Harnessing Traffic Uncertainties in Wireless