Quantifying Benefits in a Business Portfolio for Multi-Operator Spectrum Sharing

Inosha Sugathapala, *Student Member, IEEE*, Ivana Kovacevic, *Student Member, IEEE*, Beatriz Lorenzo, *Member, IEEE,* and Savo Glisic, *Senior Member, IEEE*, and Yuguang Fang, *Fellow, IEEE*

***Abstract*—Benefits of multi-operator spectrum sharing in wireless networks heavily depend on the traffic misbalance in the networks belonging to different operators. In this paper, we study the likelihood that such misbalance occurs in networks with high traffic dynamics. An extensive business portfolio for heterogeneous networks is presented to analyse the benefits due to multi-operator cooperation for spectrum sharing. High resolution pricing models are developed to dynamically facilitate price adaptation to the system state. By using queuing theory, we quantify the operators' gains in cooperative arrangements as opposed to non-cooperative independent operation. In addition, Markov model is used that can handle wider range of different distributions of traffic arrivals and service rates. A tractable analysis and quantitative results are provided for those gains as a function of the number of cooperating operators. Under the condition that there is a traffic underflow in one band, it has been shown that with capacity aggregation model, the operator operating in other band can take advantage of additional channels with probability close to 1. In capacity borrowing/leasing model, this advantage is not unconditional, and there is a risk that the operator leasing the spectra will suffer temporary packet losses. When cognitive models are used in a network with high traffic dynamics, 50-70% of the spectra may be lost due to channel corruptions caused by the return of primary users. The gains of traffic offloading from a cellular network to a WLAN are quantified by an equivalent increase in opportunistic capacity proportional to the ratio of aggregate coverage of cellular networks and WLANs. The results provide guidelines for business decision in multi-operator network management.**

***Index Terms*—Multi-operator spectrum sharing, heterogeneous networks, cognitive networks, pricing, traffic misbalance.**

I. Introduction

Wireless networks are evolving towards highly populated ones, like Internet of things, where a vast variety of applications and access network technologies coexist [1]. An important problem in this scenario is the optimization of spectrum sharing by multiple operators [2]. The variations in spectrum usage, channel quality and coverage in different operators' networks generate plenty of cooperation opportunities, which can be exploited to improve the network performance. This problem has been analyzed in the literature by focusing on specific schemes mainly for cognitive networks [3]–[10].

In the case of cellular networks, centralized architectures [11]–[13] for dynamic spectrum access have gained a lot of interest. In such models, operators bid for spectrum through a spectrum manager by using auction based sharing techniques. However, the proposed techniques allocate spectrum at the system level rather than the cell level. Thus, the traffic variations at individual cells are not considered, which limits the gains achieved with such schemes.

In this paper, we consider the traffic variations between different operators in cellular networks and develop a tractable and accurate analytical model for quantifying the benefits under different cooperation strategies. For this purpose, we present an extensive business portfolio for wireless network operators and discuss macro-economics of multi-operator cooperative networks. Both analytical and simulation results are provided to show the enhancement in spectrum utilization and performance gains by the proposed schemes.

The contributions of this paper are summarized as follows.

1) A comprehensive business portfolio for multi-operator spectrum management is presented and analyzed. The portfolio consist of the following business models: a) Capacity aggregation-A model; b) Capacity borrowing/leasing-BL model; c) Cognitive networks- C model; d) Partial cognitive networks-PC model; e) Mutually cognitive networks-MC model; f) Asymmetrical spectral aggregation in heterogeneous network- CW model or Symmetrical spectral aggregation in heterogeneous network- CWC model; g) Economic models for BL system with pricing and, h) Economical models for mutual channel BL (MBL) with high resolution pricing. All these models are compared on the basis of spectra utilization factor, representing the average percentage of spectra being used by the operator.
2) A unified analytical model based on queuing theory is presented to quantify the performance of these business plans for voice and data traffic. Probability of user benefit is used to quantify the likelihood that a user is served by another operator while facing call blocking in its own network. Although the model admits general distributions for user arrival and service rates, the closed-form expressions are obtained for exponential inter-arrival and service times.
3) A number of new fine-grained pricing models are incorporated into the system model enabling the analysis of systems' micro-economics and user dissatisfaction with

I. Sugathapala, I. Kovacevic and S. Glisic are with the Centre for wireless Communication and Department of Communication Engineering, Faculty of Information Technology and Electrical Engineering, University of Oulu, Finland. Email: {inosha.sugathapala, ivana.kovacevic, savo.glisic}@ee.oulu.fi. B. Lorenzo is with the Department of Telematics, University of Vigo, Spain. Email: blorenzo@gti.uvigo.es. Y. Fang is with Department of Electrical and Computer Engineering, University of Florida. Email: fang@ece.ufl.edu

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the service. The pricing schemes adapt automatically to the traffic volume in the system and enable changing the price and user behavior (reneging) at each new user arrival/departure.

4) By introducing an equivalent service rate in a multi-operator system, existing results in queuing theory are modified to obtain analytical results in a tractable form.

The results provide two major guidelines for the future use of the spectrum: a) for the regulatory bodies, spectra aggregation is more efficient than the concept of cognitive networks; b) for the network operators, spectrum alliances are a more efficient way to compete with the powerful operators owning larger portions of the spectrum bandwidth.

The rest of the paper is organized as follows. In Section II, we provide an overview of the related work. In Section III, we present eight different business models for multi-operator spectrum management. Performance analysis based on queuing theory is carried out in Section IV. Numerical examples and implementation details are given in Section V. Concluding remarks are summarized in Section VI.

II. Related Work

One of the first papers that introduce spectrum sharing in cellular networks is [14] where several operators aim to take advantage of the fluctuations of the incoming traffic for an opportunistic allocation of unused resources. They focus on the minimization of the call blocking probability. In [15], authors discuss K-dimensional Markov channel model with complete sharing scheme for two cellular networks. The system performance in terms of blocking probabilities and system utilization is analyzed for handoff connections and new connections. Work in [16] considers integrated voice and data networks using Markovian models with priorities. Spectrum is shared by both data and voice users. In a similar way, dynamic spectrum sharing in WiMAX and WiFi network is considered in [17] by maintaining the required QoS. The blocking probabilities with limited queuing architecture are analyzed.

With small buffer size or small number of servers, it is easier to analyze the system performance and the results can be obtained in closed-form expressions. For example, in [15], [18] and [19], multi-dimensional Markov process is used to obtain results on the blocking probabilities. In [20], authors study the system performance using two-dimensional Markov chain with handover and new calls based on the Erlang B systems. In addition, Zeng and Chlamtac [21] demonstrate such a system with different arrival rates for handover and new calls, respectively. In this paper, in addition to the previous work, we consider the scenario of two different operators with different service rates and further modify the previous model by introducing an equivalent system service rate. This enables us to analyze various kinds of multi-operator joint spectrum management schemes by using a general queuing system. The tractability of the analytical model allows us to obtain meaningful results which provide insight into the design of spectrum sharing mechanism for future wireless networks.

In terms of spectrum sharing, cognitive radio is playing a significant role in the wireless industry and intensive research has been carried out the last few years. In [4], secondary and primary users share the same spectrum. When all channels are occupied, a newly arriving secondary user joins the queue according to the pre-define priority level. As expected, a primary user with higher priority level has lower waiting time and queuing length. Our paper extends the analysis of cognitive networks to capture the channel corruption due to the return of the primary user whose channel is currently used by a secondary user. In a network with high traffic dynamics, this event is an important contribution to the network performance degradation.

The problem of designing a secondary spectrum trading market when there are multiple sellers and multiple buyers is investigated in [5]. The paper proposes a coalitional double auction mechanism for efficient spectrum allocation. Kasbekar and Sarkar [6] develop an auction-based framework that allows networks to bid for primary and secondary access based on their utilities and traffic demands. The goal is either to maximize the auctioneer's revenue or the social welfare of the bidding network, while enforcing incentive compatibility.

The above works focus on the instantaneous network status and thus, they do not characterize the long term improvement. Our work fills in this gap and proposes spectral aggregation mechanisms where operators benefit from temporal misbalance in the traffic by allowing to each other to use their free channel without any compensation. The concept is based on the expectation that in long term all operators will benefit equally from such arrangement.

Research in [7] focuses on a femtocell communications market and studies the entrant network service provider's long-term decision: whether to enter the market and which spectrum sharing technology to select to maximize its profit. Singh et al. [9] consider a network in which several service providers offer wireless access to their respective subscribed customers through potentially multihop routes. In [2] different scheduling policies for spectrum sharing are proposed.

In this paper, we present new economic models for channel borrowing/leasing to compensate the operators for leasing the unused channels. Different fine-grained pricing models are considered to deal with the change of the price even due to the small changes of the state of the system characterized by the instantaneous number of active users in the overall available spectral band. The model also captures possible decisions of the unsatisfied users to leave the system, resulting in revenue losses of the operator.

III. Multi-operator Cooperation Models

In this section, we introduce definitions and a unified notation for eight different business models for multi-operator spectra management. We will consider first two different operators and then, in Section IV-B4 the analysis will consider the multi-operator case too.

The models are applicable to different type of the networks like cellular, heterogeneous mostly cellular/WLAN as well as different types of cognitive networks with primary/secondary status of the operators. More details on the practical implementation of each model will be provided in Section V.

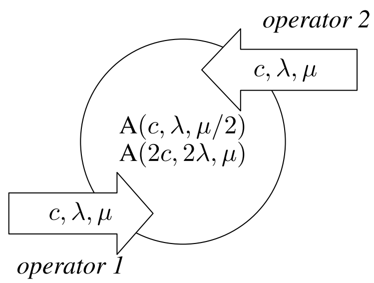

Fig. 1. Capacity aggregation – A model.

