lambda+\lambda_h}E\left[t_{ho}\right],$$

hence, (28) is straightforward. This completes the proof. □

If the call holding times are exponentially distributed, then, from Theorem 5 and (23), we have

$$\lambda_h = \frac{\eta(1-p_o)\left[1-f^*(\mu)\right]\lambda}{\mu\left[1-(1-p_f)f^*(\mu)\right]}, \tag{29}$$

and if we further assume that the nonprioritized handoff scheme is used (i.e., $p_o = p_f$), then, from Theorem 5, we have

$$\rho = \frac{\lambda}{\mu}\left\{1 - \frac{p_o\eta\left[1-f^*(\mu)\right]}{\mu\left[1-(1-p_o)f^*(\mu)\right]}\right\}. \tag{30}$$

If the call holding times are Erlang distributed according to (24), then we have

$$\lambda_h = \lambda\eta(-1)^{m+1}\alpha^m(1-p_o)\frac{g^{(m-1)}(\alpha)}{(m-1)!}, \quad \alpha = m\mu. \tag{31}$$

The cell traffic intensity can be computed from this formula and the expected channel occupancy time.

When the call holding times are Erlang distributed, we have to compute the derivatives of $g(s)$ (in (25)) for the computation of the handoff rate in a call and the handoff call arrival rate to a cell. However, the explicit expressions for the derivatives of $g(s)$ may be difficult. We therefore develop the following recursive algorithm for their computations. Let

$$h(s) = s^2[1-(1-p_f)f^*(s)].$$

Using the formula

$$(uv)^{(p)} = \sum_{i=0}^{p} u^{(i)}v^{(p-i)},$$

we obtain

$$\begin{aligned}
h^{(0)}(\alpha) &= \alpha^2\left[1-(1-p_f)f^*(\alpha)\right] \\
h^{(1)}(\alpha) &= -\alpha^2(1-p_f)f^{*(1)}(\alpha) + 2\alpha\left[1-(1-p_f)f^*(\alpha)\right] \\
h^{(2)}(\alpha) &= -\alpha^2(1-p_f)f^{*(2)}(\alpha) - 4\alpha(1-p_f)f^{*(1)}(\alpha) \\
&\quad + 2\left[1-(1-p_f)f^*(\alpha)\right] \\
h^{(p)}(\alpha) &= -\alpha^2(1-p_f)f^{*(p)}(\alpha) - 2p\alpha(1-p_f)f^{*(p-1)}(\alpha) \\
&\quad - p(p-1)(1-p_f)f^{*(p-2)}(\alpha), \quad p \ge 3.
\end{aligned}$$

Since we have $g(s)h(s) = 1 - f^*(s)$, differentiating both sides, we obtain ($p > 0$)

$$\sum_{i=0}^{p}\binom{p}{i}g^{(i)}(s)h^{(p-i)}(s) = f^{*(p)}(s).$$

From this, we obtain the following recursive algorithm to compute $g^{(m-1)}(\alpha)$:

$$g^{(0)}(\alpha) = \frac{1-f^*(\alpha)}{h(\alpha)}$$

$$g^{(p)}(\alpha) = -\frac{f^{*(p)}(\alpha) + \sum_{i=0}^{p-1}\binom{p}{i}g^{(i)}(\alpha)h^{(p-i)}(\alpha)}{h(\alpha)} \quad (p > 0).$$

5 Performance Studies

This section presents several demonstrative examples showing how the derived results can be used to study properties of channel occupancy times and handoff rate in the emerging MC/PCS networks, from which we observe how distribution models for call holding times and cell residence times affect the channel holding times, the handoff rate in a call, and handoff call arrival rate to a cell.

From [16], we know that any distribution of a nonnegative random variable can be approximated by the average summation of Erlang distributions (the so-called mixed Erlang distribution). The Erlang distribution is a special case of the mixed Erlang distribution, so, in this section, we will concentrate on Erlang distribution for the modeling of cell residence time. The mixed Erlang cases will be studied in the future.

