When Spectrum Meets Clouds: Optimal Session Based Spectrum Trading under Spectrum Uncertainty

Miao Pan, *Member, IEEE*, Pan Li, *Member, IEEE*, Yang Song, *Member, IEEE*, Yuguang Fang, *Fellow, IEEE*, Phone Lin, *Senior Member, IEEE*, and Savo Glisic, *Senior Member, IEEE*

***Abstract*—Spectrum trading creates more accessing opportunities for secondary users (SUs) and economically benefits the primary users (PUs). However, it is challenging to implement spectrum trading in multi-hop cognitive radio networks (CRNs) due to harsh cognitive radio (CR) requirements on SUs' devices, uncertain spectrum supply from PUs and complex competition relationship among different CR sessions. Unlike the per-user based spectrum trading designs in previous studies, in this paper, we propose a novel session based spectrum trading system, spectrum clouds, in multi-hop CRNs. In spectrum clouds, we introduce a new service provider, secondary service provider (SSP), to facilitate the accessing of SUs without CR capability and harvest uncertain spectrum supply. The SSP also conducts spectrum trading among CR sessions w.r.t. their conflicts and competitions. Leveraging a 3-dimensional (3-D) conflict graph, we mathematically describe the conflicts and competitions among the candidate sessions for spectrum trading. Given the rate requirements and bidding values of candidate trading sessions, we formulate the optimal spectrum trading into the SSP's revenue maximization problem under multiple cross-layer constraints. In view of the NP-hardness of the problem, we develop heuristic algorithms to pursue feasible solutions. Through extensive simulations, we show that the solutions found by the proposed algorithms are close to the optimal one.**

***Index Terms*—Cognitive radio networks, uncertain spectrum supply, link scheduling, multi-hop multi-path routing.**

Manuscript received November 16, 2012; revised April 19, 2013 and June 28, 2013. This work was partially supported by the U.S. National Science Foundation under grants CNS-1147813/1147851, ECCS-1129062/1128768, and CNS-1343356/1343361/1343220. The work of M. Pan was also partially supported by the U.S. National Natural Science Foundation under grant NSF-1137732. The work of P. Lin was supported in part by the NSC of Taiwan under grant NSC 102-2219-E-002-018, by the MoEA of Taiwan under grant 102-EC-17-A-03-S1-214, by CHT, and by ICRL/ITRI Taiwan. The work of S. Glisic was also partially supported by the Finnish Academy: Project COCAHANE-257162. The preliminary version has been presented at IEEE INFOCOM'2012 [1].

M. Pan is with the Department of Computer Science, Texas Southern University, 122 Nabrit Science Center, Houston, Texas, 77004, USA (e-mail: panm@tsu.edu). (corresponding author)

P. Li is with the Department of Electrical and Computer Engineering, Bagley College of Engineering, Mississippi State University, 229 Simrall Engineering Building, Mississippi State, MS, 39762-9571, USA (e-mail: li@ece.msstate.edu).

Y. Song is with the Storage Services Research Department, IBM Almaden Research Center, 650 Harry Rd., San Jose, CA, 95120, USA (e-mail: yangsong@us.ibm.com).

Y. Fang is with the Department of Electrical and Computer Engineering, University of Florida, 435 Engineering Building, P.O.Box 116130, Gainesville, FL, 32611, USA (e-mail: fang@ece.ufl.edu).

P. Lin is with the Computer Science & Information Engineering Department, National Taiwan University, No. 1 Sec. 4 Roosevelt Rd., Taipei, Taiwan (e-mail: mail: plin@csie.ntu.edu.tw).

S. Glisic is with the Department of Communication Engineering, University of Oulu, P.O. Box 4500, Finland (e-mail: savo.glisic@ee.oulu.fi).

Digital Object Identifier 10.1109/JSAC.2014.140320

I. Introduction

NOWADAYS, more and more people, families and companies rely on wireless services for their daily life and business, which leads to a booming growth of various wireless networks and a dramatic increase in the demand for radio spectrum. In parallel with that, current static spectrum allocation policy of Federal Communications Commission (FCC) [2]–[4] results in the exhaustion of available spectrum, while a lot of licensed spectrum bands are extremely under-utilized. Experimental tests in academia [5], [6] and measurements conducted in industries [7] both show that even in the most crowed region of big cities (e.g., Washington, DC, Chicago, New York City, etc.), many licensed spectrum bands are not used in certain geographical areas and are idle most of the time. Those studies spur the FCC to open up licensed spectrum bands and pursue new innovative technologies to encourage dynamic use of the under-utilized spectrum. As one of the most promising solutions, cognitive radio (CR) technology releases the spectrum from shackles of authorized licenses, and enables secondary users (SUs) to opportunistically access to the vacant licensed spectrum bands in either temporal or spatial domain.

The idea of opportunistic using licensed spectrum bands has initiated the spectrum trading in multi-hop cognitive radio networks (CRNs) and promoted a lot of interesting research on the design of spectrum trading systems [8]–[14]. Through spectrum trading, primary users (PUs) can sell/lease/auction their vacant spectrum for monetary gains, and SUs can purchase/rent/bid the available licensed spectrum if they suffer from the lack of radio resources to support their traffic demands. However, to trade the licensed spectrum and opportunistically access to these bands, SUs' handsets have to be frequency-agile [3], [15]. It is imperative for the SUs' devices to have the CR capability such as exploring licensed spectrum bands, reconfiguring RF, switching frequencies across a wide spectrum range (i.e., from 20 MHz to 2.5 GHz [15]–[17]), sending and receiving packets over non-contiguous spectrum bands, etc. Although some of the desired features may be realized in future, enormous amount of time and efforts must be spent in hardware designs and signal processing in order to implement these features in light weight radios [3], [15], [16]. In addition, for spectrum trading in CRNs, it is always appreciated to minimize the changes on the handsets of SUs while facilitating the spectrum trading to maximize the spectral efficiency.

Except for the harsh requirements on SUs' devices, another primary challenge for spectrum trading in multi-hop CRNs is how SUs conduct the multi-hop CR communications using the purchased spectrum. Most existing work focuses on per-user based spectrum trading [10]–[12], i.e., each SU purchases available bands from PUs and uses the purchased spectrum for communications. Unfortunately, those spectrum trading designs are confronted with several critical problems when they are deployed in multi-hop CRNs. For instance, it is not clear whom a SU communicates with (i.e., the destination SU or the receiver is not explicitly specified); it is not clear how to find a common band between two SUs to establish communications; it is not clear what kind of quality of service (e.g., throughput, delay, rate or bandwidth requirement, etc.) can be supported. Besides, although some of prior spectrum trading systems consider the impact of frequency reuse [10]–[12], they ignore almost all the other factors, such as interference mitigation, link scheduling, flow routing, etc., which may significantly affect the performance of CR sessions in multi-hop CRNs.

Another key obstacle to the spectrum trading in multi-hop CRNs lies in the uncertain licensed spectrum supply [18], [19]. Since the CR services must evacuate the licensed bands when primary services are active, the returning of PUs has significant impact on how to execute the spectrum trading in CRNs w.r.t. the opportunistic usage of licensed spectrum, interference avoidance, the access of CR sessions, quality of service (QoS), etc.

To address the challenges above, in this paper, we propose a session based spectrum trading system, *spectrum clouds*, for multi-hop CRNs. In order to facilitate the spectrum trading of SUs without CR capability, a novel network architecture and new network entities are introduced in spectrum clouds. Under the proposed architecture of CRNs, we study the session based spectrum trading instead of per-user based spectrum trading. Given the rate requirements and bidding values[1] of candidate CR sessions, we exploit the licensed band vacancy statistics and endeavor to conduct the optimal spectrum trading under multiple constraints (e.g., the availability of spectrum bands, the competition among different CR sessions, link scheduling constraints, flow routing constraints, etc.) in multi-hop CRNs. We mathematically formulate these concerns into an optimization problem and provide both the near-optimal solution and the feasible solution in this work. Our salient contributions are listed as follows.

- Different from the architecture of traditional spectrum trading systems, we introduce a new emerging service provider, called *Secondary Service Provider* (SSP), in spectrum clouds, and assume the SSP has already established some partial infrastructure with CR mesh routers[2] at low cost to provide coverage in the area of interest. Suppose that the SSP has its own bands (i.e., basic bands) and can harvest the available licensed spectrum bands. To facilitate the accessing of SUs without CR devices, all the CR mesh routers are equipped with multiple CR radios. Under the guidance of the SSP, SUs access their nearby CR mesh routers using basic bands and deliver packets via CR mesh routers using both basic bands and harvested bands.
- Inspired by the statistics of licensed band vacancy obtained on observation and experiments in [2], [5], [7], we model the uncertain spectrum vacancy of a licensed band as a random variable satisfying certain distribution. Under the proposed CRN architecture and the modeling of uncertain spectrum supply, we employ a 3-dimensional (3-D) conflict graph [20] to characterize the conflict relations among CR links in spectrum clouds. Based on the 3-D conflict graph, we mathematically describe the competition among CR sessions for radio spectrum as well as the link scheduling and routing constraints. Furthermore, we formulate the optimal session based spectrum trading into the SSP's revenue maximization problem under those cross-layer constraints. Given all the independent sets in CRNs, we can relax the integer variables in the formulation, solve the optimization problem by linear programming, and find the upper bound of the SSP's revenue for session based spectrum trading in multi-hop CRNs.
- Since the competition relationship between any two sessions is represented by binary values, it is NP-hard to solve the formulated optimization, in which these integer constraints are involved [18], [21]. To pursue feasible solutions, we develop the heuristic *relax-and-fix* algorithms to determine the values of integer variables. Briefly speaking, we divide all the CR sessions into different sets and relax-and-fix the integer variables for CR sessions in one session set after another. If there exists a feasible solution, it yields a lower bound to the original optimization problem.
- By carrying out extensive simulations in both grid topology and random topology, we demonstrate that the proposed session based spectrum trading system has great advantages over the per-user based ones in multi-hop CRNs. We also compare the upper bound and lower bounds determined by the heuristic algorithms at different data sets, and show that the feasible solutions obtained by the proposed algorithms are really close to the optimal one in terms of the SSP's revenue.

The rest of the paper is organized as follows. In Section II, we review related work in CR community. In Section III, we introduce the system architecture of spectrum clouds, corresponding network settings and related models in multi-hop CRNs. In Section IV, we mathematically describe link scheduling and routing constraints in spectrum clouds, formulate the session based spectrum trading under multiple constraints into an optimization problem and near-optimally solve it by linear programming. In Section VI, we develop the heuristic algorithms for feasible solutions. Finally, we conduct simulations and analyze the performance results in Section VII, and draw concluding remarks in Section VIII.