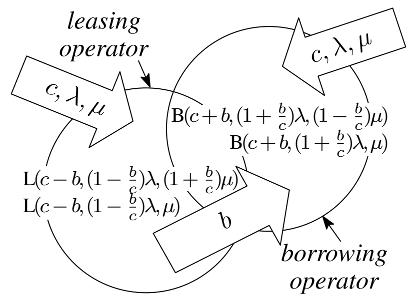

Fig. 2. Capacity borrowing/leasing – BL model.

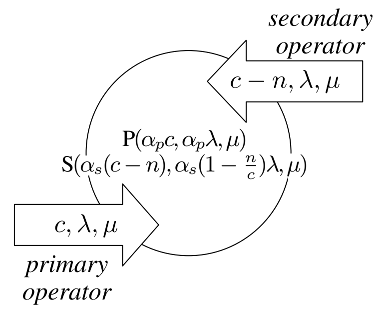

Fig. 3. Cognitive networks – C model.

A. Capacity aggregation – A model

We initially assume that each of the two operators has available c channels, and users with the same Poisson distributed arrivals with rate λ and the service rate μ [22]. They can operate independently in a non-cooperative way referred to as (c, λ, μ) *conventional* mode.

Alternatively, let us assume that each operator may make available c channels to the cooperative operation of a cellular network resulting in an aggregate number of channels $2c$, aggregate arrival rate 2λ and same service rate μ. This will be referred to as $A(2c, 2\lambda, \mu)$ aggregation model, which is symbolically presented in Fig. 1. Joining the bandwidths from the two operators may enable, for the same number of channels c and aggregate arrival rate, to reduce the service rate $\mu \rightarrow \mu/2$ (longer data messages). This will result into the *aggregation* model referred to as $A(c, \lambda, \mu/2)$. The generalization of the model to include more than two operators is straightforward although some additional details might be included like unequal contribution to the channel aggregation, as well as, unequal arrival and service rates.

The above notation in the form of the triplets (*capacity, aggregate arrival rate, service rate*) will be used throughout the paper. Occasionally, we will also use an extension of the triplet to include different arrival and service rates.

B. Capacity borrowing/leasing – BL model

The second business plan is based on the assumption that one operator is leasing b channels to another operator who is borrowing. This principle is extensively used for inter-cell traffic balancing in conventional cellular networks. In conventional, non-cooperative mode of operation this creates two independent systems referred to as $L(c-b, \lambda, \mu)$ and $B(c+b, \lambda, \mu)$. As the first step, leasing operator and borrowing operator in the non-cooperative mode can readjust the models to $L(c-b, \lambda(1-b/c), \mu)$ and $B(c+b, \lambda(1+b/c), \mu)$, respectively. Consequently, the L operator who has now reduced number of channels will proportionally reduce the arrival rate of the users. In practice, these corrections will take place in the reverse order. The operator who experiences the drop in the average arrival rate will offer to lease proportional number of channels b. Similarly, operator B who is experiencing higher arrival rates would be interested to borrow additional b channels. Instead of readjusting the arrival rate, the operators may opt for adjusting the service rate resulting into $L(c-b, \lambda, \mu(1+b/c))$ (shorter messages) and $B(c+b, \lambda, \mu(1-b/c))$ (longer messages). The combination of both is also possible, resulting into $L(c-b, \lambda(1-b/c), \mu(1+b/c))$ and $B(c+b, \lambda(1+b/c), \mu(1-b/c))$. In general, all these options can be formally represented as $BL(2c, \lambda_1, \lambda_2, \mu_1, \mu_2)$ system or equivalently, as $BL(2c, \lambda_1, \lambda_2, \mu_{eq})$ system with $\mu_{eq} = \lambda_1\mu_1/(\lambda_1+\lambda) + \lambda_2\mu_2/(\lambda_1+\lambda_2)$. This system is illustrated in Fig. 2. The equivalent service rate model will be used in the analysis of all system parameters based on the system state probability distribution function (Section IV-A), system delay (Section IV-A1) and the CDF of the system delay (Section IV-A2). Parameter $\lambda_1/(\lambda_1+\lambda)$ represents the probability that the user being served belongs to $operator 1$ and $\lambda_2/(\lambda_1+\lambda)$ that the user being served belongs to $operator 2$. With this interpretation, μ_{eq} represents the average (or equivalent) service rate of the user in the system. The introduction of μ_{eq} enables us to analyze the general queuing system with multiple operators and arbitrary arrival and service rates which was not possible by using existing queuing theory tools.

C. Cognitive networks – C model

In a cognitive network, secondary users (SUs) operating under secondary service provider (S), are sensing the spectra and potentially using $c-n$ channels. As before, c is the number of available channels and n the number of channels instantaneously occupied by the primary user (PU) operating under the primary service provider (P). A given channel will be corrupted either if the SU incorrectly detects the occupied channel as free or if the PU returns to the channel while used by the SU.

If the probability of channel corruption is $1-\alpha$ then, the effective number of channels is αc. Parameter α will be discussed in more details in Section IV-C. We suggest to characterize this phenomena by two parameters α_p and α_s representing the capacity reduction parameters for primary service provider (P) and secondary service provider (S), respectively. Then, P and S operate as $P(\alpha_p c, \alpha_p \lambda, \mu)$ and $S(\alpha_s(c-n), \lambda(\alpha_s(1-n/c)), \mu)$. This model is illustrated in Fig. 3.

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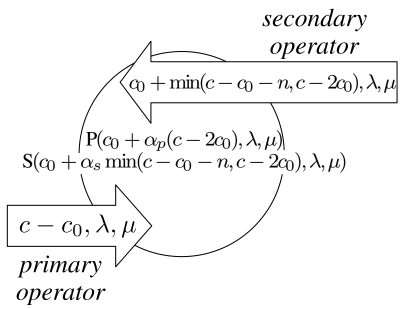

Fig. 4. Partial cognitive networks – PC model.

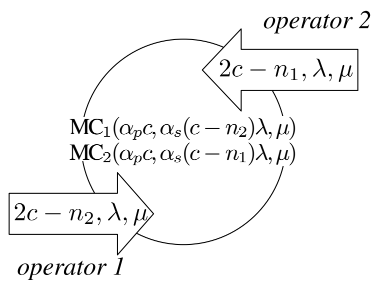

Fig. 5. Mutually cognitive networks – MC model.

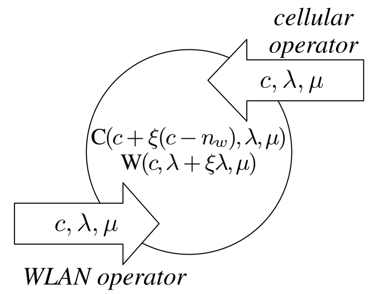

Fig. 6. Spectra Aggregation in Heterogeneous Network - CW mode.

$$P(c_0 + \alpha_p(c - 2c_0), \lambda, \mu), S(c_0 + \alpha_s \min(c - c_0 - n, c - 2c_0), \lambda, \mu), \tag{1a}$$

$$P(c_0 + \alpha_p(c - 2c_0), (c_0 + \alpha_p(c - 2c_0))\frac{\lambda}{c}, \mu), S(c_0 + \alpha_s \min(c - c_0 - n, c - 2c_0), (c_0 + \alpha_s \min(c - c_0 - n, c - 2c_0))\frac{\lambda}{c}, \mu) \tag{1b}$$

D. Partial cognitive networks – PC model

In non-cooperative mode, we assume that both, P and S operator, reserve c_0 non-cognitive channels each for their exclusive use. The remaining $c-2c_0$ cognitive channels are used by P operator and $c-2c_0-(n-c_0)$ cognitive channels can be potentially used by S operator. This results in the following business plans for the two operators, where (1a) indicates that primary operator P is using c_0 channels exclusively and the remaining $c-2c_0$ channels may be corrupted due to imperfect sensing of the channel by the SUs. This is characterized by the corruption factor in the primary channel by α_p. Then, the secondary operator S will use also c_0 channels exclusively plus the remaining channels not used by the PUs but reduced by the corruption factor α_s due to the PU return to the channel. An alternative to this model is described in (1b) where the arrival rates are increased by the operator proportional to the effective spectrum used. The system is presented in Fig. 4.

E. Mutually cognitive networks – MC model

In this concept, we have two operators each having c channels available. When one operator receives a call request and all its c channels are occupied, it samples the band of the other operator. If there is a channel available in that band, it uses that channel for as long as it is available. In other words, it behaves as S operator in that band. This operation is formally designated as

$$MC_1(\alpha_p c + \alpha_s(c - n_2), \lambda, \mu); \tag{2a}$$
$$MC_2(\alpha_p c + \alpha_s(c - n_1), \lambda, \mu), \tag{2b}$$

where (2a) indicates that operator 1 is using c channels as primary operator. Since operator 2 can also use this band as the secondary operator, the effective capacity will be modified by the corruption coefficient in primary band α_p. Operator 1 can additionally use $c-n_2$ channels from the second operator with the status of secondary user in that band. Due to the channel return probability of operator 2 in that band, the effective number of available channels will be modified by factor α_s. Equation (2b) is based on the same arguments. This system is formally presented in Fig. 5.