First, we assume that the call holding times are exponentially distributed with parameter μ. The following identity is useful for this example:

$$\int_t^\infty (\alpha x)^i e^{-\alpha x}dx = \frac{i!}{\alpha}\left(\sum_{k=0}^{i}\frac{(\alpha t)^k}{k!}\right)e^{-\alpha t}.$$

The distribution function $F(t)$ of the Erlang distribution is given by

$$F(t) = Pr(t_m \le t) = \int_o^t f(\tau)d\tau = 1 - \sum_{i=0}^{m-1}\frac{(\alpha t)^i}{i!}e^{-\alpha t}.$$

We consider the handoff call channel occupancy time distribution first. From (3), we can find the distribution function $F_{ho}(t)$ of the handoff call channel occupancy time as follows:

$$\begin{aligned}
F_{ho}(t) &= Pr(t_{ho} \le t) = Pr(t_c \le t) + \left[1 - Pr(t_c \le t)\right]F(t) \\
&= 1 - e^{-\mu t} + e^{-\mu t}\left[1 - \sum_{i=0}^{m-1}\frac{(\alpha t)^i}{i!}e^{-\alpha t}\right] \\
&= 1 - \left(\sum_{i=0}^{m-1}\frac{(\alpha t)^i}{i!}\right)e^{-(\alpha+\mu)t}.
\end{aligned} \tag{32}$$

It is important to compare the real distribution function of handoff call channel occupancy time and its exponential fitting. Since expected value must be the same, we use (13) to compute the expected value, and use its inverse as the parameter which determines the exponential function. Varying the shape parameter m from 2 to 5, we obtain the comparative plots in Fig. 3. From this figure, we observe that the exponential approximation is not good, and it becomes worse when m grows. As we know, the variance of the cell residence time is $1/(m\eta^2)$, which decreases when m increases. Hence, we can conclude that the exponential

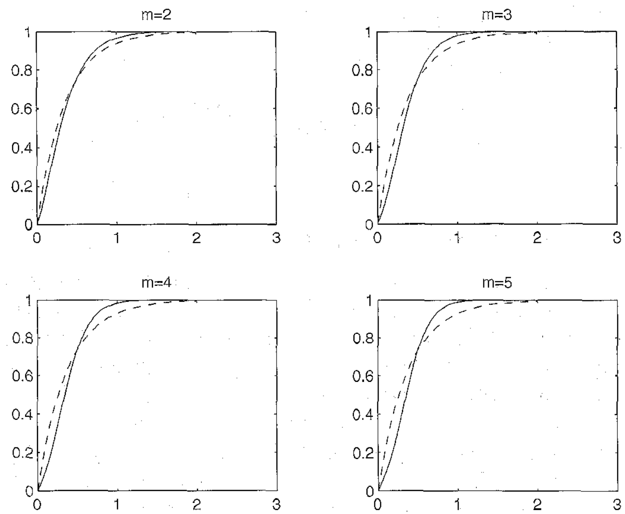


Fig. 3. Distribution of handoff call channel occupancy time (solid line) and its exponential fitting (dashed line).

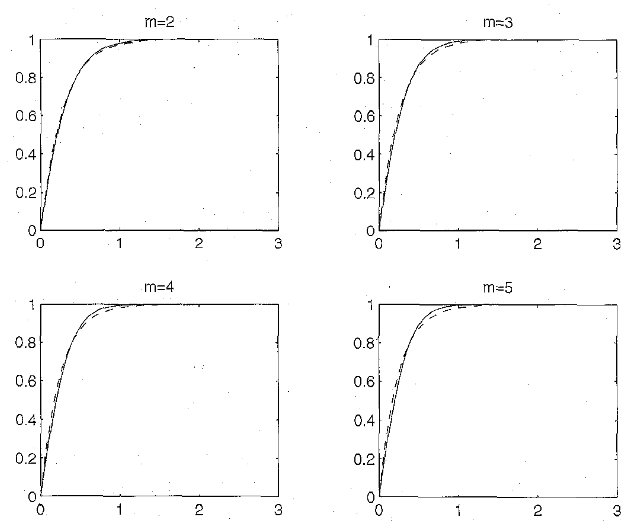


Fig. 4. Distribution of new call channel occupancy time (solid line) and its exponential fitting (dashed line).

approximation is not suitable for the handoff call channel occupancy time distribution when the variance of the cell residence time is very small.

