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Performance Analysis for a Large-scale All-optical WDM Network

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ABSTRACT

By providing several parallel high-capacity channels in the same fiber, WDM technology is commonly considered to be one of the fundamental solutions for future high speed computer networks. By means of WDM the potential THz bandwidth of the optical fiber can be multiplexed into channels with transmission speed compatible with opto/electronic and electro/optic converters - e.g., transmitters and receivers. However, a fundamental scalability problem has to be solved in order to build larger than LAN size WDM packet switched all-optical networks. Current solutions can not be scaled by simply adding nodes to the existing system due to the limited number of wavelengths and/or the available power budget. The lack of modular solutions, similar to those used in traditional electronic networks, can be attributed to the unavailability of optical technology that can support switching of WDM channels on a per packet basis.

Recently, a novel approach was proposed to obtain a packet switching WDM network which is scalable in coverage area, size and number of wavelengths. The approach is based on the combination of two ideas - use of a backbone that connects (bridges) multiple LAN segments, where bridging is done using the new concept of Photonic Slot Routing (PSR). In this concept, packets transmitted simultaneously on the WDM channels in any "photonic" slot are switched jointly in the bridge between the LAN segments and the backbone. Thus packets in a given slot remain transmitted on the same path, from their source LAN to the destination LAN and consequently, the bridge is only required to handle complete slots, i.e., all WDM channels jointly. The aim of this work is to present the first analytical model developed to evaluate the performance of the photonic slot concept applied to a single fiber ring backbone system, independently on the interconnected LAN segment topologies, e.g, ring, folded bus, star.

Keywords: WDM, All-optical systems, Performance analysis, Photonic slot routing, Blocking probabilities, Optical ring backbone

1. INTRODUCTION

All-optical networks have gained tremendous attention in the last few years that will continue growing in the near future. These networks attempt to utilize the enormous bandwidth of the optical fiber (potentially a few tens of terabits per second) by using wavelength division multiplexing (WDM) ([2,8]). It is already witnessed ([10]) that the backbone media of WANs and MANs are being replaced by optical fibers in the consideration of handling the continuously growing integrated traffic demand from/to various sources. However, the WDM solutions proposed so far to build up backbone networks require data E/O and O/E conversions at some points in the system, e.g., at the bridge connecting the backbone with the local LAN, hence the potential THz bandwidth of the fiber cannot be fully utilized due to the electronic bottleneck arising at the conversion points. The reason for lacking solutions that achieve all-optical packet switching in scalable WDM networks can be found in a fundamental technological limitation. Single-hop solutions does not scale well since the number of total nodes in the network is limited by the available number of wavelengths and/or by the power budget. The lack of all-optical multi-hop (i.e., modular)

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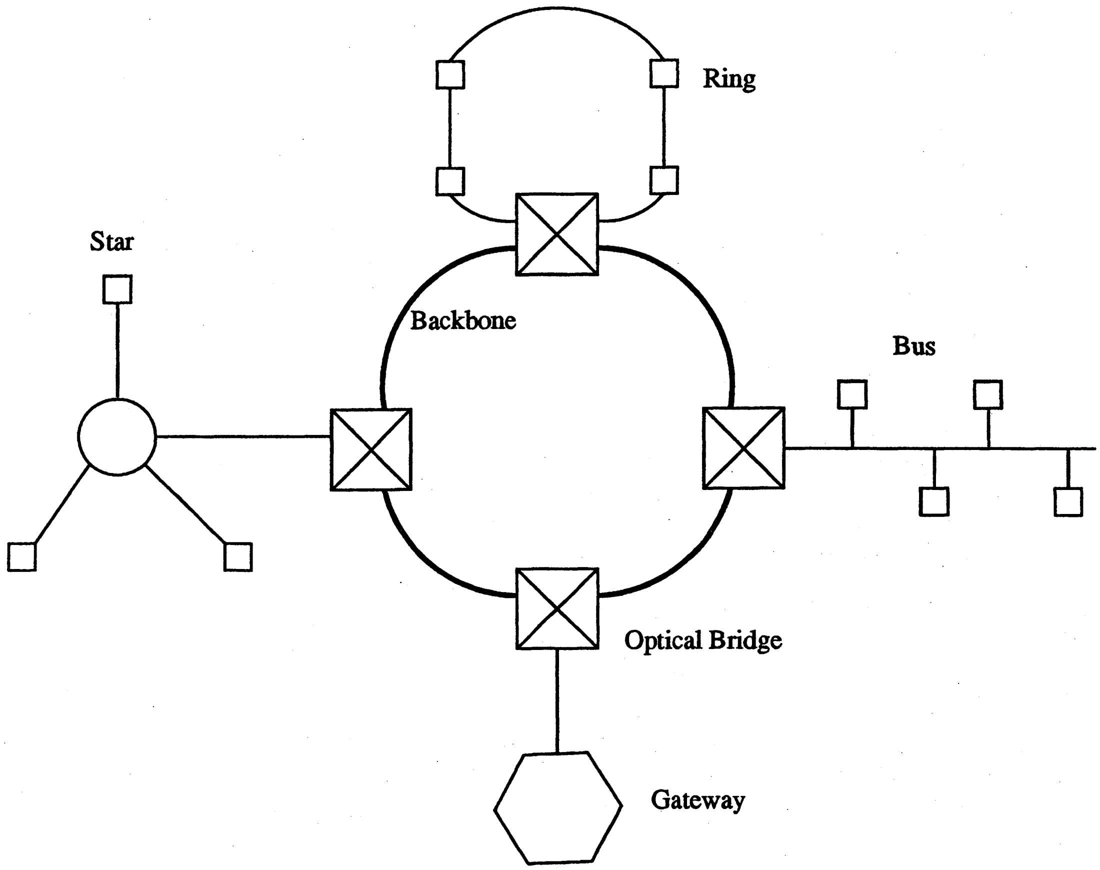