II. Related Work

Prior work has investigated spectrum trading issues from different aspects. Specifically, in [8], Grandblaise et al. gen-

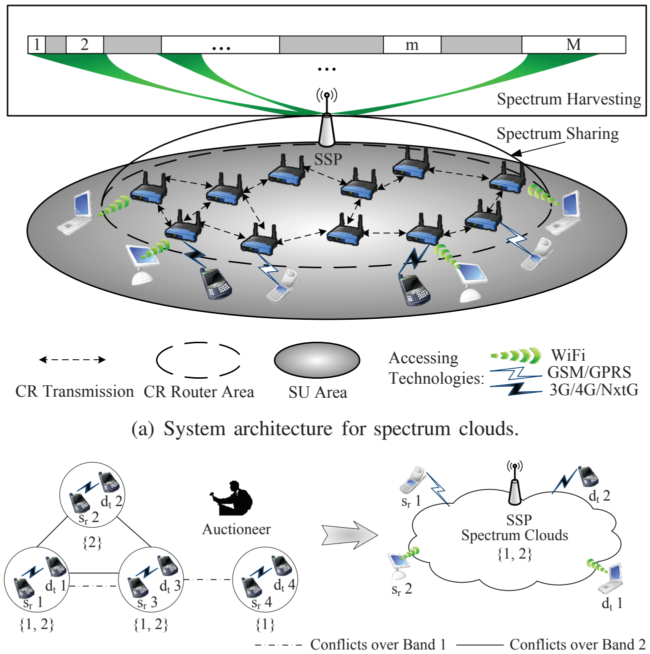
[1]In this paper, bidding values generally represent how much the SUs are willing to pay for purchasing/renting/bidding for the available spectrum, which can be used for the traffic delivery of corresponding CR sessions.

[2]In the rest of this paper, we use the words CR router/CR mesh router/router interchangeably.

erally describe the potential scenarios and introduce some microeconomics inspired spectrum trading mechanisms, and in [9], Sengupta and Chatterjee propose an economic framework for opportunistic spectrum accessing to guide the design of dynamic spectrum allocation algorithms as well as service pricing mechanisms. From the view of the PUs, Xing et al. in [22] and Niyato et al. in [23], [24] have well investigated the spectrum pricing issues in the spectrum market, where multiple PUs, whose goal is to maximize the monetary gains with their vacant spectrum, compete with each other to offer spectrum access to the SUs. From the view of the SUs, Pan et al. in [25], [26] have addressed how the SUs optimally distribute their traffic demands over the spectrum bands to reduce the risk for monetary loss, when there is more than one vacant licensed spectrum band. From the view of trading system design, models in game theory, by Wang et al. in [27], Duan et al. in [19] and Zhang et al. in [28], and auction designs in microeconomics, by Zhou et al. in [10], [29], Jia et al. in [11], Pan et al. in [30] and Wu et al. in [12], are exploited to construct spectrum trading systems with desired properties, such as power efficiency, allocation fairness, incentive compatibility, Pareto efficiency, collusion resistance and so on. Although these designs consider certain features of wireless transmissions, they are generally per-user based spectrum trading systems rather than session based ones.

The impact of multiple sessions on the performance of multi-hop wireless networks has been extensively investigated in existing literature. Jian et al. in [31] studied how the interference affects the performance of ad-hoc networks based on an NP-complete optimization problem. Zhai and Fang in [32] developed a high throughput routing metric under link scheduling and routing constraints in single-radio single-channel networks. In multi-radio multi-channel networks, Li et al. in [20] proposed a multi-dimensional conflict graph and exploited it to efficiently solve the optimal network throughput problem using linear programming. In CR research community, there have been some efforts devoted to cross-layer optimization as well. Tang et al. in [33] studied the joint spectrum allocation and link scheduling problems with the objectives of maximizing throughput and achieving certain fairness in CRNs. Hou et al. in [34] investigated the joint frequency scheduling and routing problem with the objective of minimizing the network-wide spectrum usage in CRNs. Considering the uncertain spectrum supply, Pan et al. in [18] proposed to model the vacancy of licensed bands as a series of random variables, characterized the multi-hop CRNs with a pair of (α, β) parameters and minimized the usage of licensed spectrum to support CR sessions with rate requirements at certain confidence levels. However, there remains a lack of study to incorporate these multi-hop transmission concerns into the design of spectrum trading systems.

In this work, we are trying to bridge the gap between these two active research areas in multi-hop CRNs. With the proposed spectrum trading system, spectrum clouds, we have a comprehensive study on the optimal spectrum trading problem considering multiple factors including uncertain spectrum supply, the competition among CR sessions, link scheduling, flow routing, etc. Our work effectively extends the per-user based spectrum trading into the session based spectrum trading and makes those microeconomics inspired spectrum trading mechanisms practically applicable in multi-hop CRNs.



(a) System architecture for spectrum clouds.

(b) A schematic for comparison between traditional spectrum trading mechanisms and spectrum clouds.

Fig. 1. A novel architecture for spectrum trading in multi-hop CRNs.

III. Network Model

A. System Architecture for Spectrum Clouds

We consider the proposed spectrum trading system in multi-hop CRNs, spectrum clouds, consisting of the SSP, a group of SUs, a set of CR mesh routers and a collection of available licensed spectrum bands[3] with unequal size of bandwidths as shown in Fig. 1(a). The SSP is an independent wireless service provider (e.g., a base station or an access point) with its own spectrum, i.e., the SSP's basic bands (potentially congested already), and is able to collectively harvest the available licensed bands. The SSP has also deployed some CR mesh routers at low cost to facilitate the accessing of SUs. SUs are just end-users not subscribed to primary services. No specific requirements are imposed on the SUs' communication devices. They could be any devices using any accessing technologies (e.g. laptops or desktop computers using Wi-Fi, cell phones using GSM/GPRS, smart phones using 3G/4G/NxtG accessing technology, etc.). SUs can access to the basic bands owned by the SSP, but they cannot be tuned to the harvested licensed frequency. The CR mesh routers deployed by the SSP have CR capability and are equipped with multiple CR radios.

Under spectrum clouds' architecture, the mobile SUs report their online traffic requests, which include source/destination, rate requirements and corresponding bidding values of the

[3]Taking the least-utilized spectrum bands introduced in [34] for example, we found that the bandwidth between [1240, 1300] MHz (allocated to amateur radio) is 60 MHz, while bandwidth between [1525, 1710] MHz (allocated to mobile satellites, GPS systems, and meteorological applications) is 185 MHz.

SUs' sessions, to their nearby CR mesh routers. The CR mesh routers collect these requests from different end-users and report them to the SSP. Depending on the bidding values, rate requirements and the available spectrum resources, the SSP makes decisions on the accessing/denial of the SUs' sessions, and jointly conducts link scheduling and flow routing among CR mesh routers for SUs' traffic delivery. Following the guidance of the SSP, the CR mesh routers form unicast CR communication sessions and deliver packets using both the leftover basic bands and harvested bands as shown in Fig. 1(a).

In traditional spectrum trading systems, the spectrum bands to sell/lease/auction are known to every SU. Due to broadcasting nature of wireless transmissions, the SU may also know his potential competitors and overhear their bids, so that many schemes are proposed to ensure that the spectrum trading is not manipulated in multi-hop CRNs [10], [12]. By contrast, in spectrum clouds, the SU has no idea about the specific spectrum allocation across the whole session (i.e., from the source to the destination). Even if a SU overhears the bids of other SUs, it is not helpful since the SU is not sure who are his competitors for spectrum usage. Besides, spectrum clouds can support session based spectrum trading in multi-hop CRNs, whereas the other systems can only support single-hop spectrum trading as shown in Fig. 1(b).

B. Network Configuration

Suppose there are $\mathcal{N} = \{1, 2, \cdots, n, \cdots, N\}$ CR mesh routers, each CR mesh router has $\mathcal{H} = \{1, 2, \cdots, h, \cdots, H\}$ radio interfaces, and these CR mesh routers form a set of $\mathcal{L}$ unicast communication sessions according to SUs' requests. Each session has a rate requirement and a corresponding bidding value. Denote the source/destination CR router of session $l \in \mathcal{L} = \{1, 2, \cdots, l, \cdots, L\}$ by $s_r(l)/d_t(l)$, and let $(r(l), b(l))$ be the rate requirement-bidding value pair for session $l \in \mathcal{L}$. Assume the SUs' usage of basic bands in the multi-hop CRNs is a *priori* information. The CR routers are able to use the rest of basic spectrum owned by the SSP. The CR routers are also allowed to communicate with each other by opportunistically accessing to the licensed bands when the primary services are not active, but they must evacuate from these bands when primary services become active.

Considering the geographical location of the CR routers, the available spectrum bands at one CR router may be different from another one in the network. To put it in a mathematical way, let $\mathcal{M} = \{1, 2, \cdots, m \cdots, M\}$ be the band set including the available basic bands and licensed bands with different bandwidths $\mathcal{W} = \{W^1, W^2, \cdots, W^m, \cdots, W^M\}$ for communications, and $\mathcal{M}_i \subseteq \mathcal{M}$ represent the set of available bands at CR router $i \in \mathcal{N}$. $\mathcal{M}_i$ may be different from $\mathcal{M}_j$, where j is not equal to i, and $j \in \mathcal{N}$, i.e., possibly $\mathcal{M}_i \neq \mathcal{M}_j$. Meanwhile, since primary services come back and forth, the spectrum supply from licensed bands is uncertain in the temporal domain. To capture this key feature of spectrum trading in CRNs, let $\boldsymbol{T}_{ij}^m$ denote the available time of band m at CR link (i, j) within one unit time slot, where $\boldsymbol{T}_{ij}^m$ is modeled as a random variable. As shown in [2], [18], [25], the statistical characteristics of $\boldsymbol{T}_{ij}^m$ contain abundant knowledge about band m's spectrum availability at link (i, j) for opportunistic accessing[4].

C. Other Related Models in Multi-hop CRNs

1) Transmission Range and Interference Range: Suppose all CR mesh routers use the same power P for transmission. The power propagation gain [34], [35] is

$$g_{ij} = \gamma \cdot d_{ij}^{-\beta}, \tag{1}$$

where β is the path loss factor, γ is an antenna related constant, and d_{ij} is the distance between CR routers i and j. We assume that the data transmission is successful only if the received power at the receiver exceeds the receiver sensitivity, i.e., a threshold P_{Tx}. Meanwhile, we assume interference becomes non-negligible only if it is over a threshold of P_{In} at the receiver. Thus, the transmission range for a CR router is $R_{Tx} = (\gamma P/P_{Tx})^{1/\beta}$, which comes from $\gamma \cdot (R_{Tx})^{-\beta} \cdot P = P_{Tx}$. Similarly, based on the interference threshold $P_{In}(P_{In} < P_{Tx})$, the interference range for a CR router is $R_{In} = (\gamma P/P_{In})^{1/\beta}$. It is obvious that $R_{In} > R_{Tx}$ since $P_{In} < P_{Tx}$.