F. Spectra Aggregation in Heterogeneous Network

In this model, we consider two operators each having c channels available in different type of networks. Let us assume that those operators are the cellular network and WLAN operators referred to as C and W operators, respectively. The coverage areas of the two operators are A_c and A_w, respectively. A_w refers to the overall coverage area of all WLANs within the cell. The traffic offloading coefficient for C operator is defined as $\xi_c = A_w/A_c$. Thus, a user of C operator will be in the position to offload the traffic to WLAN with probability ξ_c. For WLAN operator, $\xi_w = 1$ since C operator covers the whole area and W operator is always in position to offload its user to C network. In this scenario, we define two modes of operation:

a) CW mode refers to the case where only C operator is supposed to offload the traffic to WLAN network if such opportunity exists. This results into the following aggregation model

$$C(c + \xi_c(c - n_w), \lambda, \mu); \tag{3a}$$
$$W(c, \lambda + \xi_c\lambda, \mu), \tag{3b}$$

where (3a) indicates that C operator will have potentially $c-n_w$ channels available with n_w the number of channels used by W operator. These channels will be effectively available if the user is in the coverage area of the WLAN which is characterized by factor ξ_c resulting in an effective additional number of channels $\xi_c(c - n_w)$. Equation (3b) indicates that due to these offloads, the effective arrival rate in W network will be additional increased by $\xi_c\lambda$.

b) CWC mode includes the same opportunity of offloading the traffic in both directions from the C network to WLAN and the other way around. In the latter case, if one operator

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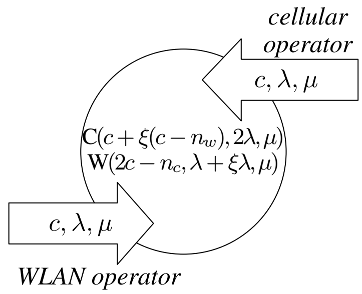


Fig. 7. Spectra Aggregation in Heterogeneous Network - CWC mode.

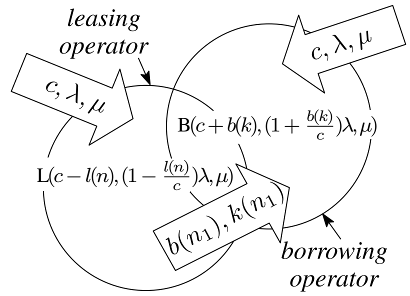


Fig. 8. Channel lending with pricing.

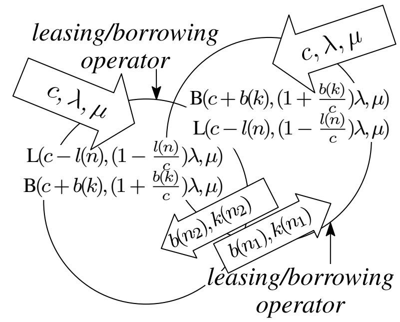


Fig. 9. Mutual channel leasing with pricing.

receives a call request when all its c channels are occupied it asks the other operator to use the channel in that band if available. This results into the following model

$$C(c+\xi_c(c-n_w), 2\lambda, \mu); \tag{4a}$$
$$W(2c-n_c, \lambda+\xi_c\lambda, \mu), \tag{4b}$$

where n_c is the number of users in C network. It should be noticed that in (4b) the increase in number of channels for W operator is $c-n_c$ and it is not modified by factor ξ_c. On the other hand, its overall arrival rate is not augmented by λ but only by $\xi_c\lambda$, since only ξ_c portion of C operator's terminals are covered by W operator. Both models are shown schematically in Fig. 6 and Fig. 7.

G. Channel borrowing/leasing (BL) with pricing

At a given moment, the state of the system is characterized by (n_1, n_2) where we assume that lending operator L has n_1 users in the system and the borrowing operator B has n_2 users. At this state of the system, the normalized price per user $k(n_1)$ required by the lender will depend on the state of the L system. This can be modeled by different pricing schemes as it will be discussed below. The B operator will make decision on whether or not to borrow the channel depending on both, $k(n_1)$ and n_2. Based on these decisions, the equivalent arrival rate of B operator will be modified to $\lambda_2 \to \lambda_2\big(k(n_1), n_2\big)$.

The behavior of this scheme is symbolically presented in Fig. 8. As we can see in the figure, after leasing $l(n)$ channels the L operator will have effectively reduced its capacity by the number of leased channels and may operate by proportionally reducing the arrival rate by $l(n)/c$. The B operator after borrowing $b(k) = l(n)$ will have the effective capacity increased by $b(k)$ channels and then, it can effectively increase the arrival rate by $b(k)/c$.

Possible options for the pricing model are given as

$$\begin{aligned} &\text{a) } k(n_1) = n_1/c; \qquad \text{b) } 1-k(n_1) = 1/(n_1+1);\\ &\text{c) } 1-k(n_1) = 1/(n_1^2+1), \end{aligned} \tag{5}$$

where in (5a) the price offered by L operator is proportional to the number of its occupied channels. In (5b) and (5c), there is different emphases on the starting price when $n_1 = 1$ and maximum price when $n_1 = c$. The possible reaction of the B operator to the price is modeled as

$$\lambda_2(k(n_1), n_2) = \begin{cases} \lambda_2, & n_2 < c, \\ (1-k(n_1))\lambda_2, & n_1 < c < n_2. \end{cases} \tag{6}$$

If the price continues to rise while the session is being served, the operator might decide to abort the transmission. If the transmission is not completed the service will not be charged. This will affect the equivalent service rate as follows $r(n) = \lim_{\Delta t \to 0} \Pr\{\text{unit reneges during } \Delta t \,|n \text{ customers present}\}/\Delta t$, $r(0) = r(1) \equiv 0$. This new process is still birth-death, but the death rate must now be adjusted to [23]

$$\mu_{n2} = \mu + r(n_1), \tag{7}$$
$$r(n_1) = e^{\alpha n_1/\mu_1}; \quad n_1 \geq 2. \tag{8}$$

A good possibility for the reneging function $r(n_1)$ is given by (8) where α is a constant. A waiting customer could estimate the average system waiting time as n_1/μ_1.

H. Mutual channel borrowing/leasing (MBL) with pricing

In this case, there is a possibility for both operators to lease/borrow the channels depending on the state of the system. In contrast to the previous model, now either of the two operators can be lender or borrower of the channels. The previous pricing models described by (5a)-(5c) can be applied in this case by assuming that operator 1 and 2 could act as leaser or borrower. In particular, (5a) could be modified as

$$k(n_1) = n_1/c; \quad n_1 < c \text{ and } c < n_2 < 2c; \tag{9a}$$
$$k(n_2) = n_2/c; \quad n_2 < c \text{ and } c < n_1 < 2c, \tag{9b}$$

where in (9a) operator 1 is the leaser since $n_1 < c$, and the operator 2 is the borrower since $c < n_2 < 2c$. Operator one is using price $k(n_1)$. In (9b) the same reasoning applies and in this case operator 1 is the borrower and 2 the leaser. The modification of (5a)-(5c) and the reaction to pricing given by (6) to this case are straightforward. If we assume that $\lambda_1 = \lambda_2 = \lambda$ then, the reaction to pricing for the borrowing operator (operator1) is

$$\lambda_2(k(n_1), n_2) = \begin{cases} \lambda, & n_2 < c, \\ (1-k(n_1))\lambda, & n_1 < c < n_2 < 2c, \end{cases} \tag{10a}$$

and similarly, for operator 2

$$\lambda_1(k(n_2), n_1) = \begin{cases} \lambda, & n_1 < c \\ (1-k(n_2))\lambda, & n_2 < c < n_1 < 2c, \end{cases} \tag{10b}$$

As before, if the price continues to rise while the session is being served the operator might decide to abort the transmission. If the transmission is not completed the service will not be charged. This will affect the equivalent service rate as in (7)-(8) when the borrower operator is operator 2. It is straightforward to obtain equivalent service rate for operator 1. The system is schematically presented in Fig. 9.

IV. Performance Analysis

In this section, we discuss system parameters used for the analysis of the different business plans. Voice and data applications are considered in parallel. We are interested to know under what condition an operator is able to temporally use a certain number of channels belonging to the other operator. In symmetrical conditions, when this opportunity is equal for both operators, there will be no need even for pricing since the average benefits in long run are the same. These advantages are quantified by the probability of benefiting (temporally gaining) the channels discussed in Section IV-B. If the two operators provide different QoS then different pricing is introduced by each operator as discussed in Section IV-D. An interesting relation is established in cognitive networks as discussed in Section IV-C where S operator benefits from the opportunity to temporally use the portion of the spectra unused by P operator. Although, there is a certain risk that effective number of channels due to channel corruptions will be reduced. Finally, channel utilization coefficient is used for the evaluation of the overall business portfolio as discussed in Section IV-E.

A. Preliminaries: Joint state probability distribution function for multi-operator system in A model

We start with some preliminary results which will be used in the rest of Section IV. Although derivation of these preliminary results might be cumbersome, the details are omitted since they are based on standard theory of Markov chains [23].

A two dimensional Markov chain is used to model the two operators system as illustrated in Fig. 10. Each state in the Markov chain represents the number of users (n_1, n_2) served by operator 1 and 2, respectively. There are in total $2c$ channels available for each operator. The arrival rate for operator 1 and 2 are denoted by λ_1, λ_2, respectively and the service rate by μ_1, μ_2, respectively.

For voice applications, first we need an expression for the joint state probability distribution function $P_{n_1 n_2}$ of a M/M/$2c$/$2c$ blocking system formed by two processes (n_1, n_2) representing the number of users being served by each operator, as shown in Fig. 10. For data applications, we use M/M/$2c$ queuing system formed by two processes (n_1, n_2). This expression as well as the expected queue size in $A(2c, \lambda_1, \lambda_2, \mu_{eq})$ system are derived in the sequel by using standard methods of Markov chain analysis. The formula for the CDF of the system waiting time is also derived.

