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ORIGINAL PAPER

Maximum flow problem in wireless ad hoc networks with directional antennas

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Abstract Directional antenna offers a variety of benefits for wireless networks, one of which is the increased spatial reuse ratio. This feature gives rise to the improved throughput in resource limited wireless ad hoc networks. In this paper, we formulate the maximum flow problem as an optimization problem in wireless ad hoc networks with switched beam directional antennas constrained by interference. We demonstrate how to solve this optimization problem. It turns out that the proposed method works for both single beam antenna and multi-beam antenna, with minor variation of the constraints.

1 Introduction

Due to the hostile wireless channel, and interference within and among flows, how to achieve the maximum throughput in multihop wireless ad hoc networks has been of great interest over the past decades. Especially for resource-constrained wireless ad hoc networks, how to improve the system capacity is even more important. With the switched beam technology, the directional antenna is shown to be an appealing option for wireless ad hoc networks. By concentrating RF energy in the intended transmission direction, the spatial transmission

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region shrinks proportionally to the beam width of a sector. By contrast, energy of traditional omni-directional antennas radiates to the whole space. Any node inside the transmission range may receive the signal. Since the area of potential collisions is reduced to a sector from a circle or sphere, the shared channel can be used by another node towards an interference-free beam. Directional antenna is able to reduce interference and energy consumption, and improve spatial reuse ratio, thus can significantly boost the channel capacity. It is feasible to equip wireless nodes with directional antennas, because the switched beam systems could be built with fairly cheap off-the-shelf components and the size is still moderately small. So this paper focuses on wireless ad hoc networks with switched beam antennas.

This paper deals with the maximum flow of a wireless ad hoc network with switched beam directional antennas. The problem to be addressed in this paper is: given a network topology and existing traffic load, how can we achieve the maximum flow between a given source-destination pair through optimal path selection?

Due to the different interference pattern induced by directional antennas in wireless ad hoc networks, constraints for the maximum flow are novel and distinct from previous work. Maximum flow problem to be addressed here is different from the classical maximum flow problem in network flow theory. In wired networks, there is no interference among transmissions. Any link can be active at any instant without interference from other links. However, the broadcasting nature of wireless medium makes the shared wireless channel bottleneck for network flow. To avoid collision, links in close neighborhood may not be active simultaneously. Furthermore, interference condition of wireless networks with directional antennas is different from those with omni-directional antennas.

Without assumptions on the network topology or homogeneity of link capacity, we attempt to solve the problem in a generalized setting. For the first time, the interference-constrained maximum flow problem in wireless ad hoc networks with directional antenna is formulated as an optimization problem. This problem is inherently a joint multipath routing and optimal scheduling problem. Generally, multipath routing is capable of supporting a larger amount of flow than single path routing. Nevertheless, the interference among multiple paths restricts the efficiency of multipath routing. Taking the advantage of mitigated interference, multipath routing is justifiable in wireless ad hoc networks with directional antennas. Yet the more involved interference pattern of multipath routing further complicates the problem because of the substantial problem size and searching space.

The paper is organized as follows. The next Section summarizes the related work. Section 3 describes the antenna model. Section 4 defines the flow contention and resource sharing graph. Then we present the problem formulation of maximum flow for switched beam antennas in Sect. 5. Section 6 demonstrates the numerical results for the maximum flow. Finally, Sect. 7 concludes the paper.

2 Related work

Many algorithms have been proposed to solve maximum network flow problem efficiently over four decades [2,4–8,11,13,17,18]. However, problem formulation may be different under different conditions. Especially for wireless networks, link condition is totally different from traditional wired links. Many papers have derived the asymptotic throughput bounds under certain assumptions on network topology and node configuration. The seminal paper by Gupta and Kumar [9] studied the network comprising of n randomly placed non-mobile nodes. The throughput per node for a randomly chosen destination is $\Theta(1/\sqrt{n \log n})$, where $\Theta(x)$ is a quantity on the same order of x. Even under the optimal node placement and communication pattern, the per-node throughput is $\Theta(1/\sqrt{n})$. In this case, the total end-to-end capacity is roughly $\Theta(n/\sqrt{n})$, which is $\Theta(\sqrt{n})$. Subsequent work [20] investigates the capacity gain of wireless ad hoc networks with directional antennas over omni-directional antennas. Kodialam and Nandagopal [12] consider the problem of joint routing and scheduling to achieve a given rate vector. The only interference constraint they take into consideration is that a node cannot transmit or receive simultaneously. They formulate the scheduling problem as an edge-coloring problem and provide a polynomial time algorithm. The approach achieves at least 67% of the optimal throughput. Jain et al. [10] model the interference between neighboring nodes using a conflict graph and present methods for computing the lower and upper bounds. They focus on the routing component alone. However, they do not propose any approximation algorithm to solve the routing problem. In [15], Peraki and Servetto study the maximum throughput in dense random wireless networks with directional antennas with bounded queue. They derive the asymptotic upper bounds on throughput by solving the minimum cut problem. An optimal resource allocation scheme is proposed based on the maximal cliques resulted from contention flows in [19]. A distributed pricing algorithm is provided to approximate global optimum and fairness among end-to-end flows.