In the widely used protocol model [18], [20], [32]–[34], [36], the interference range is typically 2 or 3 times of the transmission range, i.e., $\frac{R_{In}}{R_{Tx}} = 2$ or 3. These two ranges may vary with frequency. The conflict relationship between two links over the same frequency band can be determined by the specified interference range. In addition, if the interference range is properly set, the protocol model can be accurately transformed into the physical model as illustrated in [37].

2) Link Capacity/Achievable Data Rate: According to Shannon-Hartley theorem, if CR router i sends data to CR router j on link (i, j) with band m, the capacity of link (i, j) with band m is

$$c_{ij}^m = W^m \log_2 \left(1 + \frac{g_{ij} P}{\eta}\right), \tag{2}$$

where η is the ambient Gaussian noise power at CR mesh router j[5]. Depending on different modulation schemes, the achievable data rate is actually determined by the SNR at the receiver and receiver sensitivity [32], [34]. However, in most of existing literature [18], [20], [34], the achievable data rate is approximated by (2), even though this data rate can never be achieved in practice. In this paper, we follow the same approximation. Note that this approximation will not affect the theoretical analysis or performance comparison in this work.

[4]Chen et al. in [5] carried out a set of spectrum measurements in the 20MHz to 3GHz spectrum bands at 4 locations concurrently in Guangdong province of China. They used these data sets to conduct a set of detailed analysis on statistics of the collected data, including channel occupancy/vacancy statistics, channel utilization, also spectral and spatial correlation of these measures.

[5]Note that the denominator inside the log function contains only η. This is because of one of our interference constraints, i.e., when CR router i is transmitting to CR router j on band m, then all the other neighbors of router j within its interference range are prohibited from using this band. We will address the interference constraints in details in the following section.

IV. Optimal Spectrum Trading under Cross-layer Constraints in Multi-hop CRNs

We exploit binary value $\delta(l)$ to denote the success/failure of spectrum trading for session l, i.e.,

$$\delta(l) = \begin{cases} 1, & \text{session } l \text{ is accessed by the SSP;} \\ 0, & \text{session } l \text{ is denied by the SSP.} \end{cases} \quad (3)$$

To make the decision of accessing/denying a session $l \in \mathcal{L}$, the SSP must consider both the rate requirement and bidding value of session l. Besides, to effectively utilize the leftover basic spectrum and the harvested licensed spectrum, it is necessary for the SSP to schedule data transmission among different CR mesh routers under joint spectrum assignment, link scheduling and flow routing constraints. In the rest of this section, we first extend the conflict graph [32] to characterize the interference relationship among CR links in spectrum clouds. Then, based on the extended conflict graph, we mathematically describe link scheduling and flow routing constraints and formulate the spectrum trading into the revenue maximization problem of the SSP under multiple constraints. By relaxing the integral variables, we solve the optimization problem and provide an upper-bound of the SSP's revenue.

A. Extended Conflict Graph, Cliques and Independent Sets

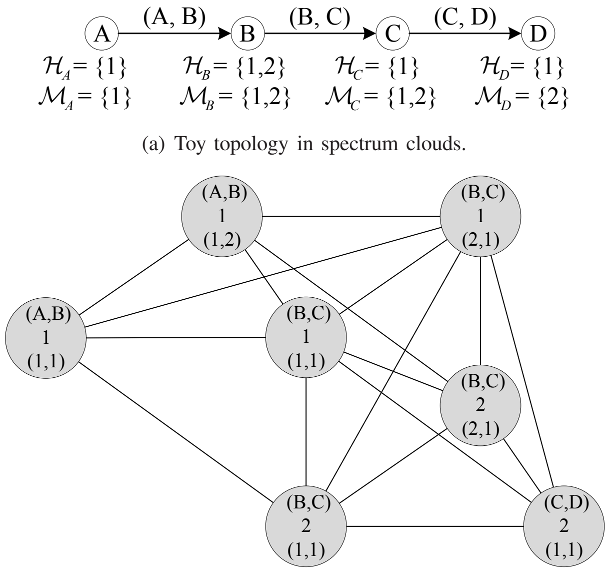
1) Construction of 3-Dimensional (3-D) Conflict Graph: Regarding the availability of spectrum bands and radios at CR mesh routers, we introduce a 3-D conflict graph to characterize the interference relationship among CR links in spectrum clouds. Following the definitions in [20], we interpret a CRN as a three-dimensional resource space, with dimensions defined by links, the set of available bands and the set of available radios. In a 3-D conflict graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, each vertex corresponds to a link-band-radio (LBR) tuple, i.e.,

$$\text{link-band-radio:} \quad ((i, j), m, (u, v)),$$

where $i \in \mathcal{N}$, $m \in \mathcal{M}_i \bigcap \mathcal{M}_j$, $j \in \mathcal{T}_i^m$, $u \in \mathcal{H}_i$ and $v \in \mathcal{H}_j$. Here, $\mathcal{T}_i^m$ is the set of CR mesh routers within CR router i's transmission range. The LBR tuple indicates that the CR router i transmits data to CR router j on band m, where radio interfaces u and v are used at sending CR router and receiving CR router, respectively. Based on the definition of LBR tuples, we can enumerate all combinations of CR mesh routers, the vacant bands and the available radios, which can potentially enable CR communication links.

Different from multi-radio multi-channel networks [20], the availability of bands and radios (i.e., the leftover radios after collecting SUs' traffic) at each CR router in CRNs may be different, i.e., for $i, j \in \mathcal{N}$, maybe $\mathcal{M}_i \neq \mathcal{M}_j$ and $\mathcal{H}_i \neq \mathcal{H}_j$. Similar to the interference conditions in [18], [20], [34], two LBR tuples are defined to interfere with each other if either of the following conditions is true: (i) if two different LBR tuples are using the same band, the receiving CR router of one tuple is in the interference range of the transmitting CR router in the other tuple; (ii) two different LBR tuples have the same radios at one or two CR routers.

Note that the first condition not only represents co-band interference but also inherently covers the following two cases: any CR router cannot transmit to multiple routers on the same band; any CR router cannot use the same band for concurrent transmission and reception, due to "self-interference" at the physical layer. Meanwhile, the second condition represents the radio interface conflicts, i.e., a single radio cannot support multiple transmissions (either transmitting or receiving) simultaneously. According to these conditions, we connect two vertices in $\mathcal{V}$ with an undirected edge in $\mathcal{G}(\mathcal{V}, \mathcal{E})$, if their corresponding LBR tuples interfere with each other.



(a) Toy topology in spectrum clouds.

(b) 3-D conflict graph.

Fig. 2. Conflict relationship represented by 3-D conflict graph in CRNs.

For illustrative purposes, we take a simple example to show how to construct a 3-D conflict graph. In this toy CRNs, we assume there are four CR routers with CR transceivers, i.e., A, B, C and D, and two bands, i.e., band 1 and band 2. Depending on the geographic locations, the set of currently available bands and radios at one CR router may be different from that at another CR router. For example, the currently available band and radio sets for A are $\mathcal{M}_A = \{1\}$ and $\mathcal{H}_A = \{1\}$, and the band and radio sets for B are $\mathcal{M}_B = \{1, 2\}$ and $\mathcal{H}_B = \{1, 2\}$. Furthermore, we use $d(\cdot)$ to represent Euclidean distance and suppose that $d(A, B) = d(B, C) = d(C, D) = d(D, E) = R_{Tx} = 0.5R_{In}$. Given the above assumptions, we can establish the corresponding 3-D conflict graph as depicted in Fig. 2(b). Here, each vertex corresponds to an LBR tuple, for example, vertex $((A, B), 1, (1, 1))$ corresponds to LBR tuple $((A, B), 1, (1, 1))$. Note that there is edge between vertices $((A, B), 1, (1, 1))$ and $((B, C), 1, (2, 1))$ because (A, B) is incident to (B, C) over band 1. There is an edge between vertices $((A, B), 1, (1, 1))$ and $((B, C), 2, (1, 1))$ because they share a radio in common at CR router B. Similar analysis applies to the other vertices in the conflict graph as well.

2) 3-D Independent Sets and Conflict Cliques: Given a 3-D conflict graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ representing spectrum clouds, we describe the impact of vertex $i \in \mathcal{V}$ on vertex $j \in \mathcal{V}$ as follows,

$$w_{ij} = \begin{cases} 1, \text{if there is an edge between vertex } i \text{ and } j \\ 0, \text{if there is no edge between vertex } i \text{ and } j, \end{cases} \quad (4)$$

where two vertices correspond to two LBR tuples, respectively.

Provided that there is a vertex set $\mathcal{I} \subseteq \mathcal{V}$ and an LBR tuple $i \in \mathcal{I}$ satisfying $\sum_{j\in\mathcal{I}, i\neq j} w_{ij} < 1$, the transmission at LBR tuple i will be successful even if all the other LBR tuples in the set $\mathcal{I}$ are transmitting at the same time. If any $i \in \mathcal{I}$ satisfies the condition above, we can schedule the transmissions over all these LBR tuples in $\mathcal{I}$ to be active simultaneously. Such a vertex/LBR tuple set $\mathcal{I}$ is called a 3-D independent set. If adding any one more LBR tuple into a 3-D independent set $\mathcal{I}$ results in a non-independent one, $\mathcal{I}$ is defined as a maximal 3-D independent set. Besides, if there exists a vertex/LBR tuple set $\mathcal{Z} \subseteq \mathcal{V}$ and any two vertexes i and j in $\mathcal{Z}$ satisfying $w_{ij} \neq 0$ (i.e., LBR tuples i and j cannot be scheduled to transmit successfully at the same time.), $\mathcal{Z}$ is called a 3-D conflict clique. If $\mathcal{Z}$ is no longer a 3-D conflict clique after adding any one more LBR tuple, $\mathcal{Z}$ is defined as a maximal 3-D conflict clique.

B. CR Link Scheduling and Flow Routing Constraints

1) CR Link Scheduling Constraints: Link scheduling can be conducted in time domain, in frequency domain or in both of them [18], [34]. In this paper, we only focus on time based link scheduling.