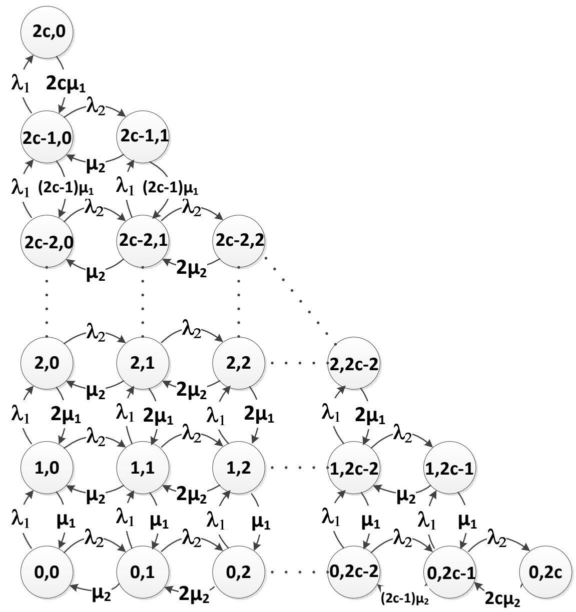


Fig. 10. Markov model representation of the two-operator voice traffic system.

1) Voice services in $A(2c, \lambda_1, \lambda_2, \mu_1, \mu_2)$: In this system, we have $2c$ channels and no queue which can be modeled as M/M/$2c$/$2c$ blocking system. Solving conventional birth-death equations for such system gives [24]:

$$\begin{aligned} P_{n_1 n_2} &= ((r_1^{n_1} r_2^{n_2})/(n_1! n_2!))P_0; \\ P_0 &= \left[\sum_{n_1=0}^{2c} (r_1^{n_1}/n_1!) \sum_{n_2=0}^{2c-n_1} (r_2^{n_2}/n_2!) \right]^{-1}, \end{aligned} \tag{11}$$

where P_0 is the probability of no user in service when $n_1 = n_2 = 0$. The remaining parameters are defined as

$$r_i = \lambda_i/\mu_i, i \in \{1,2\}; \qquad \rho_i = r_i/2c, i \in \{1,2\}. \tag{12}$$

Then, the total traffic intensity

$$\rho = \rho_1 + \rho_2 < 1; \qquad r = r_1 + r_2 = 2c\rho. \tag{13}$$

2) Data services in $A(2c, \lambda_1, \lambda_2, \mu_{eq})$: In this system, we assume $2c$ channels available and infinite length of the buffer which is modeled as M/M/$2c$ queuing system. One can show that for such system with $\mu_{eq} = \lambda_1\mu_1/(\lambda_1+\lambda_2) + \lambda_2\mu_2/(\lambda_1+\lambda_2)$, ρ_i, $i \in \{1,2\}$ given by (12), the total traffic ρ by (13) and $r_i = \lambda_i/\mu_{eq}$, $i \in \{1,2\}$ we have

$$\begin{aligned} P_{n_1 n_2} &= \begin{cases} \frac{(2c\rho)^{n_1+n_2}}{(n_1+n_2)!} P_0, & 0 \le (n_1+n_2) < 2c, \\ \frac{(2c\rho)^{n_1+n_2}}{(2c)^{(n_1+n_2-2c)}(2c)!} P_0, & (n_1+n_2) \ge 2c; \end{cases} \\ P_0 &= \left[\sum_{n_1=0}^{2c-1} \sum_{n_2=0}^{2c-n_1-1} \frac{(2c\rho)^{n_1+n_2}}{(n_1+n_2)!} + \frac{(2c\rho)^{2c}}{(2c)!(1-\rho)^2} + \frac{(2c\rho)^{2c}}{(2c-1)!(1-\rho)} \right]^{-1}. \end{aligned} \tag{14}$$

a) System Delay : The expected queue size in A$(2c, \lambda_1, \lambda_2, \mu_{eq})$ system is

$$\begin{aligned} L_q &= \frac{(2c\rho)^{2c} P_0 \rho}{(2c)!(1-\rho)^3} \left[(1+\rho) + 2c(1-\rho) \right]; \\ L_{q_1} &= \frac{\lambda_1}{\lambda_1+\lambda_2} L_q; \; L_{q_2} = \frac{\lambda_2}{\lambda_1+\lambda_2} L_q, \end{aligned} \tag{15}$$

$$
\begin{aligned}
W(t) &= W_q(0)\left(1-e^{-\mu_{eq}t}\right) + (1-W_q(0))\left(\frac{2c(1-\rho)\left(1-e^{-\mu_{eq}t}\right)}{2c(1-\rho)-1} - \frac{1-e^{-(2c\mu_{eq}-\lambda_{eq})t}}{2c(1-\rho)-1}\right) \\
&= \frac{2c(1-\rho)-W_q(0)}{2c(1-\rho)-1}\left(1-e^{-\mu_{eq}t}\right) - \frac{(1-W_q(0))}{2c(1-\rho)-1}\left(1-e^{-(2c\mu_{eq}-\lambda_{eq})t}\right).
\end{aligned} \tag{16}
$$

where P_0 and $P_{n_1n_2}$ are given by (14) and the other parameters are described above.

b) The CDF of the system waiting time: The overall CDF of M/M/$2c$ system waiting time can be written as (16) with $W_q(0) = 1 - \frac{P_0 r^{2c}}{(2c)!(1-\rho)}\left(2c + \frac{1}{(1-\rho)}\right)$, where $\lambda_{eq} = \lambda_1 + \lambda_2$ and the rest of the parameters are defined above.

B. Analysis of the system performance

In the rest of this section, we use the preliminaries defined in Section IV-A to derive the main performance measures of different business portfolios.

1) Probability $P(b)$ of benefiting b channels in A model : One operator will be able to benefit b channels from the spectra aggregation if $n_1 = c+b$ and $n_2 \le c-b$ or vice versa. In other words, operator 1 will be able to use b channels from operator's 2 band if that operator has more than b channels free.

a) Voice services $A(2c, \lambda_1, \lambda_2, \mu_1, \mu_2)$:

$$
\begin{aligned}
P(b) &= P(n_1 = c+b, n_2 \le c-b) + P(n_2 = c+b, n_1 \le c-b) \\
&= \sum_{n_2=0}^{c-b} P_{(c+b)n_2} + \sum_{n_1=0}^{c-b} P_{n_1(c+b)} \\
&= \left[\frac{r_1^{c+b}}{(c+b)!}\sum_{n_2=0}^{c-b}\frac{r_2^{n_2}}{n_2!} + \frac{r_2^{c+b}}{(c+b)!}\sum_{n_1=0}^{c-b}\frac{r_1^{n_2}}{n_1!}\right]P_0,
\end{aligned} \tag{17}
$$

where P_0 and $P_{n_1n_2}$ are given by (11) and the rest of the parameters are defined as (12)-(13).

b) Data Services $A(2c, \lambda_1, \lambda_2, \mu_{eq})$:

$$
\begin{aligned}
P(b) &= P(n_1 = c+b, n_2 \le c-b) + P(n_2 = c+b, n_1 \le c-b) \\
&= \sum_{n_2=0}^{c-b} P_{(c+b)n_2} + \sum_{n_1=0}^{c-b} P_{n_1(c+b)} \\
&= 2\left[\sum_{n=0}^{c-b}\frac{(2c\rho)^{(c+b+n)}}{(c+b+n)!}\right]P_0,
\end{aligned} \tag{18}
$$

where P_0 and $P_{n_1n_2}$ are given by (14), $\mu_{eq} = \lambda_1\mu_1/(\lambda_1 + \lambda) + \lambda_2\mu_2/(\lambda_1+\lambda_2)$, ρ_i, $i \in \{1,2\}$ is given by (12), the total traffic ρ by (13) and $r_i = \lambda_i/\mu_{eq}$, $i \in \{1,2\}$.

2) Conditional probability of benefiting from spectra aggregation P_b: An alternative way to characterize the benefits from spectra aggregation is to define the conditional probability of benefiting any number of channels given that another operator is not using certain number of channels. In other words, we are not interesting to quantify the benefit but rather to find out if an operator will benefit at all from spectra aggregation. Thus, the conditional probability of benefiting from spectra aggregation for voice and data applications is given in the sequel.

a) Voice services $A(2c, \lambda_1, \lambda_2, \mu_1, \mu_2)$:

$$
P_b = P(n_2 > c \mid n_1 < c) = \frac{P(n_2 > c, n_1 < c)}{P(n_1 < c)}, \tag{19}
$$

$$
\begin{aligned}
P(n_1 < c) &= \sum_{n_1=0}^{c-1}\sum_{n_2=0}^{2c-n_1} P_{n_1n_2} \\
&= \sum_{n_1=0}^{c-1}\frac{(2c\rho_1)^{n_1}}{n_1!}\sum_{n_2=c}^{2c-n_1}\frac{(2c\rho_2)^{n_2}}{n_2!}P_0,
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
P(n_2 > c, n_1 < c) &= \sum_{n_1=0}^{c-1}\sum_{n_2=c}^{2c} P_{n_1n_2} \\
&= \sum_{n_1=0}^{c-1}\frac{(2c\rho_1)^{n_1}}{n_1!}\sum_{n_2=c}^{2c}\frac{(2c\rho_2)^{n_2}}{n_2!}P_0,
\end{aligned} \tag{21}
$$

where P_0 and $P_{n_1n_2}$ are given by (11) and the rest of the parameters are defined as (12)-(13).

b) Data Services $A(2c, \lambda_1, \lambda_2, \mu_{eq})$:

$$
P_b = P(n_2 > c \mid n_1 < c) = \frac{P(n_2 > c, n_1 < c)}{P(n_1 < c)}. \tag{22}
$$

By using (18),

$$
\begin{aligned}
P(n_1 < c) &= \sum_{n_1=0}^{c-1}\sum_{n_2=0}^{\infty} P_{n_1n_2} \\
&= \sum_{n_1=0}^{c-1}\left[\sum_{n_2=0}^{2c-n_1-1}\frac{(2c\rho)^{n_1+n_2}}{(n_1+n_2)!} + \frac{(2c\rho)^{2c}}{(1-\rho)(2c)!}\right]P_0;
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
P(n_2 > c, n_1 < c) &= \sum_{n_1=0}^{c-1}\sum_{n_2=c}^{\infty} P_{n_1n_2} \\
&= \sum_{n_1=0}^{c-1}\left[\sum_{n_2=c}^{2c-n_1-1}\frac{(2c\rho)^{n_1+n_2}}{(n_1+n_2)!} + \frac{(2c\rho)^{2c}}{(1-\rho)(2c)!}\right]P_0,
\end{aligned} \tag{24}
$$

where P_0 and $P_{n_1n_2}$ are given by (14), $\mu_{eq} = \lambda_1\mu_1/(\lambda_1 + \lambda) + \lambda_2\mu_2/(\lambda_1+\lambda_2)$, ρ_i, $i \in \{1,2\}$ is given by (12), the total traffic ρ by (13) and $r_i = \lambda_i/\mu_{eq}$, $i \in \{1,2\}$.