Next, consider the new call channel occupancy times. Let $F_{no}(t)$ denote the distribution function of the new call channel occupancy time, then, from (1), we obtain

$$\begin{aligned}
F_{no}(t) &= Pr(t_c \le t) + [1 - Pr(t_c \le t)]Pr(r_1 \le t) \\
&= 1 - e^{-\mu t} + e^{-\mu t}\int_0^t f_r(\tau)d\tau \\
&= 1 - e^{\mu t} + e^{-\mu t}\left[1 - \int_t^{\infty} f_r(\tau)d\tau\right] \\
&= 1 - e^{-\mu t}\int_t^{\infty} f_r(\tau)d\tau
\end{aligned}$$

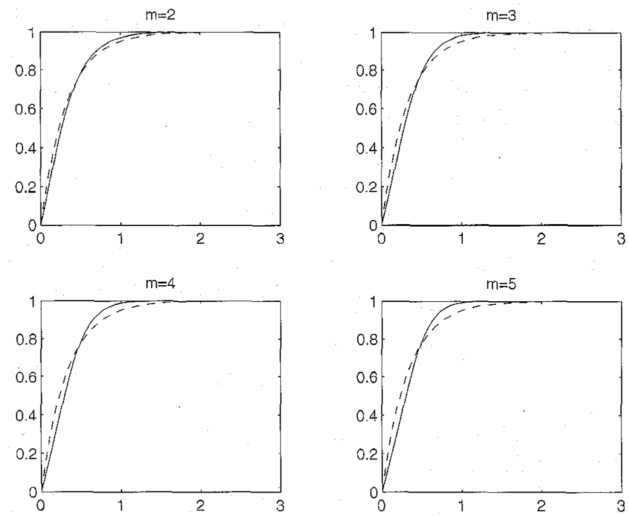


Fig. 5. Distribution of channel occupancy time (solid line) and its exponential fitting (dashed line).

$$\begin{aligned}
&= 1 - \eta e^{-\mu t}\int_t^{\infty}[1 - F(\tau)]d\tau \\
&= 1 - \eta e^{-\mu t}\int_t^{\infty}\sum_{i=0}^{m-1}\frac{(\alpha t)^i}{i!}e^{-\alpha\tau}d\tau \\
&= 1 - \eta e^{-\mu t}\sum_{i=0}^{m-1}\frac{1}{\alpha}\left(\sum_{k=0}^{i}\frac{(\alpha t)^k}{k!}\right)e^{-\alpha t} \\
&= 1 - \frac{1}{m}\sum_{i=0}^{m-1}(m-i)\frac{(\alpha t)^i}{i!}e^{-(\mu+\alpha)t} \quad (\text{noticing that } \alpha = m\mu) \\
&= 1 - \sum_{i=0}^{m-1}\left(1 - \frac{i}{m}\right)\frac{(\alpha t)^i}{i!}e^{-(\mu+\alpha)t}. \qquad (32)
\end{aligned}$$

From Fig. 4, we observe that the new call channel occupancy time is less sensitive to the variance of cell residence times than the handoff call channel occupancy time, and the exponential fitting for new call channel occupancy time is better than the handoff call channel occupancy time.

From (32), (33), and Theorem 2, we can obtain the distribution $F_{co}(t)$ of channel occupancy time as follows:

$$\begin{aligned}
F_{co}(t) &= \frac{\lambda}{\lambda + \lambda_h}F_{no}(t) + \frac{\lambda_h}{\lambda + \lambda_h}F_{ho}(t) \\
&= 1 - \sum_{i=0}^{m-1}\left(1 - \frac{\lambda i}{m(\lambda + \lambda_h)}\right)\frac{(\alpha t)^i}{i!}e^{-(\mu+\alpha)t}. \qquad (34)
\end{aligned}$$

Fig. 5 shows the channel occupancy time (used in calculating the blocking probabilities). We observe that the channel occupancy time can be appropriately approximated by the exponential distribution when $m = 2$. However, there are significant discrepancies between the distributions of the actual channel occupancy time and the exponential distribution approximations when m becomes larger (i.e., the variance of the cell residence times becomes small).

Figs. 6, 7, and 8 show the distributions of the new call channel occupancy time, handoff call channel occupancy time, and channel occupancy time, respectively, when the

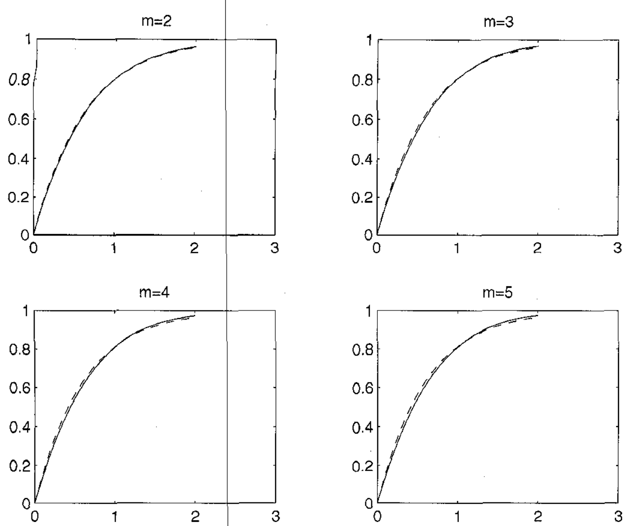


Fig. 6. Distribution of new call channel occupancy time (solid line) and its exponential fitting (dashed line): small mobility η/μ.