Several works study the multipath routing in wireless ad hoc networks using directional antennas [14,16]. Tang et al. [16] define the path interference to find the minimum single path and node-disjoint multiple paths in wireless networks equipped with directional antennas. Since interference affects the network performance, some papers attempt to reduce the interference through topology control. A recent work [3] concisely defines the interference and proposes several interference-aware topology control algorithms.

3 Antenna model

According to beam pattern (beam-radius, beam-width, beam orientation), we have omni-directional antennas, single-beam directional antennas (e.g., single-beam switched beam antennas), multi-beam directional antennas (e.g., multi-beam switched beam antennas or sectorized beam antennas). Beam-radius is the distance that a transmission reaches. Beam-width is determined by the angle

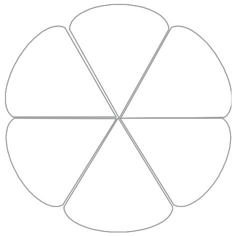


Fig. 1 An illustration of directional antenna model

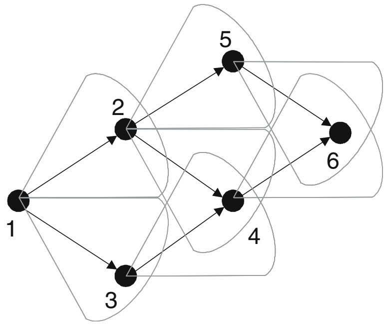


Fig. 2 A simple illustration of node graph $G = (V, E)$

of a sector. For a six-beam directional antenna, the angle of a beam is $\pi/3$. The direction a beam targeting to is defined as the beam orientation. For directional antennas, both directional transmission and directional reception are enabled. To be clear, for single-beam directional antennas, we assume only one directional transmitting beam or one directional receiving beam can be active at a time; for multi-beam directional antennas, multiple directional transmission beams or multiple directional receiving beams can be active at a time. However, a beam can only be either transmitting or receiving at any instant. An illustration of a switched beam antennas with six beams is shown as Fig. 1. Assume that the antenna is directed to discrete directions, with fixed beam-radius and beam-width. There is a link between node i and j if the distance from j to i is shorter than the beam-radius. An illustration of a node graph comprising of nodes with directional antennas is shown as Fig. 2, though a realistic node graph is always more complex. Node 1 and node 6 are considered source node and destination node, respectively.

4 Interference characterization

4.1 Flow contention graph

To study the interference pattern of directional antennas in wireless ad hoc networks, we need to learn the effect of interference through the flow contention graph. Given the toy example of node graph Fig. 2, the flow contention graphs for the single beam directional antenna case and multi-beam directional antenna case are shown as Figs. 3 and 4, respectively. The vertices in the flow

Fig. 3 Flow contention graph for single beam directional antennas

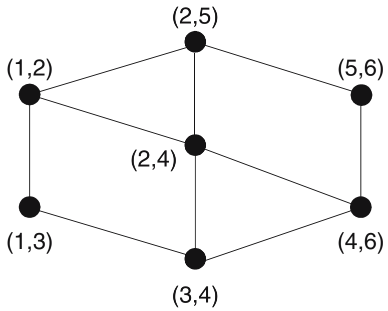
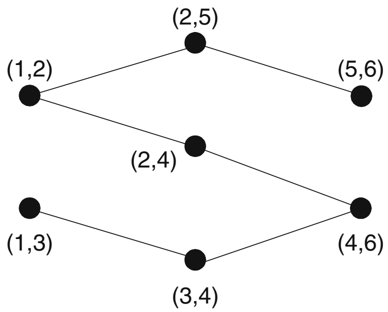