Figure 1. All optical large-scale WDM network

solutions, similar to those used in traditional electronic networks is due to the unavailability of optical technology that can support switching of WDM channels on a per packet basis.

A novel approach ([3–5]) was proposed to obtain a packet switching WDM network which is modular and scalable in coverage area, size and number of wavelengths. This approach is based on the combination of two ideas—use of a backbone that connects (bridges) multiple LAN segments, where bridging is done using the new concept of Photonic Slot Routing (PSR). In this approach, packets transmitted simultaneously on the WDM channels are grouped to form the so called "photonic" slot and are switched jointly in the bridge connecting the LAN segment with the backbone. Thus, packets in a given slot remain transmitted on the same path, from the source LAN to the destination LAN. To make this possible, all packets transmitted in the same slot need to have the same destination LAN address. To this end in every source LAN packets are transmitted in each slot according to their destination LAN using ad-hoc Sorting Access Protocols.

It is easy to see that with this approach the bridge is consequently only required to handle complete slots, i.e., all WDM channels jointly, without having to access and switch packets on different channels individually. This solution can therefore handle wavelength sensitive inter-LAN data flows using wavelength non-sensitive devices, hence overcoming the fundamental problem of switching limitations of wavelength sensitive devices at the bridges (switches).

For a better illustration of this approach, we focus on a specific network topology. Assume that the backbone topology is a ring. As shown in Figure 1, several LANs (and gateways) may be connected to the backbone network (BN) by means of optical bridges. The LANs can have different topologies. The working principle is as follows: every node in a LAN can communicate with a node in the same LAN (*intra-LAN* transmission) or in another LAN (*inter-LAN* transmission). Intra-LAN transmission requires that data be propagated only through the LAN of the originating node, possibly crossing the bridge. Inter-LAN transmission requires that data be propagated through the BN towards the intended destination LAN, thus crossing the bridges of the source and destination LANs.

The BN is a single fiber ring that can be upgraded to a multi-fiber ring if the overall network traffic exceeds the single fiber capacity. In its simplest form, the bridge connecting the LAN with the BN is a 2×2 space switch with fast switching rate (e.g., a digital switch ([1])), with one input and one output connected to the BN and the other input and other output connected to the LAN. In the ring backbone time is divided into slots of fixed length, τ. The time division is aligned over the entire set of wavelengths, thus packets transmitted in the same slot but on distinct wavelengths are aligned and form the photonic slot. The LANs connected to the backbone must use the same slot length and must synchronize their respective time slots in order to guarantee that at the bridge slots arriving from the backbone and from the LAN are perfectly aligned ([6]). A signaling associated with each slot notifies the bridge on the LAN destination of the arriving slot. Slots generated in the LAN are randomly assigned a destination LAN depending on the LAN offered traffic. Once its destination is assigned the slot is filled up by the nodes of the LAN with packets whose destination nodes are in the destination LAN of the slot. This procedure is controlled by the so called sorting access protocol that operates in the LAN. A number of such protocols have been proposed depending on the topology of the LAN ([4,6]). As each LAN independently runs its sorting protocol, slots with the same destination LAN originated in different source LAN may arrive at a bridge simultaneously. Since under this circumstance both of the slots are seeking for the same output port of the bridge, we say that a contention arises. Due to its simple structure, in case of contention the bridge can correctly route only one of the two arriving slots, and must deflect the other*. Packets borne in a deflected slot must be retransmitted. It was shown that for a number of reasons it is advantageous to give priority to the slots arriving from the backbone over the slots arriving from the LAN ([6,4]).

The aim of this paper is to present the first analytical model developed for the evaluation of the performance of the photonic slot concept applied to a single fiber ring backbone system, independently of the interconnected LAN segment topologies (ring, bus or star). For multi-fiber ring, a similar approach together with the method in [7] can be carried out. This result is achieved by modeling the system at the slot level, i.e., considering the slot as the fundamental unit, and disregarding the individual packet transmissions that occurs in every LAN. In other words, the presented model allows for the evaluation of the efficiency of the photonic slot routing in terms of successfully delivered slots per time unit. How these slots are efficiently filled up with packets by the sorting access protocols operating in the distinct LANs is beyond the scope of this work and can be found in previously published works ([6,4]).