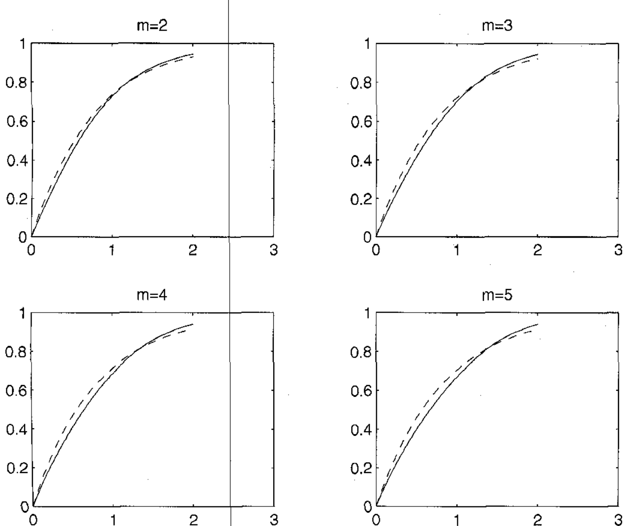


Fig. 7. Distribution of handoff call channel occupancy time (solid line) and its exponential fitting (dashed line): small mobility η/μ.

mobility η/μ is small (i.e., the customers are less mobile than the previous case). We observe that the new call channel occupancy time distribution has a good approximation by the exponential distribution, while there is still a significant mismatch between the handoff call channel occupancy time distribution and the exponential distribution. However, the channel occupancy time distribution can be better approximated by the exponential distribution.

Finally, we turn our attention to the handoff rate. Assume that the call holding times are Erlang distributed. We shall use (26) and the recursive algorithms given at the end of the previous section. The cell residence times are Gamma distributed with the density function

$$f(t) = \frac{\beta^{\gamma} t^{\gamma-1}}{\Gamma(\gamma)} e^{-\beta t}, \; f^*(s) = \left(\frac{\beta}{s+\beta}\right)^{\gamma}, \; \beta = \gamma\eta,$$

where γ is the shape parameter, β is the scale parameter,

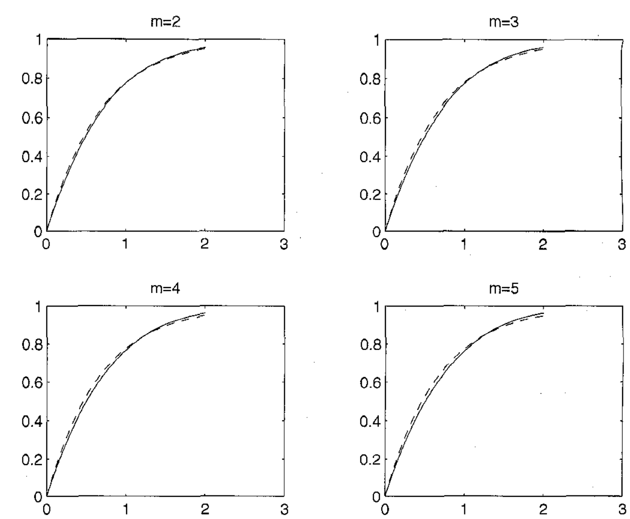


Fig. 8. Distribution of channel occupancy time (solid line) and its exponential fitting (dashed line): small mobility η/μ.

and the $\Gamma(\gamma)$ is the Gamma function. The mean and variance of this distribution are $1/\eta$ and $1/(\gamma\eta^2)$, respectively.

Figs. 9 and 10 show the handoff rate for different blocking probabilities and forced termination probabilities. We observe the following:

1) The handoff rate for fixed variance of the cell residence times (i.e., fixed m) is increasing as the mobility η/μ increases;
2) The handoff rate is increasing for fixed mobility as the variance of the cell residence times decreases (i.e., m is increasing);
3) The handoff rate is smaller than in the ideal case (when there is no blocking and no forced termination), which confirms our earlier expectations and conforms with general intuition;
4) The handoff rate is insensitive to the variance of the cell residence times when mobility η/μ is small; while it is very sensitive to the variance of the cell residence time when the mobility η/μ is high;
5) When handoff calls are given priority over new calls, the variance of the cell residence times affects the handoff rate more significantly.