Given the 3-D conflict graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ constructed from the spectrum clouds, suppose we can list all maximal 3-D independent sets[6] as $\mathscr{I} = \{\mathcal{I}_1, \mathcal{I}_2, \cdots, \mathcal{I}_q, \cdots, \mathcal{I}_Q\}$, where Q is $|\mathscr{I}|$, and $\mathcal{I}_q \subseteq \mathcal{V}$ for $1 \leq q \leq Q$. At any time, at most one maximal 3-D independent set can be active to transmit packets for all LBR tuples in that set. Let $\lambda_q \geq 0$ denote the time share scheduled to the maximal 3-D independent set $\mathcal{I}_q$, and

$$\sum_{1\leq q\leq Q} \lambda_q \leq 1, \quad \lambda_q \geq 0\,(1 \leq q \leq Q). \tag{5}$$

Let $r_{ij}^m(\mathcal{I}_q)$ be the data rate for CR link (i,j) over band m, where $r_{ij}^m(\mathcal{I}_q) = 0$ if LBR tuple $((i,j), m, (u,v)) \notin \mathcal{I}_q$; otherwise, $r_{ij}^m(\mathcal{I}_q)$ is the achievable data rate for CR link (i,j) over band m, which can be calculated from (2). Therefore, by exploiting the 3-D maximal independent set $\mathcal{I}_q$, the flow rate that link (i,j) can support over band m in λ_q is $\lambda_q r_{ij}^m(\mathcal{I}_q)\boldsymbol{T}_{ij}^m$.

Furthermore, let $f_{ij}(l)$ represent the flow rate of the session l over link (i,j), where $i \in \mathcal{N}$, $l \in \mathcal{L}$ and $j \in \bigcup_{m\in\mathcal{M}_i} \mathcal{T}_i^m$. Then, the trading CR sessions are feasible at link (i,j) if there exists a schedule of the maximal 3-D independent sets satisfying

$$\sum_{l\in\mathcal{L}}^{s_r(l)\neq j, d_t(l)\neq i} f_{ij}(l)\delta(l) \leq \sum_{q=1}^{|\mathscr{I}|} \lambda_q \sum_{m\in\mathcal{M}_i \bigcap \mathcal{M}_j} r_{ij}^m(\mathcal{I}_q)\boldsymbol{T}_{ij}^m. \tag{6}$$

Note that in the equation above, $\boldsymbol{T}_{ij}^m$ is a random variable, which represents the uncertain spectrum supply in the temporal domain as introduced in Section III-C. In order to calculate the link capacity achieved by CR link scheduling in (6), we need to quantify the temporal spectrum availability when the vacancy of the licensed band is uncertain and modeled as a random variable. Inspired by the mathematical expression of value at risk (VaR) in [40], we leverage parameter α to define temporal spectrum availability at α and denote it by $X_\alpha(\boldsymbol{T})$ as follows.

$$\begin{cases} H_{\boldsymbol{T}}(\tau) = \displaystyle\int_\tau^\infty h_{\boldsymbol{T}}(t)dt, & \tau \in \mathcal{R} \\ X_\alpha(\boldsymbol{T}) = \sup\{\tau : H_{\boldsymbol{T}}(\tau) \geq \alpha\}, \alpha \in [0,1]. \end{cases} \tag{7}$$

Similar to the definition of the X loss in [25] and that of bandwidth integration in [18], $X_\alpha(\boldsymbol{T})$ can take the best usage of the statistics of $\boldsymbol{T}$ and quantify the temporal spectrum vacancy at confidence level of α. So, based on the definition of temporal spectrum availability at α, (6) can be reformulated as

$$\begin{aligned} &\sum_{l\in\mathcal{L}}^{s_r(l)\neq j, d_t(l)\neq i} f_{ij}(l)\delta(l) \\ &\quad \leq \sum_{q=1}^{|\mathscr{I}|} \lambda_q X_\alpha\Big(\sum_{m\in\mathcal{M}_i \bigcap \mathcal{M}_j} r_{ij}^m(\mathcal{I}_q)\boldsymbol{T}_{ij}^m\Big). \end{aligned} \tag{8}$$

2) CR Routing Constraints: As for routing, the SSP will help the source CR mesh router to find the available paths and employ a number of relay CR mesh routers to forward the data packets toward its destination CR mesh router. It is obvious that there should be more than one path involved in data delivery since multi-path routing[7] is more flexible to route the traffic from a source router to its destination. Similar to the modeling in [18], [34], we mathematically present routing constraints as follows.

To simplify the notation, let $\mathcal{T}_i = \bigcup_{m\in\mathcal{M}_i} \mathcal{T}_i^m$. If CR mesh router i is the source router of session l, i.e., $i = s_r(l)$, then

$$\sum_{j\in\mathcal{T}_i} f_{ji}(l) = 0. \tag{9}$$

$$\sum_{j\in\mathcal{T}_i} f_{ij}(l)\delta(l) = r(l)\delta(l), \tag{10}$$

where $\delta(l) \in \{0,1\}$ indicates whether session l is accepted by the SSP (i.e., session l wins the opportunity for data transmission via spectrum trading) or not.

If CR mesh router i is an intermediate relay router of session l, i.e., $i \neq s_r(l)$ and $i \neq d_t(l)$, then

$$\sum_{j\in\mathcal{T}_i}^{j\neq s_r(l)} f_{ij}(l)\delta(l) = \sum_{p\in\mathcal{T}_i}^{p\neq d_t(l)} f_{pi}(l)\delta(l). \tag{11}$$

If CR mesh router i is the destination router of session l, i.e., $i = d_t(l)$, then

$$\sum_{j\in\mathcal{T}_i} f_{ji}(l)\delta(l) = r(l)\delta(l). \tag{12}$$

Note that if (9), (10) and (11) are satisfied, it can be easily verified that (12) must be satisfied. As a result, it is sufficient to list only (9), (10) and (11) as CR routing constraints in spectrum clouds.

[6] It is a NP-complete problem to find all maximal independent sets in $\mathcal{G}$ [20], [38], [39], which will be further addressed later in this paper. In this subsection, we make the assumption we could find all the maximal independent sets just for the convenience of our theoretical analysis.

[7] The multiple radios of CR routers allow for multi-path routing.

C. Optimal Spectrum Trading under Multiple Constraints

In order to optimally trade spectrum resources and determine the access/denial of certain CR sessions, the SSP must consider the rate requirements and bidding values of CR sessions, the competition among different CR sessions, the availability of bands (including the SSP's leftover spectrum and the harvested spectrum) and the efficient utilization of spectrum resources. Thus, the SSP seeks for a feasible solution to trading the available frequency bands, assigning these bands to CR mesh routers, scheduling bands for CR transmission and reception and routing those CR flows so that the revenue of the SSP is maximized and radio spectrum resources are efficiently utilized in multi-hop CRNs.

With the proposed trading system, spectrum clouds, the optimal spectrum trading problem under multiple constraints in multi-hop CRNs can be formulated as follows.

$$\text{Maximize} \sum_{l\in\mathcal{L}} b(l)\delta(l)$$

$$\text{s.t.:} \quad \sum_{j\in\mathcal{T}_i} f_{ji}(l) = 0 \qquad (l\in\mathcal{L}, i = s_r(l)) \tag{13}$$

$$\sum_{j\in\mathcal{T}_i} f_{ij}(l)\delta(l) = r(l)\delta(l) \qquad (l\in\mathcal{L}, i = s_r(l)) \tag{14}$$

$$\sum_{j\in\mathcal{T}_i}^{j\neq s_r(l)} f_{ij}(l)\delta(l) = \sum_{p\in\mathcal{T}_i}^{p\neq d_t(l)} f_{pi}(l)\delta(l)$$
$$(l\in\mathcal{L}, i\in\mathcal{N}, i\neq s_r(l), d_t(l)) \tag{15}$$

$$\sum_{l\in\mathcal{L}}^{s_r(l)\neq j, d_t(l)\neq i} f_{ij}(l)\delta(l) \le \sum_{q=1}^{|\mathscr{I}|}\lambda_q X_\alpha\Big(\sum_{m\in\mathcal{M}_i\bigcap\mathcal{M}_j} r_{ij}^m(\mathcal{I}_q)\boldsymbol{T}_{ij}^m\Big)$$
$$(i\in\mathcal{N}, j\in\mathcal{T}_i, m\in\mathcal{M}_i\bigcap\mathcal{M}_j \text{and} \mathcal{I}_q\in\mathscr{I}) \tag{16}$$

$$\sum_{q=1}^{|\mathscr{I}|}\lambda_q \le 1,\ \lambda_q\ge 0 \qquad (\mathcal{I}_q\in\mathscr{I}) \tag{17}$$

$$f_{ij}(l)\ge 0\,(l\in\mathcal{L}, i\in\mathcal{N}, i\neq d_t(l), j\in\mathcal{T}_i, j\neq s_r(l)) \tag{18}$$

$$\delta(l)\in\{0,1\} \qquad (l\in\mathcal{L}), \tag{19}$$

where $\delta(l)$, $f_{ij}(l)$ and λ_q are optimization variables, and $r(l)$ is deterministic value when session l is given. Here, (13), (14) and (15) specify the routing constraints in spectrum clouds. (16) and (17) indicate that the flow rates over link (i,j) cannot exceed the capacity of this CR link, which is obtained from the CR link scheduling as illustrated in Section IV-B. Note that $\mathscr{I}$ includes all independent sets in CRNs. Given all the maximal 3-D independent sets[8] in $\mathcal{G}(\mathcal{V},\mathcal{E})$, we find that the formulated optimization is a mixed-integer linear programming (MILP) problem, which is NP-hard to solve as proved in [21], [39].

V. The Upper Bound for the Session Based Spectrum Trading Optimization

The complexity of the optimization above arises from two parts: (i) identifying all the maximal independent sets and (ii) fixing the binary $\delta(l)$ variables. To find all the maximal independent sets/cliques itself is NP-complete, but it is not a unique problem in spectrum clouds. It has been well investigated in prior multi-hop wireless networks and many approximation algorithms have been proposed in existing literature [20], [32]. For example, one of the typical approaches is to employ K $(0\le K\le|\mathscr{I}|)$ maximal independent sets (or a number of maximal conflict cliques) for approximation instead of finding out all the maximal independent sets in $\mathcal{G}(\mathcal{V},\mathcal{E})$.

On the other hand, $\delta(l)$ variables will be involved as long as the SSP conducts the session based spectrum trading in multi-hop CRNs. Given all the maximal independent sets, we relax the binary requirement on $\delta(l)$ and replace it with $0\le\delta(l)\le 1$ to reduce the complexity for the cross-layer optimization. Due to the enlarged optimization space (caused by relaxation on $\delta(l)$), the solution to this relaxed optimization problem yields an upper bound for the SSP's revenue maximization problem. Although the upper bound may not be achieved by a feasible solution, it can play as a benchmark to evaluate the quality of feasible solutions.

VI. A Bidding Value-Rate Requirement Ratio Based Heuristic Algorithm for Spectrum Trading

In order to find feasible solutions, in this section, we propose a bidding value-rate requirement ratio (BVR^3) based heuristic algorithm for the SSP's revenue maximization problem. According to the bidding values and rate requirements of candidate trading sessions, we make the SSP classify those CR sessions into different categories in terms of decreasing access possibility. Then, we sequentially fix the $\delta(l)$-variables in different sets and give a heuristic solution, which is also a lower bound for the original MILP problem.