In particular, we will focus on the following quantities: throughput defined as the average number of slots successfully transmitted by every LAN per time unit, average packet transmission delay defined as the time necessary for a packet to be successfully transmitted without deflection, and the probability of contention occurring at the bridge. In order to characterize these quantities, we need a mathematical model for this network. It suffices to perform the analysis at each bridge where the contention arises. At each bridge, there are two traffic streams, one from the BN and the other from the LAN connected to the bridge. Since slots arriving from the BN have priority over slots arriving from the LAN, there are a few possibilities at the bridge: the BN slot is always successfully transmitted while the LAN slot is deflected if and only if it is in conflict with the BN slot. For example, if the BN slot wants to remain in the backbone, the LAN slot that is destined to the LAN itself is not deflected, whereas the LAN slot that is destined to another LAN cannot be routed through the backbone and must be dropped. Similarly, if the BN slot is destined to the LAN connected to the bridge, the LAN slot will not be deflected only if it is destined to another LAN. Analytical results for Poisson traffic and uniform traffic are presented next.

2. PERFORMANCE ANALYSIS FOR POISSON TRAFFIC

In this section, we study the case when the BN traffic and LAN traffic are Poisson distributed.

Let N be the number of the LANs connected to the BN, label them as $1, 2, \ldots, N$, where we assume that transmission direction is from 1 to $2 \ldots N$ and back to 1 (unidirectional communication). For LAN i (i.e., the ith

*Where necessary, the bridge structure can be modified to allow temporary optical buffering of the arriving slots, thus resolving the contention without deflecting one slot ([4]).

LAN), we assume that the slot generation (that includes new packet transmissions and packet retransmissions) is Poissonian with rate λ_i^I for slots destined to the same LAN (*intra-LAN traffic*) and with rate λ_i^o for slots destined for different LANs (*inter-LAN traffic*). Let τ denote the slot size. Since slots generated at distinct LANs will propagate through different portions of the BN ring, we need to specify the routing probabilities of each of these slots. Let p_{ij} denote the probability that a slot from LAN i is destined to LAN j.

We first want to estimate the probability of contentions resulting from the inter-LAN traffic. We know that in this case, contention happens only when a BN slot passes through the bridge while a LAN slot connecting to the bridge wants to access the BN. Therefore, we have to find out the arrival rate of BN slots which pass through the bridge (slots not directed to the local LAN). Because of contentions, the inter-LAN slot rate originating in LAN i that successfully enters the BN is smaller than λ_i^o. Let $\bar{P}_i$ denote the probability that a slot generated by LAN i successfully enters the BN. If we know these quantities, then other quantities can be easily obtained. Let λ_i' denote the rate of BN slots crossing the ith bridge and remaining in the BN. It is easy to see that the slots leaving LAN j have rate equal to $\bar{P}_j\lambda_j^o$, and among these slots, the slots destined to LAN k have rate $\bar{P}_j\lambda_j^o p_{jk}$. Consider now the ith bridge, if $i \neq 1$, in order for the slot from LAN 1 to pass through bridge i, it should be destined to $i+1, i+2, \ldots, N$, because if it destined to $2, 3, \ldots, i-1$, it will leave the BN before it reaches the bridge (in the BN slots only travel at most one round). From this argument, we obtain

$$\begin{aligned}
\lambda_i' = {} & \left(\sum_{l=i+1}^{N} p_{1l}\lambda_1^o\right)\bar{P}_1 + \left(p_{21}\lambda_2^o + \sum_{l=i+1}^{N} p_{2l}\lambda_2^o\right)\bar{P}_2 \\
& + \left(\sum_{l=1}^{2} p_{3l}\lambda_3^o + \sum_{l=i+1}^{N} p_{3l}\lambda_3^o\right)\bar{P}_3 + \cdots \\
& + \left(\sum_{l=1}^{i-2} p_{(i-1)l}\lambda_{i-1}^o + \sum_{l=i+1}^{N} p_{(i-1)l}\lambda_{i-1}^o\right)\bar{P}_{i-1} + \left(p_{(i+2)(i+1)}\lambda_{i+2}^o\right)\bar{P}_{i+2} \\
& + \left(\sum_{l=i+1}^{i+2} p_{(i+3)l}\lambda_{i+3}^o\right)\bar{P}_{i+3} + \cdots + \left(\sum_{l=i+1}^{N-1} p_{Nl}\lambda_N^o\right)\bar{P}_N.
\end{aligned} \tag{1}$$