One most important observation needs to be emphasized as follows. Since the handoff call arrival rate is equal to the new call arrival rate times the handoff rate in a call (see (27)), we observe from Figs. 9 and 10 that, for high mobility users, the handoff rate is much greater for the Erlang distributed call holding times ($m > 1$) than that for exponential call holding time. This implies that the exponential assumption for call holding time underestimates the handoff rate, hence, the blocking probabilities. This suggests that MC/PCS network designers have to carefully consider the distribution model for call holding time and cell residence time in order to meet the blocking probability requirement: Exponential distribution approximation for cell residence times and call holding times may not be enough for the real MC/PCS design.

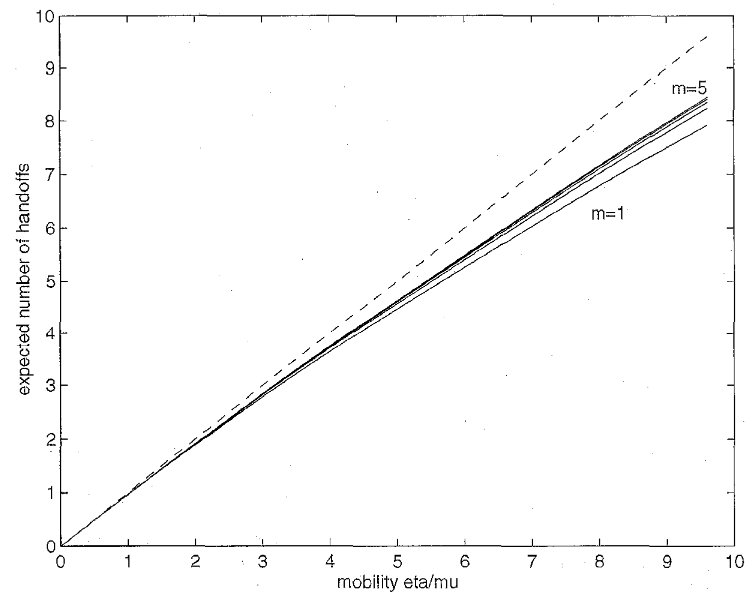


Fig. 9. Handoff rate: solid lines for $m = 1 - 5$ with $p_f = p_o = 0.02$, dashed line for the ideal case ($p_o = p_f = 0$).

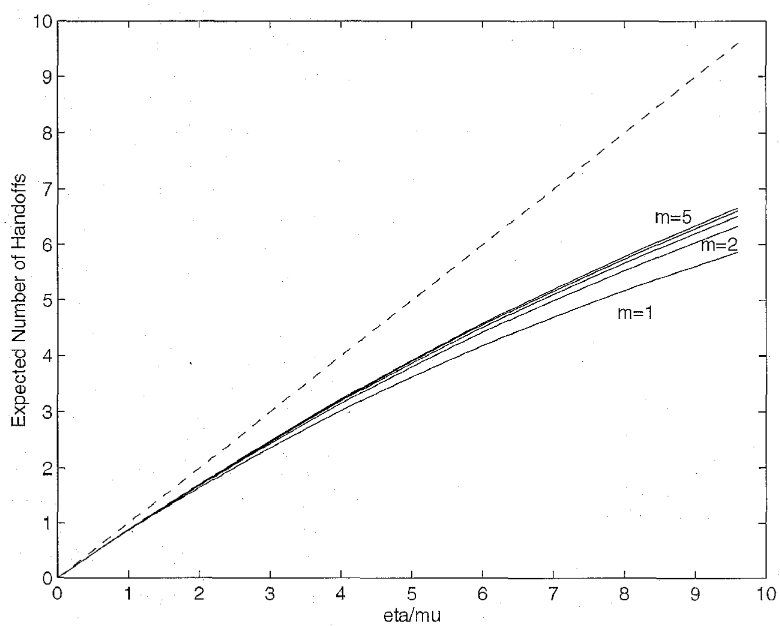


Fig. 10. Handoff rate: solid lines for $m = 1 - 5$ with $p_o = 0.1$ and $p_f = 0.05$, dashed line for the ideal case ($p_o = p_f = 0$).