Additional parameters that may be used to characterize the spectra aggregation model are:

- *Availability probability*: probability that an operator will have exactly a unused channels and so, potentially available to be used by another operator. It is defined as $P_a(a) = P_n(c-a)$.
- *Probability $P(b/a)$*: denotes the probability of benefiting b channels by operator 2 while operator 1 has a channels unused. It is given as $P(b/a) = P(n_1 = c-a, n_2 = c+b)$; $b < a$ and $P(b/a) = P(n_1 = c-a, n_2 \ge c+b)$; $b = a$.
- *Probability of benefiting from spectra aggregation* $P(n_2 > c, n_1 < c)$ is an unconditional probability of benefiting from spectra aggregation.

The above probabilities can be obtained similarly to calculating (19) and (22).

3) Joint state probability distribution function for multi-operator system in BL model and helping probabilities: Let us assume that operator two is borrowing b channels from operator one. Once channels have been borrowed, having n users in one system is independent from the state of the other system. Thus, the joint probability can be given as

$$P_{n_1 n_2} = P_{n_1} P_{n_2}, \tag{25}$$

$$P_{n_1} = \begin{cases} \frac{r_1^{n_1}}{n_1!} P_{01}, & 0 \le n_1 \le c-b \\ \frac{r_1^{n_1}}{(c-b)^{(n_1-c+b)}(c-b)!} P_{01}, & n_1 \ge c-b \end{cases};$$
$$P_{n_2} = \begin{cases} \frac{r_2^{n_2}}{n_2!} P_{02}, & 0 \le n_2 \le c+b \\ \frac{r_2^{n_2}}{(c+b)^{(n_2-c-b)}(c+b)!} P_{02}, & n_2 \ge c+b \end{cases}, \tag{26}$$

and,

$$P_{01} = \left[\sum_{n_1=0}^{c-b} \frac{{r_1}^{n_1}}{n_1!} + \frac{r_1^{c-b}}{(c-b)!(1-\rho_1)} \right]^{-1};$$
$$P_{02} = \left[\sum_{n_2=0}^{c+b} \frac{{r_2}^{n_2}}{n_2!} + \frac{r_2^{c+b}}{(c+b)!(1-\rho_2)} \right]^{-1}, \tag{27}$$

where $r_i = \lambda_i/\mu_i$ and $\rho_i = r_i/2c$ for $i \in \{1,2\}$.

The blocking probabilities for the borrowing model defined as the probabilities that all channels are busy for operator 1, P_{c_1}, and 2, P_{c_2}, are given as

$$P_{c_1} = \frac{\left({r_1}^{c-b}/(c-b)!\right)}{\sum\limits_{n_1=0}^{c-b} \left({r_1}^{n_1}/n_1!\right)}; \quad P_{c_2} = \frac{\left({r_2}^{c+b}/(c+b)!\right)}{\sum\limits_{n_2=0}^{c+b} \left({r_2}^{n_2}/n_2!\right)}. \tag{28}$$

Thus, the helping probability of the total system is defined as $P_h = P_{h_1} + P_{h_2}$ where, P_{h_1} is the helping probability of operator 2 towards operator 1 and, P_{h_2} vice versa. Both probabilities are obtained as follows

$$P_{h_1} = (1 - P_{c_2}) P_{c_1}; \qquad P_{h_2} = (1 - P_{c_1}) P_{c_2}. \tag{29}$$

In addition, the following parameters can be used to characterize this system:

- *Probability of benefiting in borrowing system*: operator 2 will have some benefit from borrowing b channels with probability $P_b = P_{n_2}(c < n_2 < c+b)$.
- *Degradation probability in borrowing system*: operator 1 will degrade its own performance when borrowing b channels with probability $P_d = P_{n_1}(c-b < n_1 < c)$.

4) Extension to K operators : Let us assume that there are K operators with the same system parameters c, λ and μ. In this case, we have the same call blocking probability per operator $P_{ci} = P_c$. The helping probability for a specific operator, given by (29), now becomes

$$p_h = \left(1 - P_c^{K-1}\right); \quad P_c = (r^c/c!)/\left(\textstyle\sum_{n=0}^{c}(r^n/n!)\right). \tag{30}$$

In the case of non-equal traffic distribution in different networks, (30) should be slightly modified as $p_{hi} = (1 - \prod_{k \ne i} P_{ck})$. This simple modification can be used in any of the equations below if needed. Given that a specific operator is blocked at the arrival of a call, the conditional probability that at least one other operator will not be blocked and can accommodate the call is given by (30). The average probability that this will happen in the network is $P_h = (\lambda/c)P_c p_h = (\lambda/c)P_c(1 - P_c^{K-1})$. This is the probability that simultaneously three events occur: 1) a specific network is blocked; 2) a call arrives in that network and; 3) at least one of the remaining networks can help to accommodate the call. The overall system helping probability is the probability that this happens to any of the K operators, $P_H = (\lambda/c)p_H = (\lambda/c)P_c K(1 - P_c^{K-1})$ where $p_H = P_c K(1 - P_c^{K-1})$.

Similarly, one can find the probability of providing the system help for an overflow of B calls in a specific network. We answer this question within a more general problem defined as: what is the probability that K_2 operators can accommodate overall traffic overflow of B calls/sessions of the remaining K_1 operators where $K = K_1 + K_2$? To answer this question, we may use the aggregate model for K_2 operators which can be represented as $A(K_2c, K_2\lambda, \mu)$ for voice and find $P_n(K_2)$ where n is the aggregate system state for K_2 operators. Then, the probability that B sessions of the remaining K_1 operators will be accommodated by K_2 operators is

$$P(K_1, B, K_2) = P(n < K_2c - B) = \sum_{n=0}^{K_2c-B} P_n(K_2). \tag{31}$$

The system utilization for individual $A(c, \lambda, \mu)$ operation in K-operator network is $u_1 = \left(K \sum_{n=1}^{c} n p_1(n)\right)/Kc = (1/c)\sum_{n=1}^{c} n p_1(n)$ where $p_1(n)$ is the steady state distribution function for a $A(c, \lambda, \mu)$ system. For K aggregated networks modeled as $A(Kc, K\lambda, \mu)$, the same parameter is defined as $u_K = (1/Kc)\sum_{n=1}^{Kc} n p_K(n)$, where $p_K(n)$ is the steady state distribution function for a $A(Kc, K\lambda, \mu)$ system. It is intuitively clear that a high utilization will be obtained for high arrival rates when the probability that the arrival will find the system full (no free channel to be allocated) is high as well. This probability is defined as $u_{1f} = \sum_{n=c+1}^{\infty} p_1(n)$ for M/M/c system and $u_{1f} = p_1(c)$ for M/M/c/c with $p_1(c) = (r^c/c!)/(\sum_{n=0}^{c}(r^n/n!))$ and $r = \lambda/\mu$. Then, the new performance measure is defined as $u_1' = u_1(1 - u_{1f})$.

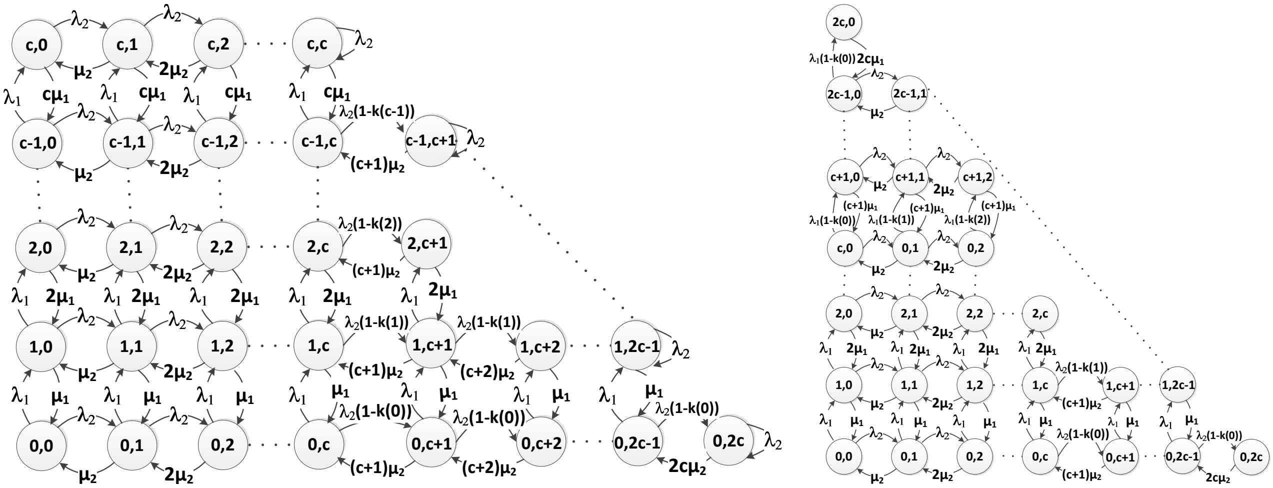
Similarly, for the K aggregated model we have $u_{Kf} = \sum_{n=Kc}^{\infty} p_K(n)$ for M/M/c and $u_{Kf} = p_K(c)$ for M/M/c/c with $p_K(c) = (r^{Kc}/(Kc)!)/(\sum_{n=0}^{Kc}(r^n/n!))$ and $r = \lambda/\mu$. The utility is now

$$u_K^{'} = u_K(1 - u_{Kf}). \tag{32}$$

The system optimization may be performed by making compromise between the utilization and call dropping or storing the message into the queue by defining the utility (for both systems) as $U(\beta) = u - \beta u_f$ and controlling λ, c or K in the system as $\lambda(\beta) = \max_\lambda U(\beta)$, $c(\beta) = \max_c U(\beta)$ and $K(\beta) = \max_K U(\beta)$, respectively.