A. The BVR^3 Based Relax-and-Fix Algorithm

The key to simplifying the NP-hard optimization, fixing flow routing (i.e., $f_{ij}(l)$-variables) and link scheduling (i.e., λ_q-variables), and attaining a feasible solution is the determination of the binary values for the $\delta(l)$-variables [18], [34]. Although we can employ the classical branch-and-bound approach to determine $\delta(l)$-variables, the number of iterations involved in that algorithm grows exponentially with $|\mathcal{L}|$. To reduce the complexity, we propose a BVR^3 based *relax-and-fix* algorithm [21]. The intuition behind the proposed algorithm is that given the leftover basic spectrum and the harvested spectrum, the SSP would like to take the best use of spectrum resources to make as much revenue as possible. That can be roughly interpreted as the SSP prefers to access the CR session with large bidding value and small rate requirements in spectrum clouds. The detailed procedure of the heuristic algorithm for the SSP's revenue maximization is presented as follows.

Based on bidding values and rate requirements of candidate CR sessions, we first sort all the CR sessions in terms of $\frac{b(l)}{r(l)}$ and partition these sessions into S disjoint session sets $\mathcal{L}^1,\mathcal{L}^2,\cdots,\mathcal{L}^S$ in the order of decreasing BVR^3, where $\bigcup_{s\in\mathcal{S}}\mathcal{L}^s=\mathcal{L}$ and $\mathcal{S}=\{1,2,\cdots,S\}$. The BVR^3 of the session in $\mathcal{L}^i$ is larger than that of the session in $\mathcal{L}^j$, if i is less than j $(\forall i,j\in\mathcal{S})$.

Then, we create auxiliary session sets by choosing subsets $\mathcal{A}^s$ with $\mathcal{A}^s\subseteq\bigcup_{u=s+1}^{S}\mathcal{L}^u$ for $s\in\{1,2,\cdots,S-1\}$. For example, in the spectrum trading problem, $\mathcal{L}^1$ may include

[8]That is a general assumption used in existing literature [20], [32], [33] for obtaining throughput bounds or performance comparison.

the $\delta(l)$-variables associated with candidate trading sessions in $\{1, 2, \cdots, l_1\}$, $\mathcal{L}^2$ may be associated with sessions in $\{l_1+1, l_1+2, \cdots, l_2\}$, and so on, whereas $\mathcal{A}^1$ would include the $\delta(l)$-variables associated with sessions in $\{l_1+1, l_1+2, \cdots, a_1\}$, and so on.

By leveraging partitioned session sets (i.e., $\mathcal{L}^s$) and auxiliary session sets (i.e., $\mathcal{A}^s$), we sequentially solve $|\mathcal{S}|$ relaxed-MILPs (R-MILPs) (denoted by *R-MILP* ^{s} with $1 \leq s \leq |\mathcal{S}|$), determine the δ-variables in $\mathcal{L}^s$ ($s \in \mathcal{S}$) and find a heuristic solution to the original MILP problem. Specifically, in the first R-MILP, *R-MILP* ^{1}, we only impose the binary requirement on the $\delta(l)$-variables for session l in $\mathcal{L}^1 \cup \mathcal{A}^1$ and relax the integrality restriction on all the other $\delta(l)$-variables for session l in $\mathcal{L}$. Thus, we have

$$\textit{R-MILP}^1 \qquad \text{Maximize} \sum_{l \in \mathcal{L}} b(l)\delta(l)$$

$$\begin{aligned} \text{s.t.:}\quad & (13),\ (14),\ (15),\ (16),\ (17),\ (18) \\ & \delta(l) \in \{0,1\} \qquad (\forall\, l \in \mathcal{L}^1 \cup \mathcal{A}^1) \\ & \delta(l) \in [0,1] \qquad (\forall\, l \in \mathcal{L} \backslash (\mathcal{L}^1 \cup \mathcal{A}^1)) \end{aligned}$$

Let $\{\hat{\delta}^1(1), \cdots, \hat{\delta}^1(l), \cdots, \hat{\delta}^1(L)\}$ be an optimal solution to *R-MILP* ^{1}. We can fix the $\delta(l)$-variables in $\mathcal{L}^1$ at their corresponding binary values, i.e., $\delta(l) = \hat{\delta}^1(l) \in \{0,1\}$ for all $l \in \mathcal{L}^1$. Then, we move to *R-MILP* ^{2}.

In the subsequent *R-MILP* ^{s} (for $2 \leq s \leq S$), we sequentially fix the binary values of the $\delta(l)$-variables for sessions in $\mathcal{L}^{s-1}$ from the solution to *R-MILP* $^{s-1}$. After that, we further add the binary restriction for the $\delta(l)$-variables in $\mathcal{L}^s \cup \mathcal{A}^s$, and we have

$$\textit{R-MILP}^s \qquad \text{Maximize} \sum_{l \in \mathcal{L}} b(l)\delta(l)$$

$$\begin{aligned} \text{s.t.:}\quad & (13),\ (14),\ (15),\ (16),\ (17),\ (18) \\ & \delta(l) = \hat{\delta}^{s-1}(l) \qquad (\forall\, l \in \mathcal{L}^1 \cup \cdots \cup \mathcal{L}^{s-1}) \\ & \delta(l) \in \{0,1\} \qquad (\forall\, l \in \mathcal{L}^s \cup \mathcal{A}^s) \\ & \delta(l) \in [0,1] \qquad (\forall\, l \in \mathcal{L} \backslash (\mathcal{L}^1 \cup \cdots \cup \mathcal{L}^s \cup \mathcal{A}^s)). \end{aligned}$$

Either *R-MILP* ^{s} is infeasible for certain $s \in \mathcal{S}$ and the heuristic algorithm has failed, or else the proposed BVR ^{3} based relax-and-fix algorithm provides a feasible solution (i.e., the solution to *R-MILP* $^{|\mathcal{S}|}$) to the original MILP problem. The procedure of the proposed heuristic algorithm is summarized in Alg. 1.

For illustrative purposes, we take a multi-hop CRN consisting of 7 candidate trading CR sessions as an example. We sort these sessions by BVR ^{3} and divide them into 4 disjoint session sets, i.e., $|\mathcal{S}| = 4$. We conduct the BVR ^{3} based relax-and-fix algorithm with the following sets $\mathcal{L}^s$ and $\mathcal{A}^s$: $\mathcal{L}^1 = \{1,2\}$, $\mathcal{L}^2 = \mathcal{A}^1 = \{3,4\}$, $\mathcal{L}^3 = \mathcal{A}^2 = \{5,6\}$, and $\mathcal{L}^4 = \mathcal{A}^3 = \{7\}$. The iterations of the heuristic algorithm are as follows.

- In the first *R-MILP* ^{1}, the $\delta(l)$-variables associated with sessions in $\{1, \cdots, 4\}$ (i.e., in $\mathcal{L}^1 \cup \mathcal{A}^1$) are restricted to be binary values, the other $\delta(l)$-variables being relaxed.
- From the solution to *R-MILP* ^{1}, we can fix the $\delta(l)$-variables corresponding to the sessions in $\{1,2\}$ (i.e., in $\mathcal{L}^1$). With the determined $\delta(l)$-variables for sessions in $\mathcal{L}^1$, we continue to solve *R-MILP* ^{2} where the $\delta(l)$-variables associated with sessions in $\{3, \cdots, 6\}$ (i.e., in $\mathcal{L}^2 \cup \mathcal{A}^2$) are now integer and $\delta(l)$-variables in $\{7\}$ (i.e., in $\mathcal{L} \backslash (\mathcal{L}^1 \cup \mathcal{L}^2 \cup \mathcal{A}^2)$) are relaxed.
- From the solution to *R-MILP* ^{2}, we can additionally fix the $\delta(l)$-variables corresponding to the sessions in $\{3,4\}$ (i.e., in $\mathcal{L}^2$). Similarly, we can solve *R-MILP* ^{3} where the $\delta(l)$-variables associated with sessions in $\{5,6,7\}$ (i.e., in $\mathcal{L}^3 \cup \mathcal{A}^3$) are now binary and there are no $\delta(l)$-variables to relax because $\mathcal{L} \backslash (\mathcal{L}^1 \cup \mathcal{L}^2 \cup \mathcal{L}^3 \cup \mathcal{A}^3) = \phi$.
- Based on the optimal solution to *R-MILP* ^{3}, we can easily determine the value of $\delta(l)$ in $\{7\}$ and determine whether there is feasible solution to the original MILP.

Algorithm 1 The BVR ^{3} based relax-and-fix algorithm

Sort all the CR sessions in terms of BVR ^{3}, i.e., $\frac{b(l)}{r(l)}$.
Partition all these sessions into S disjoint session sets, denoted by $\mathcal{L}^s$ ($s \in \mathcal{S} = \{1, 2, \cdots, S\}$ and $\mathcal{L}^s \subset \mathcal{L}$).
Create auxiliary session sets $\mathcal{A}^s \subseteq \bigcup_{u=s+1}^{S} \mathcal{L}^u$.
Set s = 1 and relax binary requirement on $\delta(l)$-variables.
for all $s \in \mathcal{S}$ **do**
  Impose binary requirement on the $\delta(l)$-variables for session $l \in \mathcal{L}^s \cup \mathcal{A}^s$.
  Using $\mathcal{L}^s$ and $\mathcal{A}^s$, solve the relaxed *R-MILP* ^{s}.
  if *R-MILP* ^{s} has a feasible solution **then**
    Determine the δ-variables in $\mathcal{L}^s$.
    $s = s + 1$. **continue**
  else
    Return there is no feasible solution.
  end if
end for
Output the solution to *R-MILP* $^{|\mathcal{S}|}$ as a feasible solution to the original MILP.

The basic idea of the BVR ^{3} based relax-and-fix algorithm is explicitly explained in the example. At each iteration, we solve a *R-MILP* ^{s} problem involving $\mathcal{L}^s \cup \mathcal{A}^s$ sessions and to avoid being too myopic we then only fix the $\delta(l)$-variables corresponding to sessions in $\mathcal{L}^s$. The auxiliary session sets $\mathcal{A}^s$ smooth the heuristic solution by creating some overlap between successive session sets.

Different from the upper bound obtained in Section V, the proposed BVR ^{3} based relax-and-fix algorithm yields a lower bound to the optimal spectrum trading problem formulated in Section IV-C, provided that there exist feasible solutions.