Fig. 4 Flow contention graph for multi-beam directional antennas



contention graph are the links in G. There is an edge between two vertices in the flow contention graph if the corresponding two links in G interfere with each other. For instance, link $(1,2)$ interferes with link $(1,3),(2,4)$ and $(2,5)$ in Fig. 2, because they cannot be active concurrently given single beam directional antenna. Hence, there are edges between vertices $(1,2)$ and $(1,3),(2,4)$ and $(2,5)$ in Fig. 3, respectively. Since the multi-beam directional antenna is able to receive or transmit towards several directions concurrently, the contention is only a portion of the single beam counterpart. For nodes with multi-beam directional antennas, link $(1,2)$ and $(1,3)$ can be active simultaneously. Only outgoing links $(2,3)$ and $(2,4)$ contend with link $(1,2)$. As a result, the flow contention graph for the network using multi-beam directional antennas is a subgraph of single beam directional antennas.

4.2 Link resource sharing graph

Since interfering links contend for channel, they share the resource at those links. Now we can derive the link resource sharing graph from a flow contention graph. Given flow contention graph Fig. 3, the link resource sharing graph for the single beam directional antenna can be represented as Fig. 5. For link $(2,4)$, the contention links are $(1,2),(2,5),(3,4)$ and $(4,6)$. Hence, no two

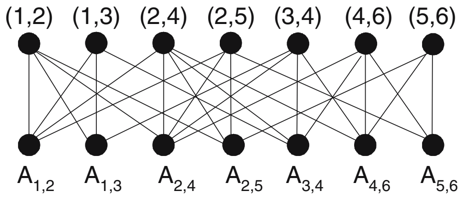


Fig. 5 Link resource sharing graph for single beam directional antennas

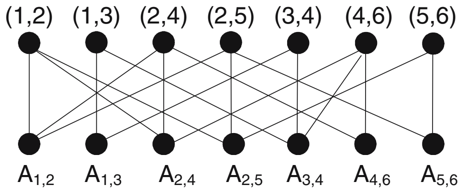


Fig. 6 Link resource sharing graph for multi-beam directional antennas

contending links are allowed to be active simultaneously. Thus the link capacity of $(2, 4)$ are shared with those links, as indicated by $A(2, 4)$. In other words, $(1, 2)$, $(2, 4)$, $(2, 5)$, $(3, 4)$ and $(4, 6)$ share the time fraction for using the common wireless channel.

When using multi-beam directional antennas, the resource sharing graph is disparate according to Fig. 4. Typically, the resource contention in networks with multi-beam directional antennas is moderate compared to single-beam directional antennas. As depicted in Fig. 6, for link $(2, 4)$, the contention links are reduced to $(1, 2)$ and $(4, 6)$. The decrease of interference level is significant.

4.3 General formulation of maximum flow

The problem here to be addressed is: given network $G(V, E)$ and existing flows, find the maximum flow supported by the network between pair *s*–*d*. Before the complete problem formulation is presented, let the constants and variables used be defined as below.

- $x_{i,j}$ indicates the flow over link (i, j).
- f is the flow from source node s to source node d.
- $b_{i,j}(i, l)$ indicates whether link (i, j) is in the l beam of node i.
- B is the total number of beams at each node.
- E is the set of edges.
- V is the set of nodes.
- θ_i^j is the beam of node i that node j resides in.

Based on the link resource sharing graph (Figs. 5 and 6), the maximum flow problem can be formulated as the following optimization problem.

$$
\begin{aligned}
&\max \quad f \\
&\text{s. t.} \\
&\sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i = s, \\ 0 & i = V - \{s,d\}, \\ -f & i = d; \end{cases} \qquad (1) \\
&\sum_{(k,l)\in A_{i,j}} x_{k,l} \le u_{i,j}, \quad \forall\, (i,j) \in E; \\
&x_{i,j} \ge 0, \quad \forall\, (i,j) \in E.
\end{aligned}
$$

where $u_{i,j}$ is the normalized remaining capacity or bandwidth ($0 \le u_{i,j} \le 1$) for link (i,j). The second constraint specifies the contention for resource of each link according to the link resource sharing graph. This is a traditional maximum flow problem with added interference constraint. To straighten the problem formulation, the interference constraint is further explored and characterized in the next section.

5 Formulation of interference-constrained maximum flow

5.1 Interference region

In wireless networks, a transmission collision occurs when a receiver is in the communication range of two transmitters, because the receiver receives both time-overlapping signals and cannot decode correctly.