C. Capacity reduction in cognitive networks

In Section III-C, we introduced parameter $1-\alpha$ referred to as the probability of channel corruption suggesting that the effective available capacity (number of non-corrupted channels) equals αc. In this section, we further elaborate this concept and define the effective available capacity for the P and S operators as $\alpha_{\mathrm{P}} c$ and $\alpha_{\mathrm{S}}(c-n)$, respectively. The channel of P operator will be corrupted if the S operator does not detect



(a) BL system model with pricing.

(b) MBL system model with pricing.

Fig. 11. System model with pricing a)BL, b)MBL

$$P_r(n) = \sum_{k=0}^{c-n} \frac{k}{c-n} p_k(t=1/\mu) + \sum_{k=c-n+1}^{\infty} p_k(t=1/\mu) \; = 1 + \left(\sum_{k=0}^{c-n} \frac{(\lambda/\mu)^k}{k!} \left(\frac{k}{c-n} - 1 \right) e^{-\lambda/\mu} \right). \tag{33}$$

$$P_r(n) = \sum_{k=0}^{c-2c_0-n} \frac{k}{c-2c_0-n} p_k(t=1/\mu) + \sum_{k=c-2c_0-n+1}^{\infty} p_k(t=1/\mu) \; = 1 + e^{-\lambda/\mu} \sum_{k=0}^{c-2c_0-n} \frac{(\lambda/\mu)^k}{k!} \left(\frac{k}{c-2c_0-n} - 1 \right). \tag{34}$$

the presence of PU in the channel. This will be characterized by P_{nd} resulting in $\alpha_{\mathrm{P}} = 1 - P_{nd}$. On the other hand, SU will not be able to use one of the remaining $c-n$ channels either because the free channel has not been detected (with probability P_{fa}) or due to return of the PU (with probability P_r). This results in $\alpha_s = 1-(1-P_{fa})(1-P_r)$ where P_r is the probability that P operator will allocate the channel used by SU. We will approximate this result by assuming that the average service time of the SU is $1/\mu$ so that, the probability of having k new PUs arriving within that time is $p_k(t=1/\mu) = \big((\lambda t)^k/k!\big)e^{-\lambda t} = \big((\lambda/\mu)^k/k!\big)e^{-\lambda/\mu}$.

The probability that a specific channel among cn channels is allocated to one of the k new arrivals is $k/(c-n)$. So, the average corruption probability due to the PU return will be (33).

For any value of μ, this result should be further averaged with respect to the distribution of this parameter. The spectra sensing quality which is characterized by P_{fa} and P_{nd} depends on the methods used for those purposes. This problem has been extensively covered in the literature and for this reason will be omitted here. Moreover, in partial cognitive networks model (PC model) as described in Section III-D, there will be $c-2c_0$ channels for cognitive users and (33) becomes (34).

As a performance measure, we define effective capacity gain in cognitive networks as $g_c = \alpha_s(c-\bar{n}) - (1-\alpha_p)c$ or the relative effective capacity gain as $g_{cr} = g_c/c = \alpha_s(1-\bar{n}/c) - (1-\alpha_p)$ where $\bar{n}$ is the average number of channels used by P operator. The first term is the effective number of channels available for the secondary user and the second term represents the loses for the P operator.

D. Joint state probability density function for BL and MBL system with pricing

For voice traffic, BL and MBL system models with pricing are presented in Figs. 11a and 11b, respectively. The arrival rates for each system are modified in accordance with (6) and (10), respectively. For the state of the system denoted as (n_1, n_2), the state transition probabilities $p(n_1, n_2; n_1', n_2')$ can be obtained from Figs. 11a and 11b, and the system can be solved for the vector of steady state probabilities $\mathbf{P}_{n_1 n_2} = [P_{n_1 n_2}]$.

E. Unified evaluation of the business portfolio: Spectra utilization

The performance measures discussed so far are business model specific and focus on the characteristic features of each individual model. For the unified evaluation of the overall business portfolio and performance comparison we need a common framework for all eight business plans. For that purposes, we use the spectra utilization factor u defined as the ratio of the average number of occupied (used) channels and the overall available number of channels.

In the case of the two operators operating independently (non-cooperative mode) and each having c channels available, this coefficient is defined as

$$u = \begin{cases} (\bar{n}_1 + \bar{n}_2)/2c, & \bar{n}_1, \bar{n}_2 < c, \\ 1, & \text{otherwise}, \end{cases} \tag{35}$$

where $\bar{n}_1$ and $\bar{n}_2$ are the average number of channels used by operator 1 and 2, respectively.

In the cooperative A-model, we can defined the channel utilization as

$$u_A = (\bar{n}_1 + \sum\nolimits_b bP(b))/c, \tag{36}$$

where probability $P(b)$ is discussed in Section IV-B1 for both data and voice traffic. In this case, the utilization is improved by using occasionally the traffic overflow from another user.

For the MBL model, we have

$$u_{MBL} = \begin{cases} \sum\limits_{n_1,n_2} (n_1+n_2)P_{n_1n_2}/2c, & \bar{n}_1 + \bar{n}_2 < 2c \\ 1, & \text{otherwise}, \end{cases} \tag{37}$$

where probability $P_{n_1n_2}$ is discussed in Section IV-B3 for both data and voice traffic.

For the cognitive network, this coefficient is defined as

$$u_c = \begin{cases} \overline{(\alpha_p(n_s)n_p + \alpha_s(n_p)n_s)}/c, & \bar{n}_p + \bar{n}_s < c, \\ 1, & \text{otherwise}, \end{cases} \tag{38}$$

where $\bar{n}_s$ and $\bar{n}_p$ are the average number of active secondary and primary users, respectively. The long over bar in (38) represents the average over n_s and n_p of the overall expression. The utilization of the channel is now expected to be higher since the secondary operator can use the spectra when available. This improvement is modified by mutual impact of the users belonging to two different operators, quantified by parameters α_p and α_s.

In the case of PC model, the utility becomes

$$u_{pc} = \begin{cases} \frac{1}{c}\left(\frac{c_0}{c}\bar{n}_p + \overline{\alpha_p(n_s)n_p}(1-\frac{2c_0}{c})\right) \\ \quad + \frac{1}{c}\left(\frac{c_0}{c}\bar{n}_s + \overline{\alpha_s(n_p)n_s}(1-\frac{2c_0}{c})\right), & \bar{n}_p + \bar{n}_s < c, \\ 1, & \text{otherwise}. \end{cases} \tag{39}$$

For mutually cognitive model, we have

$$u_{MC} = \begin{cases} \sum\limits_{n_1=1}^{c}\sum\limits_{n_2=c}^{2c} \begin{array}{l}(\alpha_p(n_2)n_1 \\ \quad + \alpha_s(n_1)n_2)\frac{P_{n_1n_2}}{c}\end{array}, & \bar{n}_p + \bar{n}_s < c, \\ 1, & \text{otherwise}. \end{cases} \tag{40}$$

For the model with mutual BL with pricing, the utility is given again by (37) with the modification of the joint state probability function as

$$u_{MBL(pricing)} = \begin{cases} \sum\limits_{n_1,n_2} \frac{(n_1+n_2)}{2c} P_{n_1n_2(pricing)}, & \bar{n}_p + \bar{n}_s < c, \\ 1, & \text{otherwise}. \end{cases} \tag{41}$$

where $P_{n_1n_2(pricing)}$ is obtained the same way as in Section IV-B3 with the only difference that the arrival rates for BL and MBL are modified in accordance with (6) and (10) respectively. Finally, for the heterogeneous network model the spectra utilization is obtained as

$$u_{W/CW} = \begin{cases} \sum\limits_{n_1=1}^{c}\sum\limits_{n_2=c}^{2c} \frac{(n_1+\xi n_2)}{c} P_{n_1n_2}, & n_1 < c < n_2, \\ 1, & \text{otherwise}, \end{cases} \tag{42}$$

where n_1 is the number of users in W network and n_2 is the same parameter in C network. Since the offloading is only from C to W network only improvement of the utility in W network can be expected. For the CWC case, we have

$$u_{W/CWC} = \begin{cases} \sum\limits_{n_1=1}^{c}\sum\limits_{n_2=c}^{2c} \frac{(n_1+\xi n_2)}{c} P_{n_1n_2}, & n_1 < c < n_2, \\ 1, & \text{otherwise}, \end{cases}$$
$$u_{C/CWC} = \begin{cases} \sum\limits_{n_1}\sum\limits_{n_2} \frac{(n_1+n_2)}{c} P_{n_1n_2}, & n_2 \le c \le n_1 \le 2c, \\ 1, & \text{otherwise}. \end{cases} \tag{43}$$

V. Numerical Results

A. Capacity aggregation – A model

The conditional probability of benefiting, P_b, defined by (22) for data traffic in A mode is shown in Fig. 12. One can see that for large misbalance of the traffic in the two operators bands, quantified by the ratio of the normalized arrival rates ρ_2/ρ_1, this probability can approach to value one. As a reminder, $\lambda_1/\mu_{eq} = r_1$, $\lambda_2/\mu_{eq} = r_2$, $r_1/2c = \rho_1$ and $r_2/2c = \rho_2$. P_b is defined as the probability that $n_2 > c$ under the condition that $n_1 < c$. For low misbalance, the traffic of the two operators is similar so, given that $n_1 < c$, the probability that simultaneously $n_2 > c$, is low. It can be also observed from Fig. 12 that P_b is high only when the traffic in the two bands is highly misbalanced (high value of ρ_2/ρ_1). From the same figure, one can see that for low misbalance, P_b is higher if the capacity per user, c, is lower. This could be expected since for small c, much lower traffic misbalance can result into the event where we have simultaneously $n_1 < c$ and $n_2 > c$.