Given that the inter-LAN traffic for LAN i is Poisson with arrival rate λ_i^o and departure rate $1/\tau$, we can derive the utilization factor for this queueing system as $\rho_i = \lambda_i^o\tau$. In order for a slot generated in LAN i to successfully enter the BN, there should not be any BN slot passing through the bridge i during one time slot. Thus, the probability that LAN i successfully transmits a slot in the BN is given by

$$\bar{P}_i = e^{-\lambda_i'\tau}. \tag{2}$$

This is equivalent to

$$-\log\left(\bar{P}_i\right) = \lambda_i'\tau. \tag{3}$$

From (1) and (3), we obtain the following set of equations: for $i = 1, 2, \ldots, N$,

$$\begin{aligned}
-\log\left(\bar{P}_i\right) = {} & \left(\sum_{l=i+1}^{N} p_{1l}\rho_1\right)\bar{P}_1 + \left(p_{21}\rho_2 + \sum_{l=i+1}^{N} p_{2l}\rho_2\right)\bar{P}_2 \\
& + \left(\sum_{l=1}^{2} p_{3l}\rho_3 + \sum_{l=i+1}^{N} p_{3l}\rho_3\right)\bar{P}_3 + \cdots \\
& + \left(\sum_{l=1}^{i-2} p_{(i-1)l}\rho_{i-1} + \sum_{l=i+1}^{N} p_{(i-1)l}\rho_{i-1}\right)\bar{P}_{i-1} + \left(p_{(i+2)(i+1)}\rho_{i+2}\right)\bar{P}_{i+2} \\
& + \left(\sum_{l=i+1}^{i+2} p_{(i+3)l}\rho_{i+3}\right)\bar{P}_{i+3} + \cdots + \left(\sum_{l=i+1}^{N-1} p_{Nl}\rho_N\right)\bar{P}_N.
\end{aligned} \tag{4}$$

Theoretically, we can solve this set of equations for $\bar{P}_1, \bar{P}_2, \ldots, \bar{P}_N$ (N equations with N unknowns) using classic recursive algorithms. Define matrix $A = (a_{ij})_{N\times N}$, where for any $i = 1, 2, \ldots, N$,

$$\begin{aligned}
a_{i1} &= \left(\sum_{l=i+1}^{N} p_{1l}\right)\rho_1 \\
a_{i2} &= \left(p_{21} + \sum_{l=i+1}^{N} p_{2l}\right)\rho_2 \\
a_{i3} &= \left(\sum_{i=1}^{2} p_{3l} + \sum_{l=i+1}^{N} p_{3l}\right) \\
&\vdots \\
a_{i(i-1)} &= \left(\sum_{l=1}^{i-2} p_{(i-1)l} + \sum_{l=i+1}^{N} p_{(i-1)l}\right)\rho_{i-1} \\
a_{ii} &= 0 \\
a_{i(i+1)} &= 0 \\
a_{i(i+2)} &= \left(p_{(i+2)(i+1)}\right)\rho_{i+2} \\
a_{i(i+3)} &= \left(\sum_{l=i+1}^{i+2} p_{(i+3)l}\right)\rho_{i+3} \\
&\vdots \\
a_{iN} &= \left(\sum_{l=i+1}^{N-1} p_{Nl}\right)\rho_N
\end{aligned} \tag{5}$$

In the above notation, we assume $\sum_{l=i}^{j} p_{kl} = 0$ if $i > j$. Thus, matrix A has the following form

$$A = \begin{pmatrix} 0 & 0 & a_{13} & a_{14} & \cdots & a_{1N} \\ a_{21} & 0 & 0 & a_{24} & \cdots & a_{2N} \\ a_{31} & a_{32} & 0 & 0 & \cdots & a_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{(N-1)1} & a_{(N-1)2} & a_{(N-1)3} & \cdots & 0 & 0 \\ 0 & a_{N2} & a_{N3} & \cdots & a_{N(N-1)} & 0 \end{pmatrix}.$$

For $N = 2$ and $N = 3$, A can be easily obtained from the above

$$A_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 0 & 0 & p_{32}\rho_3 \\ p_{13}\rho_1 & 0 & 0 \\ 0 & p_{21}\rho_2 & 0 \end{pmatrix}.$$

Define vectors $x = (x_1, \ldots, x_n)^T$ and $f(x) = (f_1(x_1), \cdots, f_n(x_n))^T$ where the supscript T denotes the matrix transpose. Equation (4) can be written using the following matrix formulation

$$f(\bar{P}) + A\bar{P} = 0 \tag{6}$$

where $\bar{P} = (\bar{P}_1, \bar{P}_2, \ldots, \bar{P}_N)^T$ and $f(\bar{P}) = (\log(\bar{P}_1), \log(\bar{P}_2), \ldots, \log(\bar{P}_N))^T$. The following lemma can be used to guarantee the existence and uniqueness of a solution for (6).