We assume that an antenna both transmits and receives directionally, but it cannot transmit and receive simultaneously. With directional antenna, two links interfere with each other if a receiver is in the transmitting beams of both transmitters, shown in Fig. 7. If transmissions from node u and i overlap in time, j cannot receive the signal from i successfully because the signal from u also arrives at j. To guarantee successful reception at node j, any node in the receiving beam of j cannot transmit towards j before current transmission finishes. The protocol model in wireless ad hoc networks with directional antennas differs from those with omni-directional antennas, because the interference region is specified not only by the transmission range or beam radius, but also the beam orientation.

The protocol model In the protocol model, the transmission from node i to node j is successful if (1) j is in the transmission range of i, $d_{ij} \le R$, where R is the transmission range; (2) any node u that in the receiving beam of j from i is not transmitting in the beam covering j (when interference range = transmission range). This means that j must be outside of transmission beam of u.

In wireless communications, only one transmission is allowed in the interference region. Therefore, the channel is occupied by one transmitter in the interference region at any instant. The link flow is the product of channel bandwidth and channel usage time. As the bandwidth of the channel is fixed, the flow is proportional to the channel busy time dedicated to the transmitter.

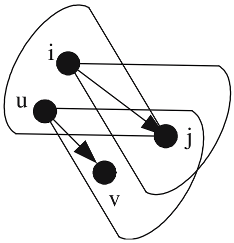


Fig. 7 An illustration of interference caused by (u, v) to (i, j)

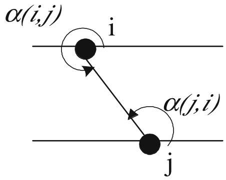


Fig. 8 Illustration of $\alpha(i,j)$ and $\alpha(j,i)$

Given a time unit, the portion assigned to a sender and receiver pair for communication indicates the flow. Instead of the circular interference area in omni-directional antenna network, the interference region in directional antenna equipped wireless networks is a beam. The smaller interference area significantly reduces the interference, thus larger amount of flows can be supported given the same channel capacity or bandwidth. In this way, the network capacity is improved comparing to the network with omni-directional antennas.

Our work is based on this protocol model. Since the interference region is a beam, the information about the beam to which a link belongs is essential for routing and scheduling. Suppose there are fixed B beams for each antenna, labeled from 1 to B counterclockwise. Then a beam is specified by the transmission range and the direction pointed to. Denote the angle between node i and another node j is $\alpha(i,j)$ as depicted in Fig. 8. The transmission and reception beams of (i,j) at node i and node j, respectively are different by $B/2$ beams. With the knowledge of $\alpha(i,j)$, (i,j) can be located in the beam $\theta_i^j = \lceil \alpha(i,j)/2\pi \times B \rceil$ of i, which is the transmission beam for link (i,j).

Now we can recapitulate condition (2) of the protocol model in the following way: (2′) when (i,j) is active, for any node u in j's receiving beam towards i, beam θ_u^j should keep silent. Denote $b(i,l)$ as the *lth* beam of node i, where $l = 1, \ldots, B$.

The problem formulation is mostly the same for the single beam and the multi-beam cases. Due to the different number of transceivers, only one constraint is different, which is the time sharing constraint as described in the following subsections. With the protocol model, we are now ready to expand the second constraint in (1).

5.2 LP formulation for single beam directional antenna

Because the single beam directional antenna can only target to one beam at a time. So the time for using the channel is shared by all links in all beams.

From the link resource sharing graph Fig. 5, the time sharing constraint is formulated as,

$$\sum_{l=1}^{B}\left(\sum_{(k,i)\in E} x_{k,i}b_{k,i}(i,l) + \sum_{(i,j)\in E} x_{i,j}b_{i,j}(i,l)\right) \leq 1, \quad \forall i \in V. \tag{2}$$

For single beam directional antennas, we can formulate the maximum flow problem as the following LP.