The same parameter for voice traffic, given by (19), is presented in Fig. 13. In general, the conditional probability of benefiting in this case is slightly lower for the same values of ρ_2/ρ_1 since for voice applications there is no possibility to keep the messages in the queue and benefit from the fact that a channel may be released while another message is waiting in the queue. In Fig. 13, we have an extra curve for significantly higher value of c ($c = 50$) which enables us an additional insight into the system behavior. For low value of ρ_1 ($\rho_1 = 0.1$), higher misbalance is needed to achieve the condition where we have simultaneously $n_1 < c$ and $n_2 > c$, and $P_b > 0$.

B. Helping Probabilities in BL system

The helping probabilities in BL system defined by (29) are shown in Fig. 14 versus the number of borrowed channels, b. To simplify the presentation the results are presented for the case of common $r = 2c\lambda/\mu$ for both operators and $c = 20$. One can see that for this set of parameters the

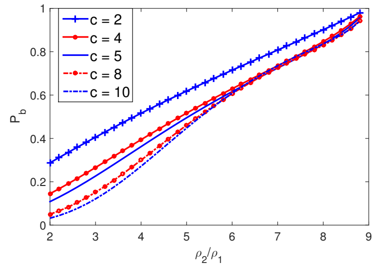


Fig. 12. P_b for data traffic in A mode.

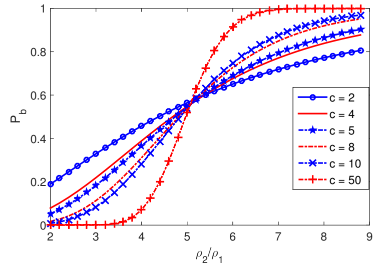


Fig. 13. P_b for voice traffic in A mode.

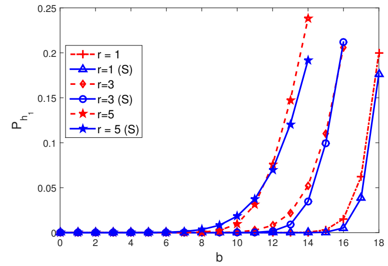


Fig. 14. Helping probabilities in B/L system

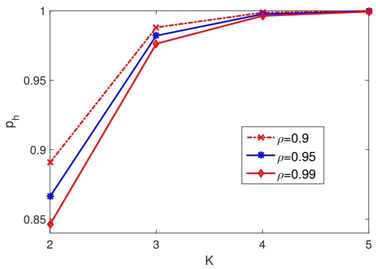


Fig. 15. Probability p_h in K-operator network

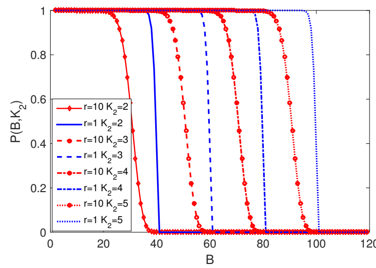


Fig. 16. $P(K_1, B, K_2)$ vs. B

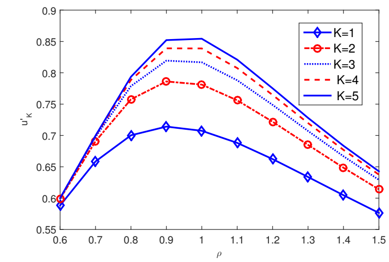


Fig. 17. Utility u' vs. ρ

helping probability can reach the value 0.2. This value might be significantly higher for misbalanced traffic densities in the two bands. The higher r the sooner the help is needed from the borrowed channels to accommodate the traffic spill over. The figure also includes the comparison of the analytical and simulation results obtained by Monte Carlo simulation (S).

In the case of K operators, in Fig. 15 we present the probability of helping an operator p_h given that he needs such help (30). One can see that in general for larger K such probability is higher. This should be expected since for larger K the probability that at least one operator will have a free channel is higher. The helping probability is also higher if the traffic in other networks is of lower intensity (lower ρ). In Fig. 16, we show the probability that a certain number of networks (K_1) with overall need of B channels can get the help from the remaining K_2 networks as in (31). The results are obtained for $r = K_2\lambda/\mu$, $\mu = 1$, $c = 20$, $r = 1$ and $r = 10$. The system utility, defined as in (32), is presented in Fig. 17.

C. Cognitive systems

The channel corruption probability in cognitive mode given by (33) is shown in Fig. 18 versus the number of channels occupied by PU, n. The results suggest that significant amount of capacity will be lost in networks with high traffic dynamics. As expected, the probability is higher for higher normalized arrival rates r of primary users. Similar effect has been confirmed in both, partially cognitive and mutually cognitive networks, but due to the limited space these results are omitted.

The system queue length L_q and waiting time CDF $W(t)$ for C and A models are shown in Fig. 19 and Fig. 20, respectively quantifying the delays for different values of the system parameters. Figure 19 demonstrates that the queue length in A mode is significantly shorter than in the conventional C mode for non-collaborative operation. Hence, in Fig. 20 we show CDF of the delay only for A mode. The improved performance is result of the opportunity for an operator to empty its buffer by using occasionally (when available) idle channels of another operator.

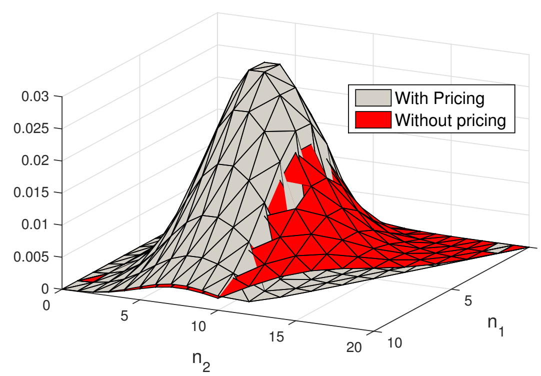


Fig. 21. $P_{n_1 n_2}$ for BL system with pricing, $\lambda_1 = 5, \lambda_2 = 8, \mu = 1$ and $c = 10$

D. BL and MBL systems with pricing

The analysis of BL and MBL system operation is identical as before with the appropriate modification of the arrival rates as given by (11) and (14). Figures 21 and 22 represent the steady state probability density function for the BL system with pricing for voice traffic. One can see that for the system with pricing $P_{n_1 n_2}$ will be lower in the range $n_2 > c = 10$ than for the system without pricing. In other words, system two will be discouraged to use additional available capacity since it has to pay for it.

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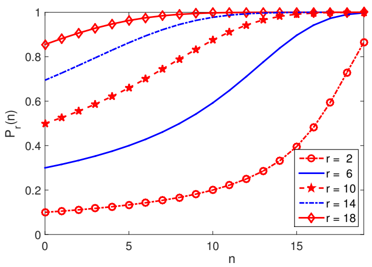

Fig. 18. $P_r(n)$ in cognitive mode for $c = 20$

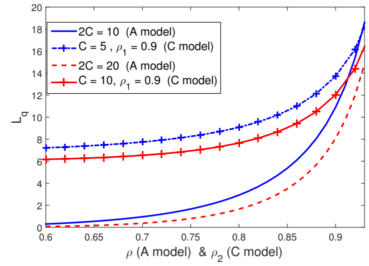

Fig. 19. L_q in A and C model

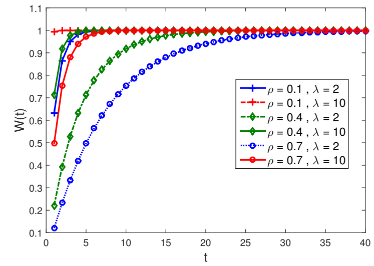

Fig. 20. Waiting time CDF for $c = 10$ in A model

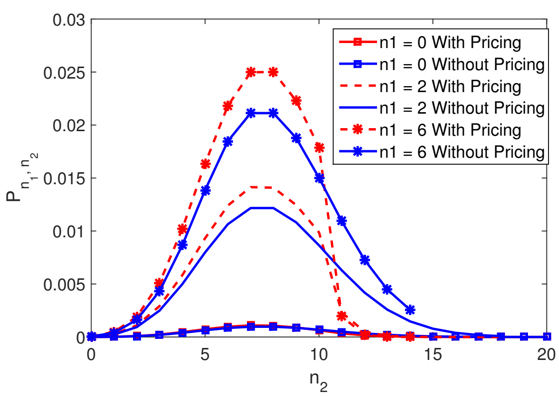

Fig. 22. 2D $P_{n_1 n_2}$ for BL system with pricing

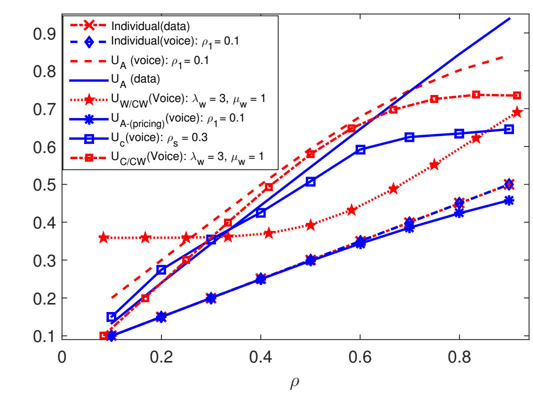

Fig. 23. Spectra utilization factor: analytical results

For clarity of presentation, in Fig. 22 we present two dimensional cuts of Fig. 21 for three different values of n_1 and the same values of the other parameters as in Fig. 21.