Lemma. ([9]) *If $f_i(t)$ $(i = 1, 2, \ldots, n)$ are continuous, strictly increasing functions of t, and matrix B has non-negative principal minors, then equation $f(x) + Bx = b$ has a unique solution for any vector b.*

Since $\log(\cdot)$ is a continuous, strictly increasing function, function $f(x)$ in (6) satisfies the condition in the lemma. Thus, if matrix A defined above has all its principal minors non-negative, equation (6) has a unique solution, which can be easily obtained using any recursive algorithm (notice that $f(x)$ is a simple function). For $N = 2$, A is a zero matrix, which obviously has nonnegative principal minors (equal to zero), so (6) has a unique solution. This is also confirmed by a direct inspection that leads to solution $\bar{P}_1 = \bar{P}_2 = 1$. When $N = 3$, the principal minors of A_3 are 0, 0 and $p_{13}p_{32}p_{21}\rho_1\rho_2\rho_3 \geq 0$, hence from the above lemma, (6) has a unique solution. However, using the natural logarithm function (log) in constructing an algorithm for equation (6) may originate numerical convergence problems. These problems are easily overcome modifying equation (6) and using the exponential function that has better smoothness property than the logarithmic function, thus equation $-\log(x) = kx$ is replaced by equation $x = e^{-kx}$.

If the traffic over the BN is light, i.e., $\lambda_i'\tau$ is sufficiently small, the exponential function in (2) can be approximated by affinely linear terms using its Taylor series, thus leading to the linear equation:

$$A\bar{P} + \bar{P} = e \tag{7}$$

where $e = (1, 1, \ldots, 1)^T$. The solution is thus

$$\bar{P} = (A + I)^{-1}e \tag{8}$$

where I denotes the identity matrix with dimension $N \times N$ (the non-singularity of matrix $A+I$ can be easily derived from the standard non-negative matrix theory).

Given that probabilities $\bar{P}_1, \ldots, \bar{P}_N$ are known from (1), we can finally obtain the BN slot passing rate λ_i' through bridge i. Returning now to the slot deflection probability of the inter-LAN traffic of LAN i, contention happens (slot deflection) if and only if there is a BN slot passing through the bridge and a LAN slot going out of the bridge at the same time, hence the slot loss probability of the outing traffic is given by

$$P_i^o = \left(1 - e^{-\lambda_i'\tau}\right)\left(1 - e^{-\lambda_i^o\tau}\right) \approx (\lambda_i'\tau)(\lambda_i^o\tau) = \rho_i'\rho_i, \tag{9}$$

where $\rho_i' = \lambda_i'\tau$.

If we assume that upon slot deflection packets borne in the deflected slot are jointly retransmitted in a new slot without the addition of any other packet, we can evaluate the average number of times an inter-LAN packet (slot) must be transmitted prior to successfully reaching the BN. A inter-LAN slot generated by LAN i is deflected by bridge i with probability ρ_i', therefore, the probability that a packet has to be transmitted n times is $\rho_i'^{n-1}(1-\rho_i')$, and the average number of transmissions for such a packet is

$$R_i^o = \sum_{n=1}^{\infty} n\rho_i'^{n-1}(1-\rho_i') = 1/(1-\rho_i').$$

Denoting with $\tau_i = \tau + prop_i$ the sum of the slot transmission time plus the round trip propagation time in LAN i, the average transmission time for an inter-LAN packet generated in LAN i prior to entering the BN is $D_i^o = \tau_i/(1-\rho_i')$. The inter-LAN throughput of LAN i is finally equal to

$$S_i^o = \rho_i e^{-\lambda_i'\tau} = \rho_i e^{-\rho_i'} \approx \rho_i(1-\rho_i').$$

Next, we consider the intra-LAN traffic. For intra-LAN traffic there exists a case for contention that may cause slot deflection, that is, the arriving BN slot is destined to the local LAN. We first need to evaluate the BN slot rate directed to a given LAN, that for LAN i is equal to

$$\lambda_i'' = \sum_{l \neq i} p_{li}\lambda_l^o\bar{P}_l. \tag{10}$$

Thus, in LAN i the probability that the intra-LAN slot is deflected is equal to

$$P_i^I = \left(1 - e^{-\lambda_i''\tau}\right)\left(1 - e^{-\lambda_i^I\tau}\right) \approx (\lambda_i''\tau)(\lambda_i^I\tau)$$

the average number of retransmissions for the intra-LAN packet is

$$R_i^I = 1/(1 - \lambda_i''\tau)$$

the average transmission time for the intra-LAN packet is

$$D_i^I = \tau_i/(1-\lambda_i''\tau)$$

and the intra-LAN throughput is

$$S_i^I = (\lambda_i^I\tau)e^{-\lambda_i''\tau} \approx \lambda_i^I\tau(1-\lambda_i''\tau).$$

As an illustrative example, we consider a special case. Assume that all LANs are identical, i.e., they have the same offered load and the same routing probabilities: each LAN has the same probability to send its slot to any other LAN. Using our notation, this is equivalent to the following conditions:

$$p_{ij} = \frac{1}{N-1}(i \neq j), \;\; \lambda_i^o = \lambda^o, \;\; \lambda_i^I = \lambda^I, \;\; \rho_i = \rho, i = 1, 2, \ldots, N.$$