6 Conclusions

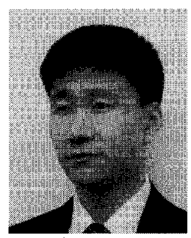
This paper investigates the channel occupancy times and handoff rates (the expected number of handoffs) for GSM based mobile computing networks and integrated MC/PCS networks. It has been shown that, except for the case of exponentially distributed cell residence times, the channel occupancy time is not exponentially distributed, the handoff traffic is not Poisson, and the merged call traffic to a cell is not Poisson, contrary to commonly made assumptions in the past. Analytical expressions were derived for channel occupancy time distributions, and comparisons were provided with channel occupancy time distributions and the exponential fitting. The results presented here can be used to evaluate the goodness of the exponential distribution approximation. The paper also studied handoff rate in a call and handoff call arrival rate to a cell; general analytical results for these two quantities under generally distributed call holding times and generally distributed cell residence times are provided. The above results were demonstrated to be useful in the traffic study, performance evaluation, management, and billing of MC/PCS networks.

ACKNOWLEDGMENTS

The authors would like to express their appreciation to the anonymous reviewers for their constructive comments which greatly improve the quality of the paper. The work of Yuguang Fang and Imrich Chlamtac was supported in part by the U.S. Army Research Office under Contract DAAH04-96-1-0308. The work of Yi-Bing Lin was supported in part by the National Science Council in Taiwan under Contract No. NSCNSC87-2213-E009-014.

