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The Analysis of Eigenvalue Assignment Robustness

Yuguang Fang

Abstract—**In this short note, we have shown that the results obtained recently in [1] are conservative. A new generalization which can overcome such conservatism is presented.**

In [1], Wang and Lin studied the robust eigenvalue assignment for systems with parameter perturbations via matrix measures. They used three special matrix measures to obtain testable conditions for robust eigenvalue assignment. In this short note, we will show how conservative their results are and derive a new generalization which overcomes the conservative nature of their approach.

To facilitate the discussion, we first give the general concept of a matrix measure. Let C^n (R^n) denote all ordered n-tuples of complex (real) numbers and $C^{n\times n}$ ($R^{n\times n}$) represent the set of all $n \times n$ matrices with complex (real) entries. Let $\|\cdot\|$ be any vector norm on C^n and $\|A\|$ denote the matrix norm of A induced by the vector norm $\|\cdot\|$, the corresponding matrix measure of A induced by the vector norm $\|\cdot\|$ is defined by

$$\mu(A) = \lim_{\xi \downarrow 0^+} \frac{\|I + \xi A\| - 1}{\xi}.$$

Detailed properties can be found in [2]. As in [1], let H denote the complex left-half plane divided by a line L, which intersects with the real axis at $(\alpha, 0)$ and has an angle θ with the imaginary axis, i.e.,

$$H = \{z = x + jy \mid y < \alpha - (\tan\theta)x\}.$$

The following theorem is the main result obtained by Wang and Lin [1].

Theorem 1: All the eigenvalues of the matrix A are located in the region H if

$$\mu_p\left(e^{-j\theta}A\right) < \alpha\cos\theta$$

where $j = \sqrt{-1}$, μ_p denote the matrix measure induced by the p-norm and $p = 1$ or 2 or ∞.

The following example illustrates the conservative nature of Theorem 1.

Example 1: Choose H to be the left-half plane (i.e., $\theta = 0$ and $\alpha = 0$). Then Theorem 1 reduces to a stability test result. Let $A = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$. It is obvious that A is stable (or its eigenvalues are in H). Theorem 1, however, cannot be used to test the stability of A. In fact, according to Table I in [1], we have

$$\mu_1(A) = 1, \quad \mu_2(A) = 0, \quad \mu_\infty(A) = 1$$

thus the conditions in Theorem 1 can not be satisfied.

To overcome this problem, we generalize Theorem 1 to the following result.

Theorem 2: All the eigenvalues of the matrix A are located in the region H if there exists a matrix measure μ such that

$$\mu\left(e^{-j\theta}A\right) < \alpha\cos\theta.$$

To prove this, we only need the following property of a matrix measure.

Lemma: For any matrix measure μ, any complex number c and any matrix A, we have

$$\mu(A + cI) = \mu(A) + \Re(c)$$

where $\Re(c)$ denotes the real part of c.

Proof: If c is complex, let $c = \Re(c) + bj$, then we have $\mu(A + cI) = \mu(A + bjI) + \mathrm{Re}(c)$, where $j = \sqrt{-1}$. So it suffices to show that $\mu(A + bjI) = \mu(A)$. In fact

$$\begin{aligned}
&\mu(A + bjI) \\
&= \lim_{\theta\downarrow 0^+} \frac{\|I + \theta(A + bjI)\| - 1}{\theta} \\
&= \lim_{\theta\downarrow 0^+} \frac{|1 + b\theta j|\|I + \frac{\theta}{1+b\theta j}A\| - 1}{\theta} \\
&= \lim_{\theta\downarrow 0^+} \frac{\sqrt{1 + b^2\theta^2}\|I + \frac{\theta}{1+b^2\theta^2}A - \frac{bj}{1+b^2\theta^2}A\theta^2\| - 1}{\theta} \\
&= \lim_{\theta\downarrow 0^+} \frac{\sqrt{1 + b^2\theta^2}\|I + \frac{\theta}{1+b^2\theta^2}A\| - 1}{\theta} = \mu(A)
\end{aligned}$$

where we have used the fact that, as $\theta \downarrow 0^+$, $\theta/(1 + b^2\theta^2) \downarrow 0^+$. This completes the proof of Lemma. □

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The author is with the Department of Electrical, Computer and Systems Engineering, Boston University, Boston, MA 02215 USA.
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Using this lemma, we can easily prove Theorem 2.