The matrix A is given by

$$\begin{aligned} A &= \begin{pmatrix} 0 & 0 & \frac{\rho}{(N-1)} & \frac{2\rho}{(N-1)} & \cdots & \frac{(N-2)\rho}{(N-1)} \\ \frac{(N-2)\rho}{(N-1)} & 0 & 0 & \frac{\rho}{(N-1)} & \cdots & \frac{(N-3)\rho}{(N-1)} \\ \frac{(N-3)\rho}{(N-1)} & \frac{(N-2)\rho}{(N-1)} & 0 & 0 & \cdots & \frac{(N-4)\rho}{(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ \frac{\rho}{(N-1)} & \frac{2\rho}{(N-1)} & \frac{3\rho}{(N-1)} & \cdots & 0 & 0 \\ 0 & \frac{\rho}{(N-1)} & \frac{2\rho}{(N-1)} & \cdots & \frac{(N-2)\rho}{(N-1)} & 0 \end{pmatrix} \\ &= \frac{\rho}{N-1} \begin{pmatrix} 0 & 0 & 1 & 2 & \cdots & (N-2) \\ (N-2) & 0 & 0 & 1 & \cdots & (N-3) \\ (N-3) & (N-2) & 0 & 0 & \cdots & (N-4) \\ \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ 1 & 2 & 3 & \cdots & 0 & 0 \\ 0 & 1 & 2 & \cdots & (N-2) & 0 \end{pmatrix}. \end{aligned} \tag{11}$$

This matrix is a circular matrix which is very easy for computation. Since each variable $\bar{P}_i$ has the same relationship with the others (determined by the perfect symmetry of the problem), we may conclude that they must have the same value. In fact, this is really the case. Let $\bar{P}_1 = \bar{P}_2 = \cdots = \bar{P}_N = p$, then from any equation of (6), we obtain

$$-\log(p) = \frac{\rho}{N-1}(1+2+\cdots+(N-2))p = \frac{(N-2)\rho}{2}p,$$

which is equivalent to

$$p = e^{-[(N-2)\rho/2]p}. \tag{12}$$

This can be iteratively solved by deriving the $(k+1)$st step from the kth step as follows: $p_{k+1} = e^{ap_k}$ where $a = -(N-2)\rho/2$. Obviously, when $N = 2$, $p = 1$ from (12). Under light traffic conditions on the BN, from (8) we obtain

$$p = \frac{1}{1+(N-2)\rho/2}.$$

The performance parameters are computed as follows:

$$\begin{aligned} \lambda' &= \lambda_i' = (N-2)\frac{\lambda^o p}{2} \\ \lambda'' &= \lambda_i'' = \lambda^o p \\ P_i^o &= \left(1-e^{-(N-2)\rho p/2}\right)\left(1-e^{-\rho}\right) \\ P_i^I &= \left(1-e^{-\rho p}\right)\left(1-e^{-\lambda^I\tau}\right) \end{aligned}$$

$$
\begin{aligned}
S_i^o &= \rho e^{-(N-2)\rho p/2} \\
S_i^I &= (\lambda^I \tau) e^{-\rho p} \\
R_i^o &= 1/(1-(N-2)\rho p/2) \\
R_i^I &= 1/(1-\rho p) \\
D_i^o &= \tau_i/(1-(N-2)\rho p/2) \\
D_i^I &= \tau_i/(1-\rho p)
\end{aligned}
$$

We can observe that slot contentions occurring at bridges are mutually affected with each other and tend to be balanced over the backbone network, in fact, the BN has self-adaptive property: when the load over BN is high, the bridge contentions are frequent, LANs then have less opportunities to enter new slots into the BN. As a consequence, the traffic in the BN diminishes, and LANs gain new access to the BN. This alternating behavior continues until convergence is reached. This aspect of the studied system is captured by equations (1)-(3) and is an interesting feature for this scalable network.

3. PERFORMANCE ANALYSIS FOR UNIFORM TRAFFIC

In this section, we evaluate the performance of a number of photonic slot routing networks under uniform traffic distribution, i.e., each LAN generates the same amount of traffic. We assume that in each time slot, LAN i transmits an inter-LAN slot with probability ρ_i and an intra-LAN slot with probability ρ_i^I (without confusion, we can still use the notation adopted in the previous section). Parameter p_{ij} denotes the probability that a slot from LAN i is directed to LAN j. Let λ_i' be the probability that a slot on the BN goes through bridge i and λ_i'' denotes the probability that a BN slot is directed to LAN i. Also, let $\bar{P}_i$ be the probability that an inter-LAN slot generated by LAN i successfully enters the BN. We obtain