Problem formulation 1:

$$\begin{aligned}
&\max \quad f \\
&\text{s. t.} \\
&\sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i = s, \\ 0 & i = V - \{s,d\}, \\ -f & i = d; \end{cases} \\
&\underbrace{\sum_{u\in b(i,l)} \sum_{(u,v)\in E} x_{u,v}b_{u,v}(u,\theta_u^i)}_{\text{contention links in l-th beam}} + \underbrace{\sum_{(k,i)\in E} x_{k,i}b_{k,i}(i,l)}_{\text{incoming flows}} \leq 1, \quad \forall\, l, i, \qquad (3) \\
&\sum_{l=1}^{B}\left(\sum_{(k,i)\in E} x_{k,i}b_{k,i}(i,l) + \sum_{(i,j)\in E} x_{i,j}b_{i,j}(i,l)\right) \leq 1, \quad \forall\, i \in V, \\
&b_{i,j}(i,l) = \begin{cases} 1, & \text{if } (i,j) \in\ b(i,l), \\ 0, & \text{otherwise} \end{cases} \\
&x_{i,j} \geq 0, \quad \forall\, (i,j) \in E.
\end{aligned}$$

The first constraint describes the in-flow and out-flow at each node. The second constraint indicates the flow interference around node i as described as condition (2′) in the protocol model. The first term represents the sum of flows causing interference to i in beam l. When those flows are active, node i must restrain from receiving. The second term stands for the total incoming flows to i in beam l. Sum of these two terms should be less than the normalized beam capacity 1 to avoid collision. By beam capacity, we mean the channel capacity or bandwidth, which is a constant. The third constraint describes the time sharing constraint. From the resource constraint graph, we observe that the second and third constraints aggregately describe the contention flows at a node. The contention region includes all link flows in the 1-hop area of a node.

Denote M the number of links in the network, N the number of nodes in the network. The number of variables and constraints in this LP are M and $O(N+M)$, respectively, where $O(x)$ indicates the variable on the order of x.

5.3 LP formulation for multi-beam directional antenna

For multi-beam directional antennas, multiple incoming flows and outgoing flows could share the time for accessing the channel, which is normalized to 1. So we obtain

$$\max_{l:1\le l\le B} \text{in-flow of beam } l + \max_{l:1\le l\le B} \text{out-flow of beam } l \le 1$$

We need max functions because several beams can transmit or receive simultaneously.

Now for a single pair of source and destination nodes in wireless networks with multi-beam directional antennas, from the resource sharing graph Fig. 6, the problem formulation in (1) can be expanded more specifically as follows,

$$\begin{aligned}
&\max \quad f \\
&\text{s. t.} \\
&\sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i=s, \\ 0 & i = V-\{s,d\}, \\ -f & i=d; \end{cases} \\
&\underbrace{\sum_{u\in b(i,l)} \sum_{(u,v)\in E} x_{u,v} b_{u,v}(u,\theta_u^i)}_{\text{contention links in l-th beam}} \quad + \\
&\underbrace{\sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l)}_{\text{incoming flows}} \le 1, \quad \forall\, l, i \\
&\max_{l:1\le l\le B} \sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l) + \max_{l:1\le l\le B} \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,l) \le 1, \quad \forall\, i\in V, \\
&b_{i,j}(i,l) = \begin{cases} 1, & \text{if } (i,j)\in\ b(i,l), \\ 0, & \text{otherwise} \end{cases} \\
&x_{i,j} \ge 0, \quad \forall\, (i,j)\in E.
\end{aligned} \tag{4}$$

The first two constraints are the same as those in (3). In the last constraint, the first maximum value is the load of the beam with the most incoming flow to node i, while the second maximum value is the load of the beam with the most outgoing flow. The last constraint guarantees that the flow is feasible because the in-flow and out-flow share the capacity at the node. This constraint also implies that the in-flow from any beam should not be greater than 1. However, the constraint is non-linear.

Notice that the relationship between the link and active beam is determined by the positions of both transmitter and receiver. Therefore

$$(i,j) \in b(i, \theta_i^j), \tag{5}$$

$b_{i,j}(i,l)$ in (4) can be calculated by

$$b_{i,j}(i,l) = \begin{cases} 1, & \text{if } l = \theta_i^j, \\ 0, & \text{otherwise} \end{cases}$$

In the formulation (4), the third constraint is non-linear because of the max function. To transform it into a linear constraint, we use the following set of constraints:

$$\sum_{(k,i)\in E} x_{k,i} b_{k,j}(i,l) + \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,m) \le 1, \quad \forall\, l, m, \ \ \forall\, i \in V$$

Observe that the constraint becomes linear at the cost of adding more constraints. The number of constraints is increased by a factor of $B^2 - 1$. Thus, the maximum flow problem can be modeled by the following LP.