Proof of Theorem 2: Using the above lemma, we have

$$\begin{aligned}\mu(e^{-j\theta}(A-\alpha I)) &= \mu(e^{-j\theta}A - \alpha e^{-j\theta}I)\\ &= \mu(e^{-j\theta}A) + \Re(-\alpha e^{-j\theta})\\ &= \mu(e^{-j\theta}A) - \alpha\cos\theta.\end{aligned}$$

Using a similar procedure as in [1], we can finish the proof.

Now, we apply this result to the previous example. Choose the vector norm $\|x\| = \|Tx\|_1$, where $T = \begin{pmatrix}1 & 0\\ 0 & 4\end{pmatrix}$, then the induced matrix measure is given by

$$\mu(A) = \mu_1(TAT^{-1}) = \mu_1\left(\begin{pmatrix}-1 & 0.5\\ 0 & -1\end{pmatrix}\right) = -0.5 < 0$$

hence from Theorem 2, we can conclude that A is stable. This illustrates that Theorem 2 is an improvement over Wang and Lin's result (Theorem 1) and can provide a way to overcome the conservatism of Theorem 1.

Because all the rest of the results presented in [1] were derived from Theorem 1, they too can be modified in a similar manner. The following result is for the generalized stability of a perturbed system.

Theorem 3: All the eigenvalues of $A + E$ remain in the region H if there exists a matrix measure μ (possibly depending on the perturbation E) such that

$$\|E\| < -\mu(e^{-j\theta}A) + \alpha\cos\theta.$$

It is observed that for any nonsingular matrix T and any vector norm $\|\cdot\|$ with the induced matrix measure μ, the matrix measure μ_T induced by the vector norm $\|Tx\|$ is given by $\mu_T(A) = \mu(TAT^{-1})$. By specifying the matrix measure μ and the transformation T, the conservative nature of the results presented in [1] can be overcome. Usually it is enough to choose μ to be one of the matrix measures μ_1, μ_2 or μ_∞, and to choose the transformation T appropriately. We formalize this observation as a corollary.

Corollary: For a nonsingular matrix T, define

$$\tilde{\mu}_i(A) = \mu_i(TAT^{-1}), \quad i = 1, 2, 3.$$

Then the following results are true:

i) All the eigenvalues of the matrix A are located in the region H if there is a nonsingular matrix T such that for $i = 1$ or 2 or ∞

$$\tilde{\mu}_i(e^{-j\theta}A) < \alpha\cos\theta.$$

ii) All the eigenvalues of $A + E$ remain in the region H if there is a nonsingular matrix T such that for $i = 1$ or 2 or ∞

$$\|TET^{-1}\|_i < -\tilde{\mu}_i(e^{-j\theta}A) + \alpha\cos\theta.$$

How to appropriately choose the transformation T to achieve the best result is still open and currently under investigation. The general guideline is to solve a minimization problem. For example, for i), we can try to solve the minimization problem

$$\min_{T\in\mathcal{N}} \mu_i(TAT^{-1})$$

where $\mathcal{N}$ is the set of nonsingular real matrices. If the minimum is less than $\alpha\cos\theta$, then all the eigenvalues will be in the region H.

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Stability and Performance of a Simple Distributed Tracking Policy for Production Control of Manufacturing Systems

Ali Sharifnia

***Abstract*— The objective of distributed tracking is to operate a production system as closely as possible to an idealized regime obtained by a continuous-flow relaxation of the actual (discrete) production control problem. The stability and performance of a large class of distributed tracking policies called "non-idling–non-exceeding (NINE)" were investigated in an earlier paper. In this work we focus on the most natural tracking policy in this class and find a tight bound on its performance for a single machine and a sufficient condition for its stability for multiple-machine systems. This condition is considerably less stringent than the one available for general NINE policies.**

I. Introduction

In a recent paper we have investigated the stability and performance of a large class of distributed tracking policies, called "non-idling–non-exceeding (NINE)," for real-time production control of manufacturing systems. These policies are guided by the solution of a continuous-flow relaxation model of the actual (discrete) production control problem [6]. This solution can be found efficiently and provides "ideal" production target trajectories for individual operations over time. A problem of special interest is the potential instability of distributed tracking due to the discreteness of the actual control space. If unstable, the tracking error can grow without limit over time.

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The author is with the Department of Manufacturing Engineering, Boston University, Boston, MA 02215 USA.

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