$$
\begin{aligned}
\lambda_i' = {} & \left(\sum_{l=i+1}^{N} p_{1l}\rho_1\right)\bar{P}_1 + \left(p_{21}\rho_2 + \sum_{l=i+1}^{N} p_{2l}\rho_2\right)\bar{P}_2 \\
& + \left(\sum_{l=1}^{2} p_{3l}\rho_3 + \sum_{l=i+1}^{N} p_{3l}\rho_3\right)\bar{P}_3 + \cdots \\
& + \left(\sum_{l=1}^{i-2} p_{(i-1)l}\rho_{i-1} + \sum_{l=i+1}^{N} p_{(i-1)l}\rho_{i-1}\right)\bar{P}_{i-1} + \left(p_{(i+2)(i+1)}\rho_{i+2}\right)\bar{P}_{i+2} \\
& + \left(\sum_{l=i+1}^{i+2} p_{(i+3)l}\rho_{i+3}\right)\bar{P}_{i+3} + \cdots + \left(\sum_{l=i+1}^{N-1} p_{Nl}\rho_N\right)\bar{P}_N.
\end{aligned} \tag{13}
$$

Observing that an inter-LAN slot can successfully enter the BN when the arriving BN slot is empty, we have

$$\bar{P}_i = 1 - \lambda_i'. \tag{14}$$

Let A denote the matrix given in (5) with the new meaning given to ρ_i in this section. We still have the matrix equation (7), and the solution for $\bar{P}$ given in (8). The related quantities are given below:

$$
\begin{aligned}
\lambda_i' &= 1 - \bar{P}_i \\
\lambda_i'' &= \sum_{l\neq i} p_{li}\bar{P}_l\rho_l \\
P_i^o &= \lambda_i'\rho_i = (1-\bar{P}_i)\rho_i \\
P_i^I &= \lambda_i''\rho_i^I = \left(\sum_{l\neq i} p_{li}\bar{P}_l\rho_l\right)\rho_i^I \\
S_i^o &= \rho_i(1-\lambda_i') = \bar{P}_i\rho_i
\end{aligned}
$$

$$
\begin{aligned}
S_i^I &= \rho_i^I(1-\lambda_i'') = \rho_i^I\left(1-\sum_{l\neq i} p_{li}\bar{P}_l\rho_l\right) \\
R_i^o &= 1/(1-\lambda_i') = 1/\bar{P}_i \\
R_i^I &= 1/(1-\lambda_i'') = 1/\left(1-\sum_{l\neq i} p_{li}\bar{P}_l\rho_l\right)
\end{aligned} \tag{15}
$$

As an illustration, consider the following special case. If the traffic is uniformly balanced, i.e., every LAN's behavior is the same, we have

$$p_{ij} = \frac{1}{N-1}\ (i \neq j),\ \rho_i = \rho,\ \rho_i^I = \rho^I,\ i = 1, 2, \ldots, N.$$

Matrix A is given in (11). Solving equation (8), we obtain (let $\bar{P}_1 = \bar{P}_2 = \cdots = \bar{P}_N = p$)

$$p = \frac{1}{1+(N-2)\rho/2}.$$

From (15), we have

$$
\begin{aligned}
\lambda_i' &= \frac{(N-2)\rho}{2+(N-2)\rho} \\
\lambda_i'' &= \frac{2\rho}{2+(N-2)\rho} \\
P_i^o &= \frac{(N-2)\rho^2}{2+(N-2)\rho} \\
P_i^I &= \frac{2\rho\rho^I}{2+(N-2)\rho} \\
S_i^o &= \frac{2\rho}{2+(N-2)\rho} \\
S_i^I &= \frac{2+(N-4)\rho}{2+(N-2)\rho}\rho^I \\
R_i^o &= 1+(N-2)\rho/2 \\
R_i^I &= \frac{2+(N-2)\rho}{2+(N-4)\rho}
\end{aligned} \tag{16}
$$

Given the symmetry of the considered traffic distribution, we can fully characterized the offered traffic with two parameters: λ, the probability that LAN i generates a slot, and α, the probability that the generated slot is used for inter-LAN traffic. We can therefore derive that $\rho = \alpha$, $\rho^I = (1-\alpha)\lambda$, and

$$
\begin{aligned}
P_i^o &= \frac{(N-2)\lambda^2\alpha^2}{2+(N-2)\lambda\alpha} \\
P_i^I &= \frac{2\lambda^2\alpha(1-\alpha)}{2+(N-2)\lambda\alpha} \\
S_i^o &= \frac{2\lambda\alpha}{2+(N-2)\lambda\alpha} \\
S_i^I &= \frac{2+(N-4)\lambda\alpha}{2+(N-2)\lambda\alpha}(1-\alpha)\lambda
\end{aligned} \tag{17}
$$

Parameter α is defined as the *inter-LAN traffic factor.* Plots in Figure 2 show the probabilities of slot deflection P_i^o and P_i^I, and the achievable throughput S_i^o and S_i^I versus α. The curves are obtained by setting $\lambda = 1$ and considering a varying number of LANs in the system, i.e., $N = 2, 4, 6, 8$.