Problem formulation 2:

$$\begin{aligned}
&\max \quad f \\
&\text{s. t.} \\
&\sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i = s, \\ 0 & i = V - \{s, d\}, \\ -f & i = d; \end{cases} \\
&\sum_{u\in b(i,l)} \sum_{(u,v)\in E} x_{u,v} b_{u,v}(u, \theta_u^i) + \sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l) \le 1, \quad \forall\, l, i \\
&\sum_{(k,i)\in E} x_{k,i} b_{k,j}(i,l) + \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,m) \le 1, \quad \forall\, 1 \le l, m \le B, \ \ \forall\, i \in V, \\
&b_{i,j}(i,l) = \begin{cases} 1, & \text{if } l = \theta_i^j, \\ 0, & \text{otherwise} \end{cases} \\
&x_{i,j} \ge 0, \quad \forall\, (i,j) \in E.
\end{aligned} \tag{6}$$

The number of variables and constraints in this LP are M and $O(N + M)$, respectively.

Until now, the maximum flow has been formulated for single beam and multi-beam directional antennas, respectively. Except one constraint, the formulation is the same. Solving the LPs in (3) and (6) is easy. So we give a brief description

Table 1 Maximum flow rate of 20-node network

	20 Nodes	
	Average maximum flow rate	Computation time (s)
Single beam	0.2674	12
Multi-beam	0.4387	25

Table 2 Maximum flow rate of 30-node network

	30 Nodes	
	Average maximum flow rate	Computation time (min)
Single beam	0.2664	5
Multi-beam	0.5066	9

Table 3 Maximum flow rate of 40-node network

	40 Nodes	
	Average maximum flow rate	Computation time (min)
Single beam	0.2048	49
Multi-beam	0.6095	112

of the algorithm. First, we can obtain $b_{i,j}(i,l)$s after establishing the neighbor node list of every beam at each node. The constants are determined by the relative positions between nodes. Then we need to calculate the remaining capacity in each beam for every node. The remaining capacity is the total capacity minus the capacity used by interfering flows. With remaining capacity in each beam, the LP can be transformed to a standard LP. Applying an existing optimization algorithm like branch and bound [1], we can obtain the optimal solution.

6 Numerical results

In this section, we give some numerical results of the two LP problems, which are solved using MATLAB [21]. Calculations are done on a machine with 3 GHz processor and 2 GB of RAM. Nodes are randomly deployed in a 10×10 square, with transmission range of 2.5 units. The link capacity is normalized to 1. The network size varies from 20 to 40 nodes. A source-destination pair is randomly chosen from all nodes. There exist other random flows which may interfere with the flow between the source-destination pair. Each instance is repeated for 30 runs. The maximum flow rates in 20, 30 and 40-node network is shown in Tables 1, 2, 3. The first column shows the average maximum flow rate supportable in the network. The right column demonstrates the average computation time.

As expected, the maximum flow in case of multi-beam directional antenna is greater than that using single beam directional antenna. An interesting observation is that the maximum flow rate decreases inversely to the network size for networks with single beam directional antenna, while the maximum flow increases for networks with multi-beam directional antenna. The reason for the different performance is that the single beam directional antenna is more sensitive to contention caused by increased flows. As the network grows, more contention among flows is introduced, so the maximum flow supportable for the given source destination pair decreases. But the multi-beam directional antenna is capable of harnessing the advantage of space reuse more efficiently, so the contention for time fraction is still low even if the network size increases. The maximum flow does not deteriorate with the densities in the computation. On the contrary, the flow increases because more space-separated paths are available. We expect the maximum flow of the network with multi-beam directional antennas to degrade when the node density reaches a certain degree.

The computational cost is also listed in the table. The computational cost is measured in time. The computational time of multi-beam case is longer than its single beam counterpart, because there are more constraints using multi-beam directional antennas.

6.1 Discussion

To sum up, we have formulated the maximum flow problem using multipath routing subject to interference as an LP for multi-hop wireless ad hoc networks using directional antennas. The problem is different from the traditional maximum flow problem because of the interference constraints. It can be solved by a centralized algorithm at an omniscient base station. This is feasible because a base station is usually available for commanding and data collection. Typically, the base station has greater computation capacity and higher energy level; thus, it is able to carry out complex computing.

Although the centralized LP solution gives the optimal multipath flow, it has the inherent and common disadvantages of all centralized algorithms—it is not scalable to the network size and cannot quickly adapt to changes in link condition and topology. The computation time shows that the computation load skyrockets steeply as the network increases. So developing a distributed algorithm for large scale ad hoc networks, which is jointly routing and scheduling, is our future work.