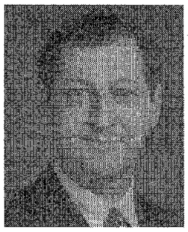
REFERENCES

[1] F. Barcelo and J. Jordan, "Channel Holding Time Distribution in Cellular Telephony," *Proc. Ninth Int'l Conf. Wireless Comm. (Wireless '97)*, vol. 1, pp. 125-134, Alberta, Canada, 9-11 July 1997.
[2] V.A. Bolotin, "Modeling Call Holding Time Distributions for CCS Network Design and Performance Analysis," *IEEE J. Selected Areas in Comm.*, vol. 12, no. 3, pp. 433-438, 1994.
[3] P.J. Burke, "The Output of a Queueing System," *Operations Research*, vol. 4, pp. 699-704, 1956.
[4] D.C. Cox, "Wireless Personal Communications: What Is It?" *IEEE Personal Comm. Magazine*, pp. 20-35, Apr. 1995.
[5] E. Del Re, R. Fantacci, and G. Giambene, "Handover and Dynamic Channel Allocation Techniques in Mobile Cellular Networks," *IEEE Trans. Vehicular Technology*, vol. 44, no. 2, pp. 229-237, 1995.
[6] ETSI/TC, *Mobile Application Part (MAP) Specification*, version 4.8.0, Recommendation GSM 09.02, 1994.
[7] Y. Fang, I. Chlamtac, and Y.B. Lin, "Modeling PCS Networks Under General Call Holding Times and Cell Residence Time Distributions," *IEEE Trans. Networking*, vol. 5, no. 6, pp. 893-906, Dec. 1997.
[8] Y. Fang, I. Chlamtac, and Y.B. Lin, "Call Performance for a PCS Networks," *IEEE J. Selected Area in Comm.*, vol. 15, no. 8, pp. 1,568-1,581, Oct. 1997.
[9] G.J. Foschini, B. Gopinath, and Z. Miljanic, "Channel Cost of Mobility," *IEEE Trans. Vehicular Technology*, vol. 42, no. 4, pp. 414-424, 1993.
[10] R.A. Guerin, "Channel Occupancy Time Distribution in a Cellular Radio System," *IEEE Trans. Vehicular Technology*, vol. 35, no. 3, pp. 89-99, 1987.
[11] D. Hong and S.S. Rappaport, "Traffic Model and Performance Analysis for Cellular Mobile Radio Telephone Systems with Prioritized and Non-Prioritized Handoff Procedures," *IEEE Trans. Vehicular Technology*, vol. 35, no. 3, pp. 77-92, 1986.
[12] C. Jedrzycki and V.C.M. Leung, "Probability Distribution of Channel Holding Time in Cellular Telephony Systems," *Proc. IEEE Vehicular Technology Conf. (VTC '96)*, pp. 247-251, 1996.
[13] N.L. Johnson, *Continuous Univariate Distributions*, vol. 1. New York: John Wiley & Sons, 1970.
[14] J. Jordan and F. Barcelo, "Statistical Modelling of Channel Occupancy in Trunked PAMR Systems," *Proc. 15th Int'l Teletraffic Conf. (ITC '15)*, V. Ramaswami and P.E. Wirth, eds., pp. 1,169-1,178. Elsevier Science B.V., 1997.
[15] I. Katzela and M. Naghshineh, "Channel Assignment Schemes for Cellular Mobile Telecommunication Systems: A Comprehensive Survey," *IEEE Personal Comm.*, vol. 3, no. 3, pp. 10-31, June 1996.
[16] F.P. Kelly, *Reversibility and Stochastic Networks*. New York: John Wiley & Sons, 1979.
[17] F.P. Kelly, "Loss Networks," *Annals of Applied Probability*, vol. 1, no. 3, pp. 319-378, 1991.
[18] L. Kleinrock, *Queueing Systems: Theory*, vol. 1. New York: John Wiley & Sons, 1975.
[19] W.C.Y. Lee, *Mobile Cellular Telecommunications: Analog and Digital Systems*, second ed. New York: McGraw-Hill, 1995.
[20] W.R. LePage, *Complex Variables and the Laplace Transform for Engineers*. New York: Dover Publications, 1980.
[21] Y.B. Lin, "Reducing Location Update Cost in a PCS Network," *IEEE/ACM Trans. Networking*, vol. 5, no. 1, pp. 25-33, Feb. 1997.
[22] Y.B. Lin and I. Chlamtac, "A Model with Generalized Holding and Cell Residence Times for Evaluating Handoff Rates and Channel Occupancy Times in PCS Networks," *Int'l J. Wireless Information Networks*, vol. 4, no. 3, pp. 163-171, 1997.
[23] Y.B. Lin, S. Mohan, and A. Noerpel, "Queueing Priority Channel Assignment Strategies for Handoff and Initial Access for a PCS Network," *IEEE Trans. Vehicular Technology*, vol. 43, no. 3, pp. 704-712, 1994.
[24] Y.B. Lin, A. Noerpel, and D. Harasty, "The Sub-Rating Channel Assignment Strategy for PCS Hand-Offs," *IEEE Trans. Vehicular Technology*, vol. 45, no. 1, Feb. 1996.
[25] F. Mademann, "General Packet Radio Services," *Telecom Report*, vol. 18, pp. 42-45, 1995.
[26] J. Medhi, *Stochastic Models in Queueing Theory*. New York: Academic Press, 1991.
[27] S. Nanda, "Teletraffic Models for Urban and Suburban Microcells: Cell Sizes and Handoff Rates," *IEEE Trans. Vehicular Techology*, vol. 42, no. 4, pp. 673-682, 1993.
[28] A.R. Noerpel, Y.B. Lin, and H. Sherry, "PACS: Personal Access Communications System—A Tutorial," *IEEE Personal Comm.*, vol. 3, no. 3, pp. 32-43, June 1996.
[29] J.E. Padgett, C.G. Gunther, and T. Hattori, "Overview of Wireless Personal Communications," *IEEE Comm. Magazine*, pp. 28-41, Jan. 1995.
[30] R. Steedman, "The Common Air Interface MPT 1375," *Cordless Telecomm. in Europe*, W.H.W. Tuttlebee, ed. New York: Springer-Verlag, 1990.
[31] S. Tekinay and B. Jabbari, "A Measurement-Based Prioritization Scheme for Handovers in Mobile Cellular Networks," *IEEE J. Selected Areas Comm.*, vol. 10, no. 8, pp. 1,343-1,350, 1992.
[32] W.C. Wong, "Packet Reservation Multiple Access in a Metropolitan Microcellular Radio Environment," *IEEE J. Selected Areas Comm.*, vol. 11, no. 6, pp. 918-925, 1993.
[33] C.H. Yoon and C.K. Un, "Performance of Personal Portable Radio Telephone Systems With and Without Guard Channels," *IEEE J. Selected Areas Comm.*, vol. 11, no. 6, pp. 911-917, 1993.
[34] T.S. Yum and K.L. Yeung, "Blocking and Handoff Performance Analysis of Directed Retry in Cellular Mobile Systems," *IEEE Trans. Vehicular Technology*, vol. 44, no. 3, pp. 645-650, 1995.