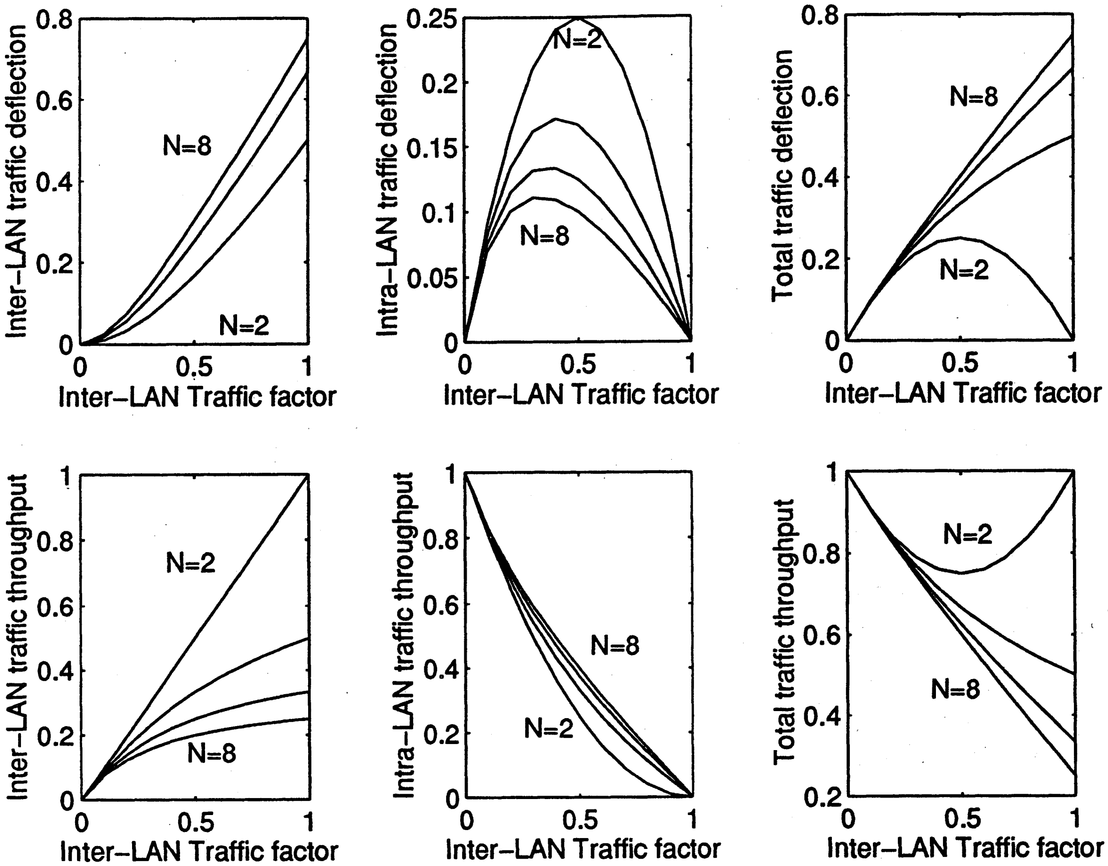

Figure 2. Numerical results obtained under uniform traffic distributions: $\lambda = 1$, $N = 2, 4, 6, 8$

Some discussions are in order. In the end of the last section, we claimed that the ring backbone has interesting self-adaptive property. This is confirmed by the curves shown in figure 2. As the inter-LAN traffic factor α changes from 0 to 1, the deflection probability for the inter-LAN slots increases, which is reasonable because the more slots attempts to access the BN, the more deflections occur. Conversely, the deflection probability for the intra-LAN slots first increases, then decreases back to zero. This latter behavior can be intuitively explained by observing that when α tends to 0 most of the traffic is local to the LANs, and only marginally affected by the inter-LAN slots that are seldom transmitted. When α tends to 1 most of the traffic is inter-LAN and rapidly saturates the backbone bandwidth. In turns, this reduces the rate of the inter-LAN slots that are successfully delivered to the destination LAN, at the advantage of the intra-LAN traffic that is again marginally affected by the inter-LAN traffic. Similar observations can be derived for the throughput. The effect of the number of LANs (N) on system performance is also clearly documented in the figure.

4. CONCLUSIONS

This paper presented the first analytical model derived for the performance evaluation of a photonic slot routing network based on a single-fiber ring backbone. Through the use of extant technology, this network achieves all-optical and packet switching transmissions among the nodes belonging to a number of WDM LANs connected via a backbone network. The model enlightened the self-adaptive property of the considered system to the distribution of the offered traffic. The numerical results obtained through the presented model showed that the single fiber backbone is sufficient when the number of LANs connected to the backbone is small or the inter-LAN traffic is limited. When none of these conditions are met in the system, the designer must upgrade the backbone with additional fiber rings.

The generalization of the proposed model to take into account multi-fiber ring backbones is currently undergoing.

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