7 Conclusion

We studied the multipath routing in wireless ad hoc networks with directional antennas in this work. The goal is to maximize the throughput between given source-destination pair over multiple paths. A key distinction of our work compared to previous work is that our approach answers the questions of what the optimal flow is and how to realize it, with a practical interference model.

Based on the protocol model, the maximum throughput problem constrained by interference is formulated as an optimization problem. By solving the LP at the powerful base station, the optimal flow can be determined.

The method applies to both single-beam and multi-beam directional antennas, with minor modifications.

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References

1. Ahuja, R.K., Magnanti, T.L.: Orlin, J.B.: Network Flows: Theory, Algorithms, and Applications. Prentice Hall (1993)
2. Armstrong, R.D., Chen, W., Goldfarb, D., Jin, Z.: Strongly polynomial dual simplex methods for the maximum flow problem. Math. Program. **80**(1), 14–33 (1998)
3. Burkhart, M., Rickenbach, P.V., Watternhofer, R., Zollinger, A.: Does topology control reduce interference?. In: Proceedings of the 5th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2004), pp. 9–19. Roppongi, Japan (2004)
4. Fernndez-Baca, D., Martel, C.U.: On the efficiency of maximum-flow algorithms on networks with small integer capacities. Algorithmica **4**(1), 173–189 (1989)
5. Ghosh, S., Gupta, A., Pemmaraju, S.V.: A self-stabilizing algorithm for the maximum flow problem. Distrib. Comput. **10**(4), 167–180 (1997)
6. Goldberg, A.V.: Recent developments in maximum flow algorithms (Invited Lecture). Letcture Notes in Computer Science (1998)
7. Goldberg, A.V., Grigoriadis, M.D., Tarjan, R.E.: Use of dynamic trees in a network simplex algorithm for the maximum flow problem. Math. Program. **50**(1–3), 277–290
8. Goldfarb, D., Hao, J.: A primal simplex algorithm that solves the maximum flow problem in at mostnm pivots and O(n 2 m) time. Math. Program. **47**(1–3), 353–365
9. Gupta, R., Kumar, P.R.: The capacity of wireless networks. Trans. IEEE Inf. Theory **46**(2), 388–404 (2000)
10. Jain, K., Padhye, J., Padmanabhan, V.N., Qiu, L.: Impact of interference on multi-hop wireless network performance. In: Proceedings of ACM MOBICOM, pp. 66–80, San Diego (2003)
11. Kim, D., Pardalos, P.M.: A dynamic domain contraction algorithm for nonconvex piecewise linear network flow problems. J. Global Optimization **17**(1–4), 225–234 (2000)
12. Kodialam, M., Nandagopal, T.: Characterizing achievable rates in multi-hop wireless networks: the joint routing and scheduling problem. In: Proceedings of ACM MOBICOM, pp. 42–54. San Diego (2003)
13. Kumar, S., Gupta, P.: An incremental algorithm for the maximum flow problem. J. Mathe. Model. Algorithms **2**(1), 1–16 (2003)
14. Li Y, Man, H.: Analysis of multipath routing for ad hoc networks using directional antennas. IEEE Vehicular Technol Conference 2004. **4**, 2759–2763 (2004)
15. Peraki, C., Servetto, S.D.: On the maximum stable throughput problem in random networks with directional antennas. In: Proceedings of the 4th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2003), pp. 76–87, Annapolis (2003)
16. Tang, J., Xue, G., Chandler, C., Zhang, W.: Interference-aware routing in multihop wireless networks using directional antennas. Proceedings of IEEE INFOCOM 2005
17. Tardos, E., Wayne, K.D.: Simple generalized maximum flow algorithms. Letcture Notes in Computer Science (1998)
18. Tuncel, L.: On the complexity of preflow-push algorithms for maximum-flow problems. Algorithmica **11**(4), 353–359 (1994)
19. Xue, Y., Li, B., Nahrstedt, K.: Optimal resource allocation in wireless ad hoc networks: a price-based approach. IEEE Trans. Mobile Comput. (in press)
20. Yi, S., Pei, Y., Kalyanaraman, S.: On the capacity improvement of ad hoc wireless networks using directional antennas. In: Proceedings of the 4th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2003), pp. 108–116, Annapolis (2003)
21. Matlab version 6.1. http://www.matlab.com/