E. Spectra Utilization Factor

Spectra utilization factors, characterized by (35) - (43) for the different business models are presented in Fig. 23 versus ρ. In all examples one can see that the individual management of the spectra is inferior compared to the joint (aggregated) spectra management. This applies for both data and voice applications. One should also notice that A model performs the best. In Fig. 24, the comparison between analytical and simulation results is presented for a number of examples when $\rho_1 = 0.1$. In Fig. 25, additional details are presented to demonstrate dependency of spectra utilization factor in cognitive networks with respect to parameter α_p which characterizes the quality of channel sampling algorithms used in the system as discussed in Section IV-C. The results presented in Fig. 25 show that even with perfect channel sensing the cognitive system is inferior compared with the joint spectra management. The performance further deteriorates when α_p is reduced. One should keep in mind that in wireless networks with high traffic dynamics keeping α_p close to one might require a significant effort.

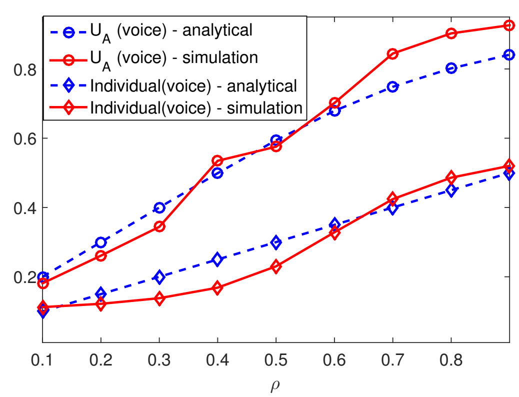

Fig. 24. Spectra utilization factor: analytical / simulationl results

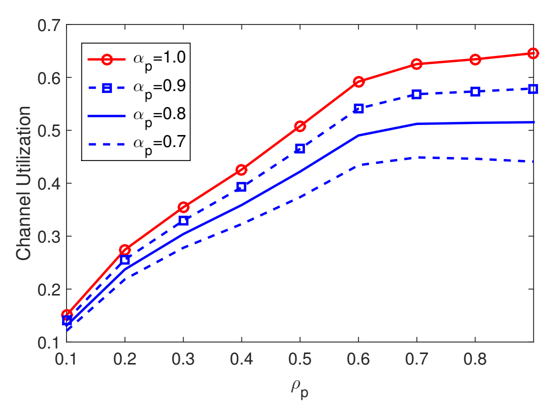

Fig. 25. Spectra utilization factor in cognitive network

F. Comments on Implementation

All spectrum aggregate models can be implemented by joint management of the spectra by multiple operators. By

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establishing a joint channel access control (JCAC) to the aggregate spectrum (overall pool of channels), the users from all operators would be served on FIFO principle. All types of terminals used in conventional networks can operate in this mode as long as they can operate in the entire aggregate spectra. All B/L schemes can be organized on the same principle with extra administration (keeping track) who is borrowing channels to whom. If the joint CAC is not established the B/L request will be exchanged on demand, when there is such a need, which can cause unnecessary delays in channel access decision. All cognitive options can be established by using primary/secondary user concept. In this case, secondary users must sample the channels which requires special type of terminals capable to monitor the spectrum and operate in very wide bandwidth. To make this technology available to all type of legacy terminals, the concept of secondary service providers (SSP) [25] can be used. The basic idea is to delegate the channel monitoring function to the network. The same principle applies to partial cognitive networks as well.

VI. Conclusions

In this paper, we have presented a unified modeling of eight different business plans for joint spectra management in multi-operator wireless networks. Each plan is different with respect to terms and conditions of the mutual contract between the operators. A meaningful advantage in such systems can be expected when there is a misbalance in the traffic densities in the two bands. The contribution of this paper is in the analysis that quantifies these benefits. Depending on the amount of the traffic misbalance, the conditional probability of benefiting can be close to one which means that it is almost certain that such benefit will be materialized. Both voice and data traffic are analyzed. Besides the system benefiting probability, a number of other parameters are defined to quantify these advantages including the probability that specifies the number of channels that can be harvested in other band. The Borrowing/Lending (BL mode) is also modeled and helping probability is used to quantify the system performance. In aggregate (A) mode, the overflow in one band can be served in another band under the condition that there is simultaneous underflow in that band. In BL mode, a certain number of channels are borrowed in advance to serve such overflows for certain, but this can cause losses in another band if there is no simultaneous underflow in such band. All these cases are modeled precisely and quantified by their probabilities of occurrences.

The family of solutions including, conventional cognitive, partially cognitive and mutually cognitive systems is characterized by the channel corruption probability and equivalent losses in the spectra. The mutual traffic offload mechanisms in heterogeneous networks are modeled by defining the equivalent available spectra for the two operators related to the relative coverage of each network.

The spectra utilization factor for all business plans in the business portfolio is significantly higher compared with the performance of the "individual spectra management option" which represents the conventional technique used so far.

The results presented in this paper suggest two major guidelines for organizing the business and regulations in the field of spectrum management: 1) There are still examples where large operators, owing larger portions of the spectrum, demonstrate a certain reluctance to accept joint spectra management principles in order to monopolize the business. In such cases, smaller operators may consider to form the business alliances, use the options from the proposed business portfolio and jointly compete with large operators more efficiently; 2) Spectrum control regulators may in the future favor joint spectra management rather than cognitive networks as it provides better performance and operates with all type of existing terminal technologies.

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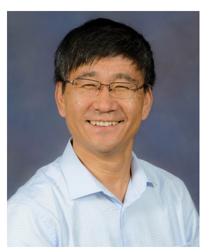
Inosha Sugathapala (S'13) received her B. Sc. degree in Electrical and Electronic Engineering from the University of Peradeniya, Sri Lanka in 2007 and the M. Eng. degree from the Asian Institute of Technology, Thailand in 2011. She is currently working on her Dr. Tech. degree in Communications Engineering in University of Oulu, Finland. Inosha is also a member of the research staff of the Centre for Wireless Communications (CWC), Oulu, Finland. Her main research interests are in heterogeneous networks, optimization, spectrum sharing and queuing theory.

Ivana Kovacevic (S'15) received the M. Sc. degree in communications engineering from the University of Oulu, Oulu, Finland, in 2015. In 2015, she joined the Centre for Wireless Communications (CWC), University of Oulu, where she is currently pursuing the Ph.D. degree in the area of wireless networks. Her research interest is in the area of spectrum sharing, multihop wireless networks, optimization theory, cognitive networks and game theory, radio resource management, QoS and queuing theory. The latest interest is in the area of networks information theory, and mean-field theory.

Beatriz Lorenzo (M'08) received the M.Sc degree in Telecommunication Engineering from University of Vigo, Spain, in 2008 and the Ph.D. degree from University of Oulu, Finland, in 2012. From 2008 to 2014 she worked at the Centre for Wireless Communications (CWC) at University of Oulu as researcher and project manager. In June 2014, she joined Atlantic Research Center for Information and Communication Technologies (AtlantTIC) at the University of Vigo where she currently works as senior researcher. Her research interests include multi-hop communications, heterogeneous networks, network economics, opportunistic communications, and complex networks.

Savo Glisic (SM'94) is professor of telecommunications at University of Oulu and Head of the networking group at the Department of Communication Engineering. He is running a postgraduate program in Networking at University of Oulu, the most comprehensive program in Europe covering wireless networking. He was a Visiting Scientist at the Cranfield Institute of Technology, Cranfield, U.K. (1976-1977), and the University of California at San Diego, San Diego, CA (1986-1987). He has been active in the field of wireless communications for 30 years and has published a number of papers and books. The latest books "Advanced Wireless Networks: 5G/6G technology, 3rd ed., John Wiley and Sons, scheduled for 2015" covers the enabling technology for the definition of incoming 5G/6G systems. Glisic holds a number of patents in this field and is consulting in this field for industries and governments. He has served as Technical Program Chairman of the Third IEEE ISSSTA 1994, the Eight IEEE PIMRC 1997, and IEEE ICC 2001. He was Director of the IEEE ComSoc Membership Developments Programs, and the Organizer and the First Chair of the ComSoc Chapter in Finland. His latest research interest is in the area of spectrum sharing, robust network design, and complex network theory.



Yuguang "Michael" Fang (F'08) received an MS degree from Qufu Normal University, China, in 1987, and Ph.D. degrees from both Case Western Reserve University and Boston University in 1994 and 1997, respectively. He joined the Department of Electrical and Computer Engineering at University of Florida since 2000 and has been a full professor since 2005. Dr. Fang received the US NSF CAREER Award in 2001, the US ONR Young Investigator Award in 2002, 2015 IEEE Communications Society CISTC Technical Recognition Award, 2014 IEEE Communications Society WTC Recognition Award, and is a recipient of the Best Paper Award in IEEE International Conference on Network Protocols in 2006. He also received a 2010-2011 UF Doctoral Dissertation Advisor/Mentoring Award. He served as the Editor-in-Chief of IEEE Wireless Communications and is currently serving as the Editor-in-Chief of IEEE Transactions on Vehicular Technology. He is a Fellow of IEEE.