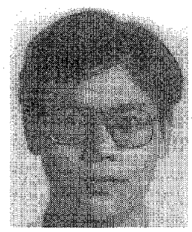


Yuguang Fang (S'92-M'94-S'96-M'98) received the BS and MS degrees in mathematics from Qufu Normal University, Qufu, Shandong, People's Republic of China, in 1984 and 1987, respectively, the PhD degree in systems and control engineering from the Department of Systems, Control, and Industrial Engineering at Case Western Reserve University, Cleveland, Ohio, in January 1994, and the PhD degree in wireless networks and mobile communications from the Department of Electrical and Computer Engineering at Boston University in May 1997.

From 1987 to 1988, he held research and teaching positions in both the Department of Mathematics and the Institute of Automation at Qufu Normal University. From September 1989 to December 1993, he was a teaching and research assistant in the Department of Systems, Control, and Industrial Engineering at Case Western Reserve University, where he held a research associate position from January 1994 to May 1994. He held a postdoctoral position in the Department of Electrical Engineering at Boston University from June 1994 to August 1995. From September 1995 to May 1997, he was a research assistant in the Department of Electrical and Computer Engineering at Boston University. Since June 1997, he has been a visiting assistant professor in the Department of Electrical Engineering at the University of Texas at Dallas. He has published more than 30 papers in professional journals and refereed conferences. He is the area editor for *ACM Mobile Computing and Communications Review*, the program vice-chair for the Seventh International Conference on Computer Communications and Networking (IC^{3}N '98), and a member of the technical program committee for the 1998 International Conference on Computer Communications (INFOCOM '98). His research interests include wireless networks and mobile communications, personal communication services (PCS), stochastic and adaptive systems, hybrid systems in integrated communications and controls, robust stability and controls, nonlinear dynamical systems, and neural networks. He is a member of the IEEE.



Imrich Chlamtac (M'86-SM'86-F'93/ACM F'96) received his PhD in computer science from the University of Minnesota (1979), and his BSc and MSc degree in mathematics awarded with the Highest Distinction. He currently holds the Distinguished Chair in Telecommunications at the University of Texas at Dallas, and is the president of Boston Communications Networks. He is the author of more than 200 papers in refereed journals and conferences, multiple books, and book chapters. Dr. Chlamtac is the founding editor-in-chief of the *ACM-URSI-Baltzer Wireless Networks* (*WINET*) and the *ACM-Baltzer Mobile Networking and Nomadic Applications* (*MONET*) journals. He served on the editorial board of *IEEE Transactions on Communications* and other leading journals. He served as the general chair and program chair of several ACM and IEEE conferences and workshops, was a Fulbright Scholar, and an IEEE, Northern Telecom, and BNR distinguished lecturer. He is the founder of ACM/IEEE MobiCom and of the ACM Sigmobile, of which he is the current chairman. Dr. Chlamtac is an IEEE fellow and an ACM fellow and is an honorary member of the senate of the Technical University of Budapest.



Yi-Bing Lin (S'80-M'96-SM'96) received his BSEE degree from National Cheng Kung University in 1983, and his PhD degree in computer science from the University of Washington in 1990. From 1990 to 1995, he was with the Applied Research Area at Bell Communications Research (Bellcore), Morristown, New Jersey. In 1995, he was appointed a professor in the Department of Computer Science and Information Engineering (CSIE), National Chiao Tung University (NCTU). In 1996, he was appointed as deputy director of the Microelectronics and Information Systems Research Center, NCTU. In 1997, he was elected chairman of CSIE, NCTU. His current research interests include design and analysis of personal communications services network, mobile computing, distributed simulation, and performance modeling.

Dr. Lin is a subject area editor of the *Journal of Parallel and Distributed Computing*, an associate editor of the *International Journal in Computer Simulation*, an associate editor of *IEEE Network*, an associate editor of *SIMULATION* magazine, an area editor of *ACM Mobile Computing and Communication Review*, a columnist of *ACM Simulation Digest*, a member of the editorial board of the *International Journal of Communications Systems*, a member of the editorial board of *ACM/Baltzer Wireless Networks*, a member of the editorial board of *Computer Simulation Modeling and Analysis*, an editor of *Journal of Information Science and Engineering*, program chair for the Eighth Workshop on Distributed and Parallel Simulation, general chair for the Ninth Workshop on Distributed and Parallel Simulation, program chair for the Second International Mobile Computing Conference, the publicity chair of ACM Sigmobile, guest editor for the *ACM/Baltzer MONET* special issue on Personal Communications, and guest editor for *IEEE Transactions on Computers* special issue on mobile computing. He is a senior member of the IEEE.