B. A Coarse-Grained Relax-and-Fix Heuristic Algorithm

Following the same procedure in Section V, we first relax the original MILP into LP and find the optimal solution to the relaxed LP, in which $\delta(l)$'s value is in $[0,1]$. By employing a threshold $0.5 \leq \theta < 1$, we coarsely set the $\delta(l)$-variables exceeding θ to 1 and the other $\delta(l)$-variables to 0. Denote the value of $\delta(l)$ in this solution as $\tilde{\delta}(l) \in \{0,1\}$. In addition, we keep the same decomposition of session sets as the BVR ^{3} based relax-and-fix algorithm, i.e., $\mathcal{L}^s$ and $\mathcal{A}^s$ for $s \in \mathcal{S}$.

Then, at each step s ($s \in \mathcal{S}$), all $\delta(l)$-variables are fixed at their $\tilde{\delta}(l)$ values in the best solution found so far (or in the last solution encountered), except the $\delta(l)$-variables in the set

$\mathcal{L}^s \cup \mathcal{A}^s$ which are restricted to binary values. Therefore, the problem solved at step s is

$$\textit{R-MILP}^s \qquad \text{Maximize} \sum_{l \in \mathcal{L}} b(l)\delta(l)$$

$$\begin{aligned} \text{s.t.:} \quad & (13),\ (14),\ (15),\ (16),\ (17),\ (18) \\ & \delta(l) = \tilde{\delta}(l) \qquad (\forall\, l \in \mathcal{L} \backslash (\mathcal{L}^s \cup \mathcal{A}^s)) \\ & \delta(l) \in \{0, 1\} \qquad (\forall\, l \in \mathcal{L}^s \cup \mathcal{A}^s). \end{aligned}$$

If a better solution is found, $\tilde{\delta}(l)$ is updated and the fixing procedure continues. Compared with the BVR^3 based relax-and-fix algorithm, different steps s ($s \in \mathcal{S}$) in coarse-grained relax-and-fix heuristic are independent of one another, and any subset of $\mathcal{S}$ can be performed in any order.

VII. Performance Evaluation

A. Simulation Setup

We consider a spectrum clouds in multi-hop CRNs consisting of a SSP, $|\mathcal{N}|$ = 36 CR mesh routers and $|\mathcal{L}|$ = 18 candidate trading sessions, each of which has a random rate requirement within $[10, 30]$ Mb/s. The bidding values of these sessions are within $[100, 300]$. All CR mesh routers use the same power P = 10 W for transmission. Considering the AWGN channel, we assume the noise power η is 10^{-10} W at all routers. Moreover, suppose the path loss factor $\beta = 4$, the antenna parameter $\gamma = 3.90625$, the receiver sensitivity $P_{Tx} = 100\eta = 10^{-8}$ W and the interference threshold $P_{In} = 6.25 \times 10^{-10}$ W. According to the illustration in Section III-C, we can calculate the transmission range R_{Tx} and the interference range R_{In}, which are equal to 250 m and 500 m, respectively. For illustrative purposes, we assume all the bands have identical bandwidth, which is set to be 10 MHz, i.e., W^m = 10 MHz for all $m \in \mathcal{M}$. Based on the observed data and the statistical analysis in [5], the available time of a licensed band follows the truncated exponential distribution within $[0, 1]$, i.e., $h_{\boldsymbol{T}}(t, \xi) = \frac{\frac{1}{\xi}e^{-\frac{t}{\xi}}}{1-e^{-\frac{1}{\xi}}}$, where $\xi \in (0, 3]$. As for the confidence level, we set α = 0.85. Besides, for the simplicity of computation, we set $K = 1 \times 10^4$, i.e., if the total number of the maximal independent sets in $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is less than or equal to 1×10^4, we employ all the maximal independent sets for the solution; otherwise, we employ 1×10^4 maximal independent sets for approximation.

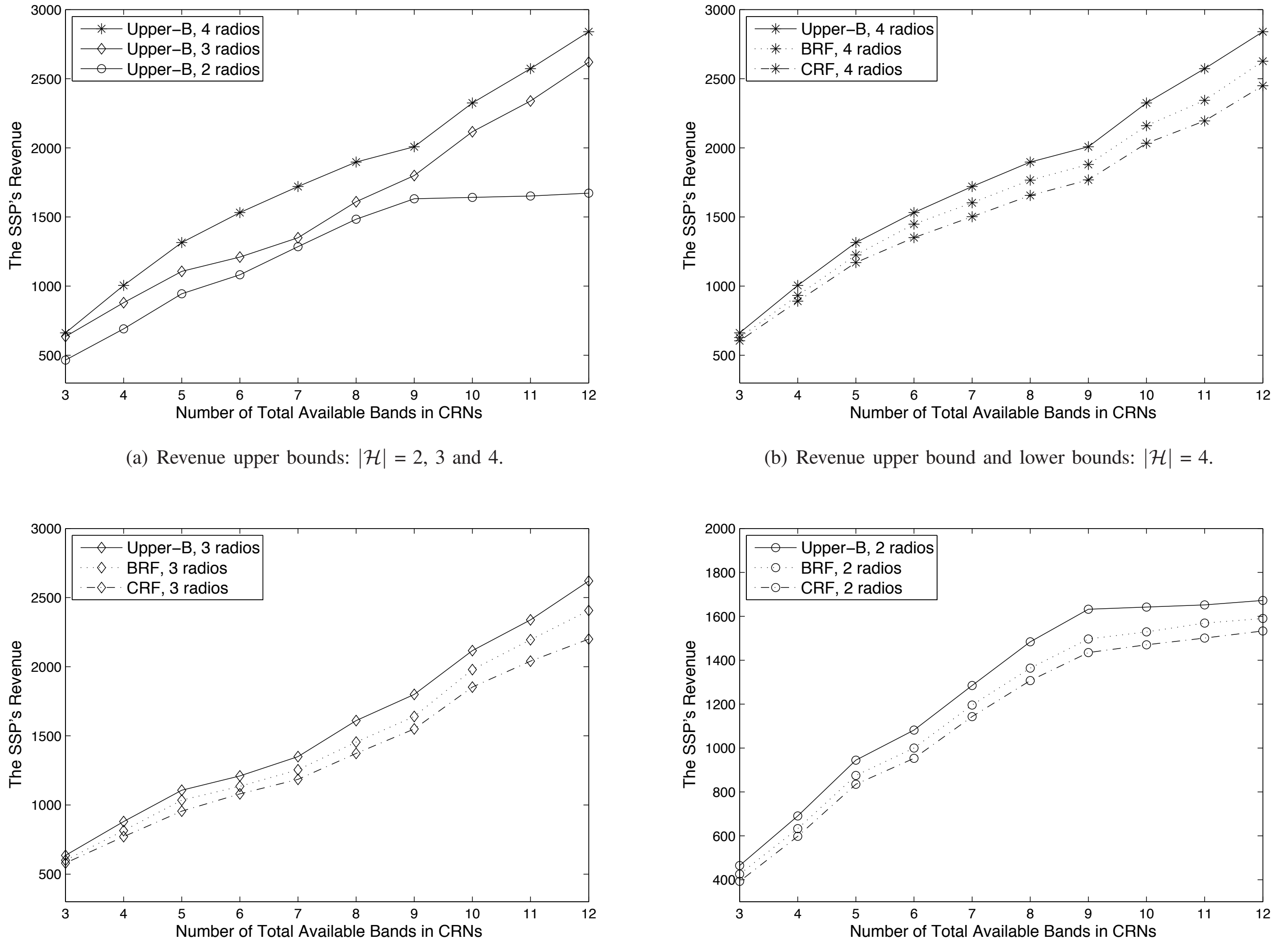
Based on the simulation settings above, we conduct simulations to study the optimal spectrum trading problem in spectrum clouds with the following two topologies: i) a grid topology, where 36 CR mesh routers are distributed within 1000×1000 m^2 area and the area is divided into 25 square cells in 200×200 m^2; ii) a random topology, where 36 CR mesh routers are randomly deployed in a 1000×1000 m^2 area forming a connected network. Note that we employ CPLEX [41] to solve the relaxed optimization problems to obtain the upper bound and lower bounds of the SSP's revenue.

B. Results and Analysis

In Fig. 3 and Fig. 4, we compare the upper bound of the SSP's revenue with the lower bounds determined by the heuristic BVR^3 based relax-and-fix algorithm (denoted by BRF in figures) and the coarse-grained relax-and-fix algorithm (denoted by CRF in figures) at different number of available bands (i.e., $|\mathcal{M}|$) and radios (i.e., $|\mathcal{H}|$) in multi-hop CRNs. We relax the $\delta(l)$-variables and employ $K = 1 \times 10^4$ maximal independent sets to solve the problem as illustrated in Section V, which also yields the upper bound. To develop the lower bounds, we equally divide the 18 candidate trading sessions into 6 session sets (i.e., $|\mathcal{S}|$ = 6 and each set has 3 sessions) for the BVR^3 based relax-and-fix algorithm, and set θ = 0.7 for the coarse-grained relax-and-fix algorithm as shown in Section VI. Given the number of available bands $|\mathcal{M}|$ in CRNs and radios $|\mathcal{H}|$ at CR routers, we employ 50 data sets that can produce feasible solutions and take the average value as a result. For each data set, we re-generate available bands $\mathcal{M}_i$ at CR router i, $s_r(l)$/$d_t(l)$ and $(r(l), b(l))$ pair of session l, and the random network topology (we keep the same grid topology for each data set), which follows the guideline of simulation setup.

From the results shown in Fig. 3 and Fig. 4, four observations can be made in order. First, the upper bound is close to the lower bounds obtained from the proposed BVR^3 based relax-and-fix algorithm and the coarse-grained relax-and-fix algorithm, no matter how many available bands and radios are there in the spectrum clouds. We will further present the ratio of the upper bound to lower bounds with 50 data sets in Fig. 5, analyze the statistical results and show the closeness between those bounds. Second, as the number of available bands and the number of CR mesh router's radios increase, the SSP's revenue increases as well. The reason is that more bands and radios available create more LBR tuples, so that more CR links in spectrum clouds may be activated for transmission simultaneously and more opportunities can be leveraged for spectrum trading in CRNs. However, the increment of the SSP's revenue basically stops when $|\mathcal{M}|$ is over 9 for $|\mathcal{H}|$ = 2 case in both grid topology and random topology, which leads to the third observation. That is, the CR mesh router has to equip a reasonable number of radios to utilize all the available bands efficiently (at least 3 radios for our simulation scenarios). This observation also gives a good suggestion on the design and deployment of CR mesh routers for spectrum clouds in practice. Fourth, the performance of the grid topology generally outperforms that of the random topology in terms of the SSP's revenue. The performance gap stems from the differences in topological structure. For the grid topology, each CR link has the same topological information if we ignore the border effect. The performance improvement of spectrum trading is mainly determined by the number of radios and the available bands at different CR routers. By contrast, the random topology is non-uniformed topology. The performance improvement of spectrum trading is not only hindered by the number of bands and radios, but also bottlenecked by the critical cliques in the random topology.

Fig. 5 presents the ratio of the upper bound to the lower bounds obtained from the proposed heuristic algorithms in both grid topology and random topology, where $|\mathcal{H}|$ = 3 and $|\mathcal{M}|$ = 9. As shown in Fig. 5(a) and Fig. 5(b), the ratio of the upper bound to lower bound in the grid topology is near to 1 with 50 different data sets, where the lower bounds



(a) Revenue upper bounds: $|\mathcal{H}|$ = 2, 3 and 4.

(b) Revenue upper bound and lower bounds: $|\mathcal{H}|$ = 4.

(c) Revenue upper bound and lower bounds: $|\mathcal{H}|$ = 3.

(d) Revenue upper bound and lower bounds: $|\mathcal{H}|$ = 2.

Fig. 3. Impact of the number of available bands $|\mathcal{M}|$ and radio interfaces $|\mathcal{H}|$ on spectrum trading in multi-hop CRNs: grid topology.

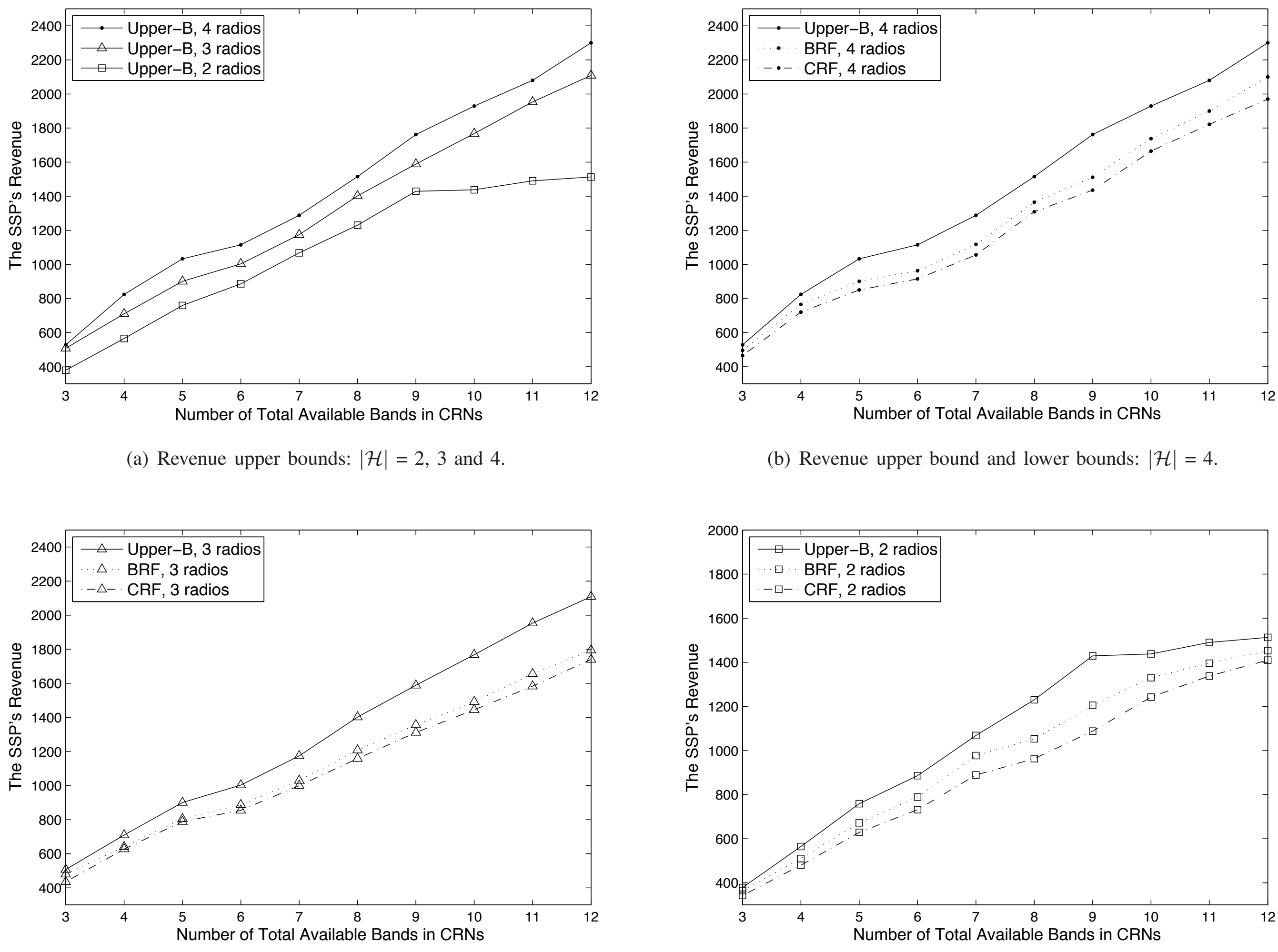
are determined by the BVR^3 based relax-and-fix algorithm and the coarse-grained relax-and-fix algorithm, respectively. Specifically, the mean ratio of the upper bound to the BVR^3 based lower bound for all the data sets is 1.0973, and the standard deviation is 0.0707; the mean ratio of the upper bound to the coarse-grained based lower bound for all the data sets is 1.1462, and the standard deviation is 0.1255. Similar analysis applies to the random topology as well. As shown in Fig. 5(c) and Fig. 5(d), the mean ratio of the upper bound to the BVR^3 based lower bound for all the data sets is 1.1722, and the standard deviation is 0.1365; the mean ratio of the upper bound to the coarse-grained based lower bound for all the data sets is 1.2113, and the standard deviation is 0.1595. All these statistical results indicate that the solutions found by the heuristic algorithms must be close to the optimum, since the optimal solution lies between the upper bound and the lower bound.

Given the specific data set at $|\mathcal{H}| = 3$ and $|\mathcal{M}| = 9$, Table I(a) and Table I(b) present the trading status of the 18 candidate sessions w.r.t. BVR^3 values in the grid topology and the random topology, respectively. The results[9] demonstrate that unlike per-user based spectrum trading in CRNs, it is not necessary for the SSP to accommodate the CR sessions with high BVR^3 values in order to maximize the SSP's revenue. Some other critical factors may also affect the results of the session based spectrum trading in multi-hop CRNs, e.g., the location of source/destination CR routers of a session, the interference a session incurs to the existing flows, etc. As shown in the formulation, the proposed spectrum clouds gives a comprehensive consideration on those factors. The data in Table I further verify this statement and explicitly show the advantages of our design over the per-user based spectrum trading systems in multi-hop CRNs.

[9]We exploit the proposed BVR^3 based relax-and-fix algorithm to derive these results in both the grid topology and the random topology.

VIII. Conclusion

In this paper, we have proposed a novel spectrum trading system, i.e., spectrum clouds, and presented a theoretical study on the optimal session based spectrum trading problem under uncertain spectrum supply and multiple cross-layer constraints in multi-hop CRNs. We introduce a new service provider, SSP, and let the SSP provide coverage in CRNs with low-cost CR mesh routers in order to facilitate the accessing of SUs without CR capability. To capture the special features of spectrum uncertainty, we exploit the statistics of licensed spectrum



(a) Revenue upper bounds: $|\mathcal{H}|$ = 2, 3 and 4.

(b) Revenue upper bound and lower bounds: $|\mathcal{H}|$ = 4.

(c) Revenue upper bound and lower bounds: $|\mathcal{H}|$ = 3.

(d) Revenue upper bound and lower bounds: $|\mathcal{H}|$ = 2.

Fig. 4. Impact of the number of available bands $|\mathcal{M}|$ and radio interfaces $|\mathcal{H}|$ on spectrum trading in multi-hop CRNs: random topology.

vacancy and model it as random variables. Under the proposed CRN architecture and the modeling of uncertain spectrum supply, we employ the 3-D (link-band-radio) conflict graph to characterize the conflicts among CR links and mathematically describe the competitions among candidate trading sessions in spectrum clouds. Given the rate requirements and bidding values of candidate trading sessions, we formulate the optimal spectrum trading into the SSP's revenue maximization problem under the availability of spectrum in both temporal and spatial domains, link scheduling and flow routing constraints in multi-hop CRNs. Since the formulated problem is NP-hard to solve, we derive an upper bound for the optimization by relaxing the integer variables. Furthermore, we propose heuristic algorithms for feasible solutions (low bounds as well). Through simulations, we show that: i) the proposed session based spectrum trading has superior advantages over the per-user based one in multi-hop CRNs; ii) the solutions attained by the proposed heuristic algorithms are near-optimal under different data sets in both the grid topology and the random one.

REFERENCES

[1] M. Pan, P. Li, Y. Song, Y. Fang, and P. Lin, "Spectrum clouds: A session based spectrum trading system for multi-hop cognitive radio networks," in *Proc. IEEE Conf. Comput. Commun. (INFOCOM)*, Mar. 2012.

[2] *IEEE 802.22-2011(TM) Standard for Cognitive Wireless Regional Area Networks (RAN) for Operation in TV Bands*, July 2011.

[3] J. Mitola, "Cognitive radio: An integrated agent architecture for software defined radio," Ph.D. Thesis, Royal Institute of Technology, Sweden, May 2000.

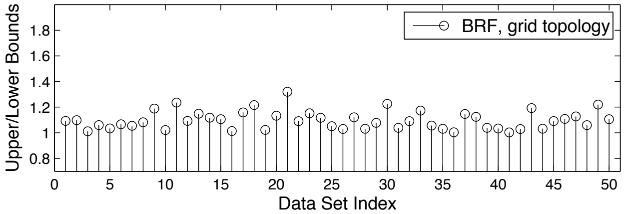
[4] I. Akyildiz, W. Lee, M. Vuran, and M. Shantidev, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Netw. (Elsevier) J.*, vol. 50, no. 4, pp. 2127–2159, Sep. 2006.

[5] D. Chen, S. Yin, Q. Zhang, M. Liu, and S. Li, "Mining spectrum usage data: A large-scale spectrum measurement study," in *Proc. ACM Int. Conf. Mobile Comput. Netw. ACM (Mobicom)*, Sep. 2009.

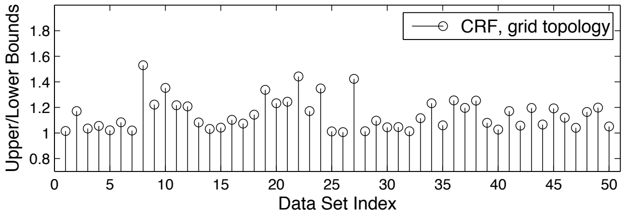
[6] S. M. Mishra, D. Cabric, C. Chang, D. Willkomm, B. V. Schewick, A. Wolisz, and R. W. Brodersen, "A real time cognitive radio testbed for physical and link layer experiments," in *Proc. IEEE Int. Symp. New Frontiers Dynamic Spectrum Access Netw. (DySPAN)*, Nov. 2005.

[7] M. A. McHenry, P. A. Tenhula, D. McCloskey, D. A. Roberson, and C. S. Hood, "Chicago spectrum occupancy measurements and analysis and a long-term studies proposal," in *Proc. Int. Conf. Theory Practice Algorithms (Computer) Syst. (TAPAS)*, Aug. 2006.

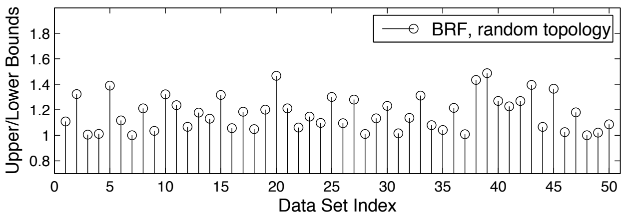
[8] D. Grandblaise, P. Bag, P. Levine, K. Moessner, and M. Pan, "Microeconomics inspired mechanisms to manage dynamic spectrum allocation,"



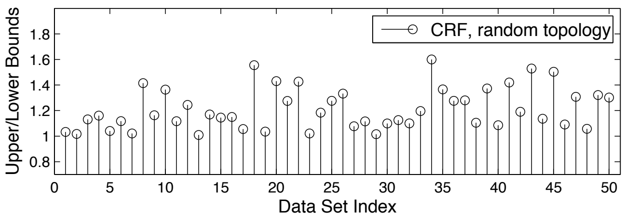
(a) Ratio of the upper bound to the lower bound determined by the BVR^3 based relax-and-fix algorithm: grid topology.



(b) Ratio of the upper bound to the lower bound determined by the coarse-grained relax-and-fix algorithm: grid topology.



(c) Ratio of the upper bound to the lower bound determined by the BVR^3 based relax-and-fix algorithm: random topology.



(d) Ratio of the upper bound to the lower bound determined by the coarse-grained relax-and-fix algorithm: random topology.

Fig. 5. Ratio of the upper bound to lower bounds determined by the proposed algorithms at $|\mathcal{H}| = 3$ and $|\mathcal{M}| = 9$.

in *Proc. IEEE Int. Symp. New Frontiers Dynamic Spectrum Access Netw. (DySPAN)*, Apr. 2007.

[9] S. Sengupta and M. Chatterjee, "An economic framework for dynamic spectrum access and service pricing," *IEEE/ACM Trans. Netw.*, vol. 17, no. 4, pp. 1200–1213, Aug. 2009.

[10] X. Zhou, S. Gandhi, S. Suri, and H. Zheng, "Ebay in the sky: Strategy-proof wireless spectrum auctions," in *Proc. ACM Mobile Comput. Netw. Conf. (Mobicom)*, Sep. 2008.

[11] J. Jia, Q. Zhang, Q. Zhang, and M. Liu, "Revenue generation for truthful spectrum auction in dynamic spectrum access," in *Proc. ACM Int. Symp. Mobile Ad Hoc Netw. Comput. (MobiHoc)*, May 2009.

[12] Y. Wu, B. Wang, K. J. Liu, and T. Clancy, "A multi-winner cognitive spectrum auction framework with collusion-resistant mechanisms," in *Proc. IEEE Int. Symp. New Frontiers Dynamic Spectrum Access Netw. (DySPAN)*, Oct. 2008.

[13] L. Gao, X. Wang, Y. Xu, and Q. Zhang, "Spectrum trading in cognitive radio networks: A contract-theoretic modeling approach," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 4, pp. 843–855, Apr. 2012.

[14] X. Wang, Z. Li, P. Xu, Y. Xu, X. Gao, and H. Chen, "Spectrum sharing in cognitive radio networks - An auction based approach," *IEEE Trans. Syst. Man Cybern. B, Cybern.*, vol. 40, no. 3, pp. 587–596, May 2010.

[15] J. H. Reed, *Software Radio: A Modern Approach to Radio Engineering*. New York: Prentice Hall, May 2002.

[16] H. So, A. Tkachenko, and R. W. Brodersen, "A unified hardware/software runtime environment for FPGA based reconfigurable computers using borph," in *Proc. IEEE/ACM/IFIP Int. Conf. Hardware-Software Codesign Syst. Synthesis*, Oct. 2006.

[17] Defense Advanced Research Projects Agency (DARPA), "The neXt generation program (XG) official website," [Online]. Available: http://www.darpa.mil/sto/smallunitops/xg.html

[18] M. Pan, C. Zhang, P. Li, and Y. Fang, "Joint routing and scheduling for cognitive radio networks under uncertain spectrum supply," in *Proc. IEEE Conf. Comput. Commun. (INFOCOM)*, Apr. 2011.

[19] L. Duan, J. Huang, and B. Shou, "Cognitive mobile virtual network operator: Investment and pricing with supply uncertainty," in *Proc. IEEE Conf. Comput. Commun. (INFOCOM)*, Mar. 2010.

[20] H. Li, Y. Cheng, C. Zhou, and P. Wan, "Multi-dimensional conflict graph based computing for optimal capacity in MR-MC wireless networks," in *Proc. IEEE 30th Int. Conf. Distrib. Comput. Syst. (ICDCS)*, June 2010.

[21] Y. Pochet and L. A. Wolsey, *Production Planning by Mixed Integer Programming*. Secaucus, NJ: Springer-Verlag New York, Inc., 2006.

[22] Y. Xing, R. Chandramouli, and C. Cordeiro, "Price dynamics in competitive agile spectrum access markets," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 613–621, Apr. 2007.

[23] D. Niyato and E. Hossain, "Competitive pricing for spectrum sharing in cognitive radio networks: Dynamic game, inefficiency of Nash

TABLE I
SPECTRUM TRADING STATUS OF THE CANDIDATE SESSIONS W.R.T. THE DESCENDING BVR^3 VALUES IN MULTI-HOP CRNS.

(a) Grid topology with 3 radios and 9 bands

S-Index	BVR^3 Val.	Status		S-Index	BVR^3 Val.	Status
1	25.001	$\surd$		10	15.353	$\times$
2	23.422	$\surd$		11	14.742	$\times$
3	21.811	$\surd$		12	14.071	$\times$
4	21.489	$\surd$		13	12.996	$\times$
5	20.014	$\surd$		14	12.159	$\times$
6	19.125	$\surd$		15	10.016	$\surd$
7	17.475	$\times$		16	8.287	$\times$
8	17.212	$\times$		17	6.883	$\times$
9	16.135	$\times$		18	5.295	$\times$

(b) Random topology with 3 radios and 9 bands

S-Index	BVR^3 Val.	Status		S-Index	BVR^3 Val.	Status
1	24.211	$\surd$		10	12.587	$\surd$
2	22.023	$\surd$		11	11.233	$\times$
3	20.835	$\surd$		12	10.489	$\times$
4	19.333	$\surd$		13	10.038	$\times$
5	18.025	$\surd$		14	8.955	$\times$
6	15.667	$\times$		15	7.122	$\times$
7	14.511	$\times$		16	6.734	$\times$
8	13.936	$\times$		17	5.533	$\times$
9	13.012	$\times$		18	4.327	$\times$

equilibrium, and collusion," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 192–202, Jan. 2008.

[24] D. Niyato, E. Hossain, and Z. Han, "Dynamics of multiple-seller and multiple-buyer spectrum trading in cognitive radio networks: A game theoretic modeling approach," *IEEE Trans. Mobile Comput.*, vol. 8, no. 8, pp. 1009–1022, Aug. 2009.

[25] M. Pan, H. Yue, Y. Fang, and H. Li, "The x loss: Band-mix selection for opportunistic spectrum accessing with uncertain supply from primary service providers," *IEEE Trans. Mobile Comput.*, vol. 11, no. 12, pp. 2133–2144, Dec. 2012.

[26] M. Pan, Y. Song, P. Li, and Y. Fang, "Reward and risk for opportunistic spectrum accessing in cognitive radio networks," in *Proc. IEEE Global Telecommun. Conf. (Globecom)*, Dec. 2010.

[27] B. Wang, Z. Han, and K. J. R. Liu, "Distributed relay selection and power control for multiuser cooperative communication networks using buyer/seller game," in *Proc. IEEE Conf. Comput. Commun. (INFOCOM)*, May 2007.

[28] J. Zhang and Q. Zhang, "Stackelberg game for utility-based cooperative cognitive radio networks," in *Proc. ACM Int. Symp. Mobile Ad Hoc Netw. Comput. (MobiHoc)*, May 2009.

[29] X. Zhou and H. Zheng, "Breaking bidder collusion in large-scale spectrum auctions," in *Proc. ACM Int. Symp. Mobile Ad Hoc Netw. Comput. (MobiHoc)*, Sep. 2010.

[30] M. Pan, J. Sun, and Y. Fang, "Purging the back-room dealing: Secure spectrum auction leveraging Paillier cryptosystem," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 4, pp. 866–876, Apr. 2011.

[31] K. Jain, J. Padhye, V. N. Padmanabhan, and L. Qiu, "Impact of interference on multi-hop wireless network performance," in *Proc. ACM Int. Conf. Mobile Comput. Netw. ACM (Mobicom)*, Sep. 2003.

[32] H. Zhai and Y. Fang, "Impact of routing metrics on path capacity in multirate and multihop wireless ad hoc networks," in *Proc. IEEE Int. Conf. Netw. Protocols (ICNP)*, Nov. 2006.

[33] J. Tang, S. Misra, and G. Xue, "Joint spectrum allocation and scheduling for fair spectrum sharing in cognitive radio wireless networks," *Comput. Netw. (Elsevier) J.*, vol. 52, no. 11, pp. 2148–2158, Aug. 2008.

[34] Y. T. Hou, Y. Shi, and H. D. Sherali, "Spectrum sharing for multi-hop networking with cognitive radios," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 146–155, Jan. 2008.

[35] A. Goldsmith, *Wireless Communications*. New York: Cambridge University Press, 2005.

[36] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[37] Y. Shi, Y. T. Hou, and S. Kompella, "How to correctly use the protocol interference model for multi-hop wireless networks," in *Proc. ACM Int. Symp. Mobile Ad Hoc Netw. Comput. (MobiHoc)*, May 2009.

[38] R. Diestel, *Graph Theory*. New York: Springer-Verlag New York, Inc., 2005.

[39] M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*. New York: W. H. Freeman and Company, 1979.

[40] G. Holton, *Value-at-Risk: Theory and Practice*. San Diego: Academic Press, 2003.

[41] IBM ILOG CPLEX Optimizer [Online]. Available: http://www-01.ibm.com/software/integration/optimization/cplex-optimizer/