Cooperative Communication Aware Link Scheduling for Cognitive Vehicular Networks

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Abstract-Throughput maximization is a key challenge for wireless applications in cognitive Vehicular Ad-hoc Networks (C-VANETs). As a potential solution, cooperative communications, which may increase link capacity by exploiting spatial diversity, has attracted a lot of attention in recent years. However, if link scheduling is considered, this transmission mode may perform worse than direct transmission in terms of end-to-end throughput. In this paper, we propose a cooperative communication aware link scheduling scheme and investigate the throughput maximization problem in C-VANETs. Regarding the features of cooperative communications and the availability of licensed spectrum, we extend the links into cooperative links/general links, define extended link-band pairs, and form a 3-dimensional (3-D) cooperative conflict graph to characterize the conflict relationship among those pairs. Given all cooperative independent sets in this graph, we mathematically formulate an end-to-end throughput maximization problem and near-optimally solve it by linear programming. Due to the NP-completeness of finding all independent sets, we also develop a heuristic pruning algorithm for cooperative communication aware link scheduling. Our simulation results show that the proposed scheme is effective in increasing end-to-end throughput for the session in C-VANETs.

Index Terms—Throughput Maximization, Cognitive Vehicular Ad-hoc Networks, Cooperative Communications, Link Scheduling.

I. INTRODUCTION

W ITH the maturity of road infrastructure and the increasing number of motorists, highway traveling has become a part of life for people in US and many other countries. Various broadband vehicular communication applications in Vehicular Ad-hoc Networks (VANETs), which can entertain passengers and make long journeys enjoyable, are envisioned to be prevalent in the near future. However, proliferation of vehicular applications beyond safety requires additional radio resources to support, which makes the already crowed licensed spectrum even worse. Meanwhile, for all these passengeroriented applications [1]–[3], no matter vehicle-to-vehicle (V2V) communication based applications (e.g., network gaming among passengers in different cars, file transfers, virtual

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meetings among coworkers, etc.) or vehicle-to-roadside (V2R) communication based ones (e.g., web browsing, cooperative downloading, online video, etc.), the most critical and essential requirement is the data transmission with high end-to-end throughput, which is also a challenging task in VANETs.

In view of the radio spectrum demands from VANETs, Federal Communications Commission (FCC) opens the underutilized licensed TV spectrum (i.e., the UHF television frequency spanning over 470-806 MHz) and allows the opportunistic accessing of unlicensed users. By exploiting cognitive radio (CR) technology, the vehicles/nodes (the words vehicles/nodes will be used in this paper interchangeably) as well as the roadside unit (RSU) in VANETs can sense the vacant spectrum and opportunistically use these licensed bands temporally/geographically, when/where primary services are not active. We call such a VANET with CR capability [3], [4] as a cognitive VANET (C-VANET).

On the other hand, by employing multiple antennas, e.g., multiple-input and multiple-output (MIMO), spatial diversity has been shown to be effective in lowering bit error rate, enhancing power efficiency and improving throughput in VANETs. However, equipping a wireless node with multiple antennas may not always be practical. To achieve spatial diversity without requiring multiple transceiver antennas on the same node, the so-called cooperative communications has been introduced in [5], [6]. The idea of cooperative communications can be best illustrated by a three-node example [5], [6] shown in Fig. 1(a). In this sub-figure, node i transmits to node i via one-hop, and node r acts as a cooperative relay node. Cooperative transmission from i to j is done on a frame-byframe basis. Within each frame, there are two time slots [1], [5], [7]–[9]. In the first time slot (solid lines), i makes a transmission to destination j. Due to the broadcast nature of wireless transmissions, transmission by i is also overheard by relay node r. In the second time slot (dash lines), r forwards the data it overheard in the first time slot to j. Thus, under cooperative communications, each node is equipped with only a single antenna and relies on the antennas of neighboring cooperative nodes to achieve spatial diversity.

If the cooperative relay node is appropriately selected, cooperative communications can effectively increase the link capacity [7], [10]. However, if we take time-frame based link scheduling into consideration, cooperative communications is not necessarily helpful to improving the end-to-end throughput. Take the toy topology shown in Fig. 1(b) as an example. If node *i* directly transmits packets to node *j*, link (i, j) will have no interference with link (u, v), so that they can be scheduled to transmit simultaneously. By contrast, if (i, j) employs *r* for



(a) A 3-node schematic for coop- (b) A schematic for the interference inerative communications. curred by cooperative communications.

Fig. 1. Illustrative toy topologies for cooperative communications.

cooperative communications, (i, j) will conflict with (u, v)since the transmissions of cooperative relay r cast interference on the receiving node v of (u, v). As a result, (i, j)and (u, v) cannot be scheduled to transmit simultaneously, which may decrease the end-to-end throughput from s_r to d_t . In terms of throughput, the benefit brought by cooperative communications may be offset, or even overwhelmed by the loss of opportunities for scheduling more links to transmit at the same time. Based on that observation, there appear several interesting questions for the throughput maximization problem in C-VANETs: When link scheduling is considered, does there exist an optimal approach to maximize the benefit brought by cooperative communications in terms of the endto-end throughput? Does the availability of licensed bands have any impact on transmission mode selection (i.e., direct transmissions or cooperative communications) as well as the throughput? Can we find a simple and feasible way to solve this problem in practice?

To address these issues, in this paper, we propose a cooperative communication aware link scheduling scheme, with the objective of maximizing the throughput for a session in C-VANETs. We let the RSU schedule the multi-hop data transmissions among vehicles on highways by sending smallsize control messages. Jointly considering availability of licensed spectrum, transmission modes and link scheduling, we mathematically formulate the throughput maximization problem, near-optimally solve it by linear programming, and provide a simple heuristic algorithm to give feasible results. Our salient contributions are summarized as follows.

- Regarding the features of cooperative communications, we novelly extend a link using cooperative communications into a cooperative link. To keep notation consistent, we leverage a dummy cooperative relay and extend a link using direct transmissions into a general link.
- Inspired by the link conflict graph in prior work [11]– [15], we propose a 3-dimensional (3-D) cooperative conflict graph to describe the interference relationship among the extended links in C-VANETs. Similar to the methodology used in [13]–[15], we interpret each vertex in the graph as a basic resource point for scheduling and represent each resource point with an *extended linkband pair*. Based on these extended link-band pairs, we establish the 3-D cooperative conflict graph and re-define the cooperative independent sets and conflict cliques.
- With the help of 3-D cooperative conflict graph, the RSU can mathematically formulate the throughput maximization problem under multiple constraints (i.e., availability of bands, selection of transmission modes and

link scheduling). Given all cooperative independent sets in C-VANETs, the RSU can relax the integer variables in the formulation, solve the optimization problem by linear programming and obtain the optimal end-to-end throughput between the source and destination nodes.

- Since it is NP-complete to find all the cooperative independent sets in C-VANETs [12]–[16], we employ a number of maximum cooperative conflict cliques and develop a heuristic pruning algorithm to approximate the optimal end-to-end throughput. We let the RSU select the band and transmission mode for the extended link-band pairs in those cliques, prune the pairs not selected and update clique transmission time until the largest clique transmission time among all cliques cannot be further decreased. The throughput is estimated as the reciprocal of the largest clique transmission time.
- By carrying out numerical simulations, we demonstrate the impact of the number of available bands and the distance between source and destination nodes on the performance of throughput in C-VANETs. We also show that i) the CR capability creates more opportunities for using cooperative communications; ii) the performance of cooperative communication aware link scheduling is better than that purely relying on one transmission mode; iii) the proposed pruning algorithm is close to the optimal one in terms of end-to-end throughput in C-VANETs.

The rest of the paper is organized as follows. In Section II, we introduce the settings and related models in C-VANETs. In Section III, we describe the 3-D cooperative conflict graph and present the concept of cooperative independent sets and conflict cliques. In Section IV, we mathematically formulate the throughput maximization problem in C-VANETs and near-optimally solve it by linear programming. In Section V, we develop a heuristic pruning algorithm for cooperative communication aware link scheduling. Finally, we conduct simulations and analyze the performance results in Section VI, and draw concluding remarks in Section VII.

II. NETWORK MODEL

A. Network Setting of C-VANETs

We consider a multi-hop C-VANETs [3], [4] consisting of multiple vehicles operating on different vacant licensed frequency bands and a RSU (e.g., a base station (BS), a gateway, an access point (AP), etc.) who serves this group of nodes $\mathcal{N} = \{1, 2, \cdots, n, \cdots, N\}$ on (one way) highways. Let s_r/d_t denote the source/destination node for a session in C-VANETs. Our objective is to maximize end-to-end throughput of this session. By exchanging small-size control messages with the vehicles, the RSU¹ can schedule the transmissions of largesize data packets for multi-hop V2V communications [4]. The scheduling period is set to τ considering the vehicles merging into/exiting from the highway as well as the availability of licensed bands. Suppose that the set of licensed spectrum bands $\mathcal{B} = \{1, 2, \cdots, b, \cdots, B\}$ have the identical bandwidth, where the size of the bandwidth is equal to W. Both direct transmissions and cooperative communications can be used for

¹The RSU can also be interpreted as a group of associated RSUs connected by the backbone network, if the length of the path is long.

packets delivery. To distinguish two types of relay nodes [10] in C-VANETs, we call a relay node used for cooperative communications purpose as a cooperative relay and a relay node used for multi-hop relaying in the traditional sense as multi-hop relay². Considering the concept of cooperative communications as well as the inherent hardware limitation of CR devices, we also assume that each node has only one radio, but the radio can be tuned into any available frequency band for packet delivery.

Each node $i \in \mathcal{N}$ employs certain spectrum sensing techniques (e.g., [17], [18]) to identify a set of available licensed bands, which are not occupied by primary services. Depending on the geographical locations of nodes, the available bands at one node may be different from another one in C-VANETs. To put it in a mathematical way, let $\mathcal{B}_i \subseteq \mathcal{B}$ represent the set of available licensed bands at CR node $i \in \mathcal{N}$. \mathcal{B}_i may be different from \mathcal{B}_j , where j is not equal to i, and $j \in \mathcal{N}$, i.e., possibly $\mathcal{B}_i \neq \mathcal{B}_j$.

For a link (i, j) using cooperative relay r, we assume the transmission from i to j and the transmission from r to j use the same band. Thus, we have $\mathcal{B}_{(i,r,j)} = \mathcal{B}_{(i,j)} = \mathcal{B}_i \cap \mathcal{B}_j$. Besides, the time share³ assigned by the RSU will be measured in time frames, and each time frame will be equally divided into two time slots for the transmission from i to j and that from r to j, if cooperative communications is employed.

B. Transmission Modes

In this subsection, we give expressions for achievable data rate under different transmission modes. For cooperative communications, we consider both AF and DF modes [5], [7].

1) Amplify-and-Forward (AF): Under this transmission mode, cooperative relay r receives, amplifies and forwards the signal from node i to node j [5], [7], [10]. Let h_{ij} , h_{ir} , h_{rj} capture the effects of path-loss, shadowing and fading between nodes i and j, i and r, and r and j, respectively. Denote z_j and z_r the zero-mean background noise at nodes j and r, with variance σ_j^2 and σ_r^2 , respectively. Besides, denote P_i and P_r the transmission powers at nodes i and r, respectively. Since the results are valid for all the bands, we omit the band notations in this subsection.

Following the same notations in [5]–[7], [10], the achievable data rate under AF can be expressed as

$$C_{\rm AF}(i,r,j) = W \cdot I_{\rm AF}(i,r,j), \tag{1}$$

where $I_{AF}(i,r,j) = \frac{1}{2}\log_2\left(1 + \text{SNR}_{ij} + \frac{\text{SNR}_{ir}\cdot\text{SNR}_{rj}}{\text{SNR}_{ij} + \frac{P_i}{\sigma_j^2}|h_{ij}|^2}\right)$, $\text{SNR}_{ij} = \frac{P_i}{\sigma_j^2}|h_{ij}|^2$, $\text{SNR}_{ir} = \frac{P_i}{\sigma_r^2}|h_{ir}|^2$, $\text{SNR}_{rj} = \frac{P_r}{\sigma_j^2}|h_{rj}|^2$, and W is the available bandwidth at nodes i and r.

2) Decode-and-Forward (DF): Under this transmission mode, relay node r decodes and estimates the received signal from node i in the first time slot, and then transmits the estimated data to node j in the second time slot [5]–[7]. As

in [5]–[7], [10], the achievable data rate under DF transmission mode is given as

$$C_{\rm DF}(i,r,j) = W \cdot I_{\rm DF}(i,r,j), \qquad (2)$$

where $I_{\text{DF}}(i, r, j) = \frac{1}{2} \min\{\log_2(1 + \text{SNR}_{ir}), \log_2(1 + \text{SNR}_{ij} + \text{SNR}_{rj})\}$.

Note that $I_{AF}(\cdot)$ and $I_{DF}(\cdot)$ are increasing functions of P_i and P_r , respectively. This suggests that, in order to achieve the maximum data rate under either mode, both node *i* and node *r* should transmit at the maximum power. In this paper, we let $P_i = P_r = P$.

3) Direct Transmission: When cooperative communications is not used, the achievable data rate from node i to node j is

$$C_{\text{DTx}}(i,j) = W \cdot \log_2(1 + \text{SNR}_{ij}).$$
(3)

Based on the above results, we have two observations. First, comparing C_{AF} (or C_{DF}) to C_{DTx} , it is hard to say that cooperative communication is always better than the direct transmission. In fact, a poor choice of relay node could make the achievable data rate under cooperative communications be lower than that under direct transmissions [7]. Second, although AF and DF are different mechanisms, the capacities for both of them have the same form, i.e., a function of SNR_{ij} , SNR_{ir} , and SNR_{rj} . Therefore, a cooperative communication aware link scheduling algorithm designed for AF can be readily extended for DF. Therefore, it is sufficient to focus on one of them, where we choose AF in this paper.

C. Transmission/Interference Range

The interference in wireless networks can be defined according to the protocol model or the physical model [21]. In protocol model [12], [21], there will be a fixed transmission range and a fixed interference range, where the interference range is typically 1.5 to 3 times of the transmission range. These two ranges may vary with the frequency bands. Let \mathcal{T}_i^b denote the set of neighboring nodes within node *i*'s transmission range over licensed band $b \in \mathcal{B}_i$. For a link (i, j)using *r* for cooperative communications over band *b*, we have $r \neq j$ and $r \in \mathcal{T}_{(i,j)}^b = \mathcal{T}_i^b \cap \mathcal{T}_j^b$.

III. COOPERATIVE CONFLICT GRAPH, CONFLICT CLIQUES AND INDEPENDENT SETS IN C-VANETS

In this section, we first extend the links in C-VANETs into cooperative links/general links with respect to (w.r.t.) the special features of cooperative communications. Then, we establish a 3-D cooperative conflict graph to describe the interference relationship among these extended links. Besides, we also re-define independent sets and conflict cliques [11], [12] to show which links can be activated at the same time and which links cannot, when cooperative communications is involved in C-VANETs.

A. Extending Links into Cooperative/General Links

For a link (i, j), if node r is the best cooperative relay for it, we calculate the achievable data rate for cooperative communications (i.e., $C_{AF}(i, r, j)$) as illustrated in (1). If $C_{AF}(i, r, j) > C_{DTx}(i, j)$, we can extend link (i, j) into (i, r, j)

 $^{^{2}}$ Note that a cooperative relay operates at the physical layer while a multi-hop relay operates at the network layer.

³In this paper, time period refers to the scheduling period, i.e., τ ; time share refers to the active time scheduled for an independent set, i.e., $\lambda_m \tau$, as illustrated in Sec. IV-B; time frame refers to the basic unit of time for link scheduling; time slot refers to the two time slots defined in cooperative communications [5], [7].

and define (i, r, j) as a cooperative link. To keep the link notation consistent, we exploit (i, ϕ, j) to denote a link using direct transmissions, where ϕ is a dummy cooperative relay, and define (i, ϕ, j) as a general link. The same procedure can be done for each link in the C-VANET. Define $\mathcal{R}_{(i,j)}^b =$ $\{\phi\} \bigcup \mathcal{T}_{(i,j)}^b$. Then, we can extend each link (i, j) into the form of (i, r, j) over band b, where $r \in \mathcal{R}_{(i,j)}^b$. Note that for a link qualified to be a cooperative link, the RSU can choose to use it as a cooperative link or a general link, when the RSU considers the interference relationship among different links and schedules the transmissions over these links.

B. Establishing the 3-D Cooperative Conflict Graph

Regarding the availability of licensed bands and the features of cooperative communications, we introduce a 3-D cooperative conflict graph to characterize the interference relationship among multiple links in C-VANETs.

Specifically, in a 3-D cooperative conflict graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, each vertex corresponds to an *extended link-band pair*, where a extended link-band pair is defined as ((i, r, j), b). The linkband pair indicates that the extended link (i, r, j) operates on available licensed band b. Note that it includes the general link when the cooperative relay $r = \phi$, and includes the cooperative link when the cooperative relay $r \neq \phi$. It also includes cooperative communications in single-radio singlechannel networks as a special case when the number of available licensed bands $|\mathcal{B}| = 1$.

Two extended link-band pairs are defined to interfere with each other, if any of the following conditions is true:

- **Condition 1**: Two different extended link-band pairs have nodes in common.
- **Condition 2**: If the two extended link-band pairs are using the same band, their transmissions interfere with each other when either the receiving node or the cooperative relay node of one pair is in the interference range of either the transmitting node or the cooperative relay node in the other pair.

Based on these conditions, we connect two vertices in \mathcal{V} with an undirected edge in $\mathcal{G}(\mathcal{V}, \mathcal{E})$, if their corresponding link-band pairs interfere with each other. Note that cooperative communication may increase the achievable data rate of a link, but it also incurs additional interferences. The reason is that we must consider both the nodes within the interference range of the transmitting node and the nodes within the interference range of the cooperative relaying node, when this cooperative link-band pair is scheduled for transmissions⁴.

C. Cooperative Independent Sets and Conflict Cliques

Given a 3-D cooperative conflict graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ representing C-VANETs, we describe the impact of vertex $u \in \mathcal{V}$ on vertex $v \in \mathcal{V}$ as follows,

 $w_{uv} = \begin{cases} 1, \text{ (if there is an edge between vertex } u \text{ and } v) \\ 0, \text{ (if there is no edge between vertex } u \text{ and } v), \end{cases} (4)$

where the two vertices correspond to two link-band pairs.

Provided that there is a vertex/extended link-band set $\mathcal{I} \subseteq \mathcal{V}$ and an extended link-band $u \in \mathcal{I}$ satisfying $\sum_{v \in \mathcal{I}, u \neq v} w_{uv} < v$



Fig. 2. Possible cases for relay selection collisions w.r.t. link scheduling.

1, the transmission at link-band pair u will be successful even if all the other link-band pairs belonging to the set \mathcal{I} are transmitting at the same time. If any $u \in \mathcal{I}$ satisfies the condition above, we can schedule the transmissions over all these extended link-band pairs in \mathcal{I} to be active simultaneously. Such a vertex/extended link-band pair set \mathcal{I} is called a cooperative independent set. If adding any one more extended link-band pair into a cooperative independent set \mathcal{I} results in a non-independent one, \mathcal{I} is defined as a maximum cooperative independent set. Besides, if there exists a vertex/extended linkband pair set $\mathcal{Z} \subseteq \mathcal{V}$ in \mathcal{G} and any two extended link-band pairs u and v in Z satisfying $w_{uv} \neq 0$ (i.e., vertex u and v cannot be scheduled to transmit successfully at the same time.). \mathcal{Z} is called a cooperative conflict clique. If \mathcal{Z} is no longer a conflict clique after adding any one more extended link-band pair, Z is defined as a maximum cooperative conflict clique.

IV. OPTIMAL COOPERATIVE COMMUNICATION AWARE LINK SCHEDULING FOR HIGH END-TO-END THROUGHPUT

After we construct the 3-D cooperative conflict graph, in this section, we first discuss the possible collisions of relay selection w.r.t. link scheduling in C-VANETs. Then, we address how to calculate the path capacity and describe flow routing constraints for the single-radio based nodes. According to the cross-layer constraints, we mathematically formulate the throughput maximization problem in C-VANETs and nearoptimally solve it by linear programming.

A. Collisions of Relay Selection w.r.t. Link Scheduling

Before we discuss cooperative communication aware link scheduling, we need to clarify two issues related to the collisions of relay selection w.r.t. link scheduling. As introduced in [10], two kinds of relay selection collisions may happen when cooperative communications is incorporated into multihop wireless networks. The first one is the collision between cooperative relay selection and multi-hop relay selection (i.e., a node is chosen both as a cooperative relay and a multihop relay), as shown in Case 1 and 2 in Fig. 2; the second one is the collision among different links for cooperative relay selection (i.e., different links choose the same node as cooperative relay), as shown in Case 3 in Fig. 2.

If there is only one band available in the network, it can easily be proved that the relay selection collisions can never happen w.r.t. link scheduling⁵. However, if there are multiple bands available in the network (e.g. in C-VANETs),

⁴For specific examples, please refer to the technical report posted at http://plaza.ufl.edu/miaopan/TR-CCLSVNETs.pdf.

⁵The hint is that for any two links having relay selection collision, these two links inherently interfere with each other if there is only one band available. They cannot be scheduled to transmit simultaneously.

both collisions exist as shown in Fig. 2. Fortunately, the 3-D cooperative conflict graph can perfectly describe all the relay selection collisions in C-VANETs (e.g., all three cases in Fig. 2 satisfying interference **Condition 1**), so that the RSU can exploit it to conduct the cooperative communication aware link scheduling. Note that a node in C-VANETs can alternate its role between cooperative relay and multi-hop relay at different time shares, which is different from the node's fixed role in [10].

B. Path Capacity with Link Scheduling Consideration

For a given path \mathcal{P} , we can establish the 3-D cooperative conflict graph $\mathcal{G}_{\mathcal{P}} = (\mathcal{V}_{\mathcal{P}}, \mathcal{E}_{\mathcal{P}})$ following the same approach illustrated in Sec. III-B. Then, we can list all independent sets as $\mathcal{I}_{\mathcal{P}} = \{\mathcal{I}_1, \mathcal{I}_2, \cdots, \mathcal{I}_m, \cdots, \mathcal{I}_M\}$, where M is $|\mathcal{I}_{\mathcal{P}}|$, and $\mathcal{I}_m \subseteq \mathcal{V}_{\mathcal{P}}$ for $1 \leq m \leq M$. Although it is a NP-complete problem to find all independent sets [13], [16], [22], some brute-force algorithm can finish it in polynomial time if the number of extended link-band pairs in $\mathcal{V}_{\mathcal{P}}$ is not large [12].

At any time, at most one independent set can be activated to transmit packets for all link-band pairs in that set. Let $\lambda_m \ge 0$ denote the time share scheduled to independent set \mathcal{I}_m , and

$$\sum_{1 \le m \le M} \lambda_m \le 1, \quad \lambda_m \ge 0 \ (1 \le m \le M).$$
⁽⁵⁾

Let $r^b_{(i,r,j)}(\mathcal{I}_m)$ be the data rate for the extended link (i,r,j) over band b, where $r^b_{(i,r,j)}(\mathcal{I}_m) = 0$ if link-band pair $((i,r,j),b) \notin \mathcal{I}_m$. Otherwise, if (i,r,j) is a cooperative link and $((i,r,j),b) \notin \mathcal{I}_m$, $r^b_{(i,r,j)}(\mathcal{I}_m)$ is the achievable data rate for (i,r,j) over band b when cooperative communications is leveraged. Under AF transmission mode, $r^b_{(i,r,j)}(\mathcal{I}_m)$ can be calculated from (1); if (i,r,j) is a general link and $((i,r,j),b) \in \mathcal{I}_m, r^b_{(i,r,j)}(\mathcal{I}_m)$ is the achievable data rate for (i,r,j) over band b using direct transmissions, which can be calculated as illustrated in (3).

By exploiting the independent set \mathcal{I}_m , the flow rate that an extended link (i, r, j) can support over band b in the time share λ_m is $\lambda_m r^b_{(i,r,j)}(\mathcal{I}_m)$. Let s represent the aggregated traffic demands. Considering the availability of licensed bands in C-VANETs, the traffic is feasible at the extended link (i, r, j) if there exists a schedule of the independent sets satisfying

$$s \le s_{(i,r,j)} = \sum_{m=1}^{|\mathscr{I}_{\mathcal{P}}|} \lambda_m \sum_{b=1}^{|\mathscr{B}_{(i,r,j)}|} r_{(i,r,j)}^b(\mathcal{I}_m).$$
(6)

To maximize the path capacity of \mathcal{P} , we have

$$C_{\mathcal{P}} = \max\min_{\substack{(i,r,j) \in \mathcal{P} \\ \text{flow Routing Constraints in } C-VANETs}} s_{(i,r,j)}. \tag{7}$$

As for routing, the RSU will help the source node to find the available paths to the destination node for data delivery. Similar to the modeling in [19], we mathematically present those routing constraints as follows.

Let $f_{(i,r,j)}^b$ represent the flow rate of the extended link (i,r,j) over band b, where $i \in \mathcal{N}, j \in \mathcal{T}_i^b, r \in \mathcal{R}_{(i,j)}^b$ and $r \neq j$. If node i is the source node, i.e., $i = s_r$, then

$$\sum_{b \in \mathcal{B}_{(j,r,i)}} \sum_{j \in \mathcal{I}_i^b}^{r \neq i, r \in \mathcal{R}_{(j,i)}^o} f_{(j,r,i)}^b = 0.$$
(8)

Regarding the single-radio requirement of cooperative communications and the inherent single-radio constraint of CR devices, we focus on the unicast and single-path routing problem. Thus, we have

$$\sum_{b \in \mathcal{B}_{(i,r,j)}} \sum_{j \in \mathcal{T}_i^b} f^b_{(i,r,j)} \, \delta^b_{(i,r,j)} = s, \tag{9}$$

where $\delta^b_{(i,r,j)}$ indicates that the extended link (i,r,j) can only have a nonzero flow at a time due to the single-radio constraint, i.e.,

$$\sum_{b \in \mathcal{B}_{(i,r,j)}} \sum_{j \in \mathcal{T}_i^b} \sum_{j \in \mathcal{T}_i^b} \delta_{(i,r,j)}^b \le 1, \quad \delta_{(i,r,j)}^b \in \{0,1\}.$$
(10)

If node *i* is an intermediate multi-hop relay node (not a cooperative relay node), i.e., $i \neq s_r$ and $i \neq d_t$, then

$$\sum_{b \in \mathcal{B}_{(i,r,j)}} \sum_{j \in \mathcal{T}_{i}^{b}}^{r \neq j, r \in \mathcal{R}_{(i,j)}^{b}} f_{(i,r,j)}^{b} \delta_{(i,r,j)}^{b}$$
$$= \sum_{b \in \mathcal{B}_{(j,q,i)}} \sum_{j \in \mathcal{T}_{i}^{b}}^{q \neq i, q \in \mathcal{R}_{(j,i)}^{b}} f_{(j,q,i)}^{b} \delta_{(j,q,i)}^{b}.$$
(11)

If node *i* is the destination node, i.e., $i = d_t$, then

=

$$\sum_{b \in \mathcal{B}_{(j,r,i)}} \sum_{j \in \mathcal{T}_i^b} f^b_{(j,r,i)} \delta^b_{(j,r,i)} = s.$$
(12)

D. Maximizing the Throughput under Multiple Constraints

To maximize the end-to-end throughput between the source node and the destination node, the RSU needs to find a feasible solution to jointly assigning the available frequency bands, conducting cooperative communication aware link scheduling bands, and routing the traffic for transmission and reception in multi-hop C-VANETs. Thus, the end-to-end throughput maximization problem under multiple constraints in C-VANETs can be formulated as follows.

Maximize s

s.t. (8), (9), (10), (11), (12), (5), and

$$0 \leq \sum_{b=1}^{|\mathcal{B}_{(i,r,j)}|} f_{(i,r,j)}^{b} \leq \sum_{m=1}^{|\mathscr{I}|} \lambda_{m} \sum_{b=1}^{|\mathcal{B}_{(i,r,j)}|} r_{(i,r,j)}^{b}(\mathcal{I}_{m})$$
 $(i \in \mathcal{N}, j \in \mathcal{T}_{i}^{b}, r \in \mathcal{R}_{(i,j)}^{b}, b \in \mathcal{B}_{(i,r,j)} \text{ and } \mathcal{I}_{m} \in \mathscr{I}), (13)$

where (8), (9), (10), (11), and (12) specify that there is at most one outgoing link from each node with a nonzero flow, and that there is a path selected by the RSU between the source and the destination; (5) and (13) indicate that the flow rate of traffic over (i, r, j) cannot exceed the capacity of this extended link, which is obtained from the cooperative communication aware link scheduling as illustrated in Sec. IV-B.

Note that \mathscr{I} includes all independent sets in C-VANETs. Given all independent sets⁶ in the network, we find that the

⁶That is a general assumption used in existing literature [11]–[15] for obtaining throughput bounds or performance comparison, where both link scheduling and flow routing are considered.

formulated optimization is a mixed-integer linear programming problem since δ_{ij} only has binary values. It can nearoptimally be solved in polynomial time by some typical algorithms (e.g., sequential fixing algorithm [19], [20], branch and bound [23], etc.) or softwares (e.g., CPLEX [24]), provided that all the cooperative independent sets can be found in $\mathcal{G}(\mathcal{V}, \mathcal{E})$.

V. A HEURISTIC PRUNING ALGORITHM FOR COOPERATIVE COMMUNICATION AWARE SCHEDULING

As we know, to find all cooperative independent sets in $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is NP-complete [11]–[14], [25]. Compared with complex path selection in other wireless networks, it is much more simple in C-VANETs because there are only a few paths between the source and destination nodes due to the limited spatial redundancy and fixed direction of highways⁷. However, even for a given path, it is too complex for the RSU to find all cooperative independent sets along the path, if the number of extended links or the number of available licensed bands along the path is large. Instead of using cooperative independent sets, in this section, we employ a number of maximum cooperative cliques and propose a 7-step pruning algorithm to approximate the maximum throughput for a session in C-VANETs. **Step 1:** Establishing the 3-D cooperative conflict graph

Given a candidate path \mathcal{P} , we first set up a 3-D cooperative conflict graph $\mathcal{G}_{\mathcal{P}}(\mathcal{V}_{\mathcal{P}}, \mathcal{E}_{\mathcal{P}})$ as illustrated in Sec. III-B. **Step 2:** Searching for the maximum conflict cliques

With the established 3-D cooperative conflict graph of the given path \mathcal{P} , we try to find all the maximum cooperative conflict cliques in $\mathcal{G}_{\mathcal{P}}(\mathcal{V}_{\mathcal{P}}, \mathcal{E}_{\mathcal{P}})$ and form the set \mathscr{Z} consisting of the maximum cooperative conflict cliques. If \mathcal{P} involves with too many extended links or available bands, and it is impossible to find all the maximum cliques, we can employ K maximum cliques for approximation when K is large enough. **Step 3:** Calculating the conflict clique transmission time

Then, we let the RSU employ the maximum cooperative conflict cliques to estimate the benchmark path capacity for the path \mathcal{P} . Similar to the illustration in [11], [12], we define the cooperative conflict clique transmission time $T_{\mathcal{Z}}$ for a cooperative conflict clique \mathcal{Z} as

$$T_{\mathcal{Z}} = \sum_{((i,r,j),b)\in\mathcal{Z}} T_{((i,r,j),b)}$$
(14)

where $T_{((i,r,j),b)}$ is the transmission time for one unit of traffic over the extended link (i,r,j) using the available licensed band b. Specifically, $T_{((i,r,j),b)}$ can be written as

$$T_{((i,r,j),b)} = \frac{1}{r^b_{(i,r,j)}(\mathcal{Z})},$$
(15)

where $r^b_{(i,r,j)}(\mathcal{Z})$ is equal to the achievable data rate of link (i,r,j) over band b, if $((i,r,j),b) \in \mathcal{Z})$. Otherwise, $r^b_{(i,r,j)}(\mathcal{Z}) = \infty$.

Step 4: Sorting the maximum cooperative conflict cliques

For $\mathcal{Z} \in \mathscr{Z}$, we sort the maximum cooperative conflict cliques in terms of the cooperative conflict clique transmission

time $T_{\mathcal{Z}}$. Let $T_{\mathcal{P}}$ be the maximum value of the transmission time for all cooperative conflict cliques. $T_{\mathcal{P}}$ can be written as

$$T_{\mathcal{P}} = \max_{\mathcal{Z} \in \mathscr{X}} T_{\mathcal{Z}}.$$
 (16)

Considering an extended link-band pair ((i, r, j), b) in $\mathcal{Z} = \underset{Z \in \mathscr{Z}}{\operatorname{argmax}} (T_{\mathcal{Z}})$ and one unit of traffic successfully delivered from the source to the destination, it takes time $T_{\mathcal{P}}$ to travel through all the extended link-band pairs in $\hat{\mathcal{Z}}$, and ((i, r, j), b) cannot be scheduled to do any other transmission during $T_{\mathcal{P}}$. That indicates that the throughput at the extended link-band pair ((i, r, j), b) is less than or equal to $\frac{1}{T_{\mathcal{P}}}$. Since the end-to-end throughput cannot be larger than the throughput of any

link along the path, the benchmark path capacity $C_{\mathcal{P}}$ can be estimated as⁸ $C_{\mathcal{P}} = \frac{1}{T_{\mathcal{P}}}$. **Step 5:** Selecting the optimal band for the high throughput

If there are multiple available licensed bands for an extended link to access, one of them must be chosen due to the single-radio constraint. From (14) and (16), we find that if the size of \hat{Z} shrinks, the throughput of the path may increase. It is obvious that if some of the co-band interference between the extended links can be mitigated, the size of \hat{Z} can be effectively reduced. As we know, the CR devices can be tuned into different frequencies and allow the extended links to operate on different bands. This special CR feature will help to reduce co-band interference between the extended links so that the end-to-end throughput may be improved. Following this thread, we conduct the optimal band selection as follows.

First, for an extended link (i, r, j) with multiple accessing bands, we randomly select an extended link-band pair ((i, r, j), b) in \hat{Z} and temporarily delete other $((i, r, j), \cdot)$ pairs as well as the conflict edges associated with $((i, r, j), \cdot)$. Then, we find the maximum cooperative conflict clique in the leftover graph cut from \hat{Z} and calculate the clique transmission time $T_{\hat{Z}}^{((i,r,j),b)}$ as in (14). For $b \in \mathcal{B}_{(i,r,j)}$, the same process is conducted and the values of clique transmission time are stored. After that, we update $T_{\hat{Z}}$ as

$$T_{\hat{\mathcal{Z}}} = \min\{T_{\hat{\mathcal{Z}}}^{((i,r,j),1)}, T_{\hat{\mathcal{Z}}}^{((i,r,j),2)}, \cdots, T_{\hat{\mathcal{Z}}}^{((i,r,j),|\mathcal{B}_{(i,r,j)}|)}\}.$$
 (17)

We identify the band reaching the value of $T_{\hat{z}}$, put that band into (i, r, j)'s usage and prune all the other $((i, r, j), \cdot)$ pairs as well as the conflict edges associated with $((i, r, j), \cdot)$.

The same procedure above is repeated by all the extended link-band pairs in $\hat{\mathcal{Z}}$ one after another, and the $T_{\hat{\mathcal{Z}}}$ is continuously updated.

If all the available licensed bands are identical to an extended link in terms of band condition (i.e., bandwidth, the propagation gain, etc.), it will be much more simple to select the optimal band for this extended link. As for such an extended link, we just need to keep the extended link-band pair with the least conflict edges and eliminate the other link-band pairs associated with this extended link. Meanwhile, we also prune the corresponding conflict edges and update $T_{\hat{z}}$ based on the leftover graph cut from \hat{z} .

⁷In [8], Ding and Leung even employ string topology to investigate the cross-layer routing problem in VANETs.

⁸Actually, the benchmark path capacity $C_{\mathcal{P}}$ should be upper-bounded by $\frac{1}{T_{\mathcal{P}}}$, i.e., $C_{\mathcal{P}} \leq \frac{1}{T_{\mathcal{P}}}$. The equal sign holds if there are no odd cycles [22] in $\mathcal{G}_{\mathcal{P}}$ as illustrated in [12]. In this paper, we just consider the general paths without odd cycles.

Step 6: Pruning the cooperative/general link-band pairs

After the band selection for an extended link, it is necessary to determine which type of transmission (i.e., cooperative communications or direct transmission) should be used by this extended link. In \hat{Z} , there may be two coupled link-band pairs extended from the same link (i, j): a general link-band pair $((i, \phi, j), u)$ and a cooperative link-band pair ((i, r, j), v), $(r \in \mathcal{R}_{(i,j)} \text{ and } r \neq \phi)$, where u and v are the available bands selected for (i, ϕ, j) and (i, r, j) in **Step 5**, respectively.

From (1), (2), (3), (15) and (14), we can easily calculate the transmission time for the clique $\hat{Z} \setminus \{((i, \phi, j), u)\}$ and $\hat{Z} \setminus \{((i, r, j), v)\}$, i.e., $T_{\hat{Z} \setminus \{((i, \phi, j), u)\}}$ and $T_{\hat{Z} \setminus \{((i, r, j), v)\}}$, respectively. We compare $T_{\hat{Z} \setminus \{((i, \phi, j), u)\}}$ and $T_{\hat{Z} \setminus \{((i, r, j), v)\}}$ and make the decision of pruning the cooperative/general linkband pairs as follows.

- If T_{ẑ\{((i,φ,j),u)}} > T_{ẑ\{((i,r,j),v)}</sub>, the RSU will keep the general link-band pair ((i, φ, j), u) and prune the cooperative link-band pair ((i, r, j), v) as well as the conflict edges associated with ((i, r, j), v). That is, the RSU chooses the direct transmission instead of cooperative communications for the link (i, j). In addition, the RSU will update T_{x̂} by setting T_{x̂} = T_{x̂\{((i, r, i), v)}.
- will update $T_{\hat{z}}$ by setting $T_{\hat{z}} = T_{\hat{z} \setminus \{((i,r,j),u)\}}$. • If $T_{\hat{z} \setminus \{((i,\phi,j),u)\}} \leq T_{\hat{z} \setminus \{((i,r,j),v)\}}$, the RSU will keep the cooperative link-band pair ((i,r,j),v) and prune the general link-band pair $((i,\phi,j),u)$ as well as the conflict edges associated with $((i,\phi,j),u)$. That is, the RSU chooses cooperative communications instead of the direct transmission for the link (i,j). In addition, the RSU will update $T_{\hat{z}}$ by setting $T_{\hat{z}} = T_{\hat{z} \setminus \{((i,\phi,j),u)\}}$.

The same procedure is repeated by any two coupled extended link-band pairs in \hat{z} associated with the same link, and the $T_{\hat{z}}$ is continuously updated.

Step 7: Iterating the procedure and estimating the throughput

Jump back to **Step 4**, resort the maximum cooperative conflict cliques in terms of the cooperative conflict clique transmission time (with the updated $T_{\hat{z}}$), find new \hat{z} and iterate the following steps with this clique. Iterations continue until $\hat{T}_{\mathcal{P}}$ cannot be decreased further. Then, the RSU can set $T_{\mathcal{P}}^* = T_{\mathcal{P}} = T_{\hat{z}}$ and estimate the throughput of \mathcal{P} as $C_{\mathcal{P}}^* = \frac{1}{T_{\mathcal{P}}^*}$. Similarly, the RSU can maximize the throughput of the other paths via cooperative communication aware link scheduling, and select the one with the highest throughput⁹.

VI. PERFORMANCE EVALUATION

We consider a C-VANET consisting of $|\mathcal{N}| = 30$ vehicles randomly distributed along a 3 km two lane straight highway. All the vehicles are moving in the same direction. The bandwidth for each band W is set to be 8 MHz, schedule period τ is set to be 10 s, the maximum transmission power at each node is set to be 5 W, the transmission range is set to be 250 m, and the interference range¹⁰ is set to be 400 m. For simplicity, we assume that h_{ij} only includes the propagation gain between node *i* and *j* and is given by



Fig. 3. Comparison between cooperative communications and direct transmissions for a three-node schematic.

 $|h_{ij}|^2 = d(i, j)^{-4}$, where d(i, j) is the distance (in meters) between nodes *i* and *j* and path loss index is 4. For the AWGN channel, we assume the variance of noise is 10^{-10} W at all nodes. Besides, we set K = 200, i.e., if the total number of maximum cooperative cliques in $\mathcal{G}_{\mathcal{P}}$ is less than or equal to 200, we employ all the maximum cooperative cliques; otherwise, we employ 200 maximum cooperative cliques for approximation. For illustrative purposes, we investigate the throughput maximization problem in C-VANETs with the following two scenarios: i) all the vehicles move at the speed of 75 mph (i.e., 120.7 km/h, the typical speed limit); ii) vehicle speed follows a Gaussian distribution with a mean of 75 mph and a standard deviation of 10 mph (i.e., 16.09 km/h).

By fixing the leftmost node as the source and the rightmost node as the destination, we compare the results of different throughput maximization algorithms. These results include the optimal throughput considering both transmission mode selection and band selection (i.e., "Optimal CC/Dtx w/ CR"), the throughput obtained from the proposed pruning algorithm (i.e., "Pruning CC/Dtx w/ CR"), the optimal throughput considering band selection under different transmission modes (i.e., "Optimal CC w/ CR" and "Optimal Dtx w/ CR") and the single-band based optimal throughput under different transmission modes (i.e., "Optimal CC w/o CR" and "Optimal Dtx w/o CR") [12]. Note that given the independent sets, we can employ CPLEX [24] to solve the optimization problems and obtain near-optimal results. Besides, we demonstrate the impact of the number of available licensed bands on the throughput in C-VANETs and present the results in Fig. 4. For the sessions from the source node to all the other nodes along the highway, we also conduct simulations to evaluate the impact of distance with different throughput maximization algorithms and show the corresponding results in Fig. 5.

In Fig. 3, we compare two transmission modes in terms of link capacity. Here, we assume the transmitter, the cooperative relay and the receiver are on the same lane, and the distance between the transmitter and the receiver is 250 m. We find that cooperative communications is not necessarily better than direct transmissions in terms of link capacity, and the benefit brought by cooperative communications highly depends on the location of the cooperative relay.

Figure 4 demonstrates the impact of the number of available licensed bands on the end-to-end throughput in C-VANETs. From the results shown in Fig. 4(a) and Fig. 4(b), four observations can be made in order. First, "Optimal CC/Dtx w/ CR" and the heuristic pruning algorithm outperform the other algorithms in terms of end-to-end throughput. It is not

⁹For specific examples, please refer to the technical report posted at http://plaza.ufl.edu/miaopan/TR-CCLSVNETs.pdf.

¹⁰As illustrated in [20], the transmission range and interference range can be determined by the receiver sensitivity and the threshold of interference tolerance, respectively.



(b) Scenario 2: vehicle speed follows Gaussian distribution with a mean of 75 mph and a standard deviation of 10 mph.



surprising because both of them have a joint consideration of transmission mode selection and the band selection, when the transmissions are scheduled. In addition, the throughput obtained from the proposed pruning algorithm is close to that from the optimal one. Second, considering link scheduling, cooperative communications may incur extra interference and hinder the end-to-end throughput, especially when the number of available bands is limited. Third, the CR capability of the nodes creates more opportunities to use cooperative communications and therefore improve the throughput. As for those algorithms considering the CR capability of nodes, the endto-end throughput increases as the number of available bands increases. The reason is that more licensed bands available give more opportunities for nodes' accessing, so that more cooperative links can be utilized without incurring additional interference and more links can be activated for transmission simultaneously. The increment of throughput stops when the number of available bands is large enough, i.e., the throughput



(b) Scenario 2: vehicle speed follows Gaussian distribution with a mean of 75 mph and a standard deviation of 10 mph.

Fig. 5. Impact of distance between the source and destination nodes on the end-to-end throughput in C-VANETs.

cannot be further increased since both cooperative communications and link scheduling are fully exploited. Fourth, the deviation of vehicle speed leads to performance degradation of link scheduling. That is because speeding up/slowing down may result in certain changes of network topology (e.g., overtaking) in C-VANETs.

Figure 5 shows the impact of distance between the source and destination nodes on the throughput in C-VANETs. For the simplicity of computing independent sets [13], we assume there are 2 licensed bands available in the network. Except for the observations we already have made in Fig. 4, we find that the longer distance the path spans, the more likely the throughput is affected by the band selection, transmission mode selection and link scheduling. For a short-distance path which includes only a few links, cooperative communications is always preferred since there is no link scheduling involved. By contrast, a long-distance path includes more links, which implies that more links could be scheduled to transmit at the same time. Thus, the end-to-end throughput maximization of such a path depends more on band selection, transmission mode selection and link scheduling.

VII. CONCLUSION

In this paper, we have studied the throughput maximization problem in C-VANETs under multiple constraints (i.e., CR devices' inherent single-radio constraint, the availability of licensed spectrum, transmission mode selection and link scheduling). Considering the special features of cooperative communications, we first extend the links and classify them into cooperative links/general links. Then, depending on the available bands at different extended links, we define extended link-band pairs and form a 3-D cooperative conflict graph to describe the conflict relationship among those pairs. After that, we mathematically formulate the end-to-end throughput maximization problem. Given all cooperative independent sets in C-VANETs, we can relax the formulated optimization problem and near-optimally solve it by linear programming. Due to the NP-completeness of finding all independent sets, we provide a heuristic pruning algorithm for the cooperative communication aware link scheduling as well. By numerical simulations, we demonstrate that: i) the CR capability creates more opportunities for using cooperative communications; ii) the performance of link scheduling with appropriately selected transmission mode is better than that purely relying on one transmission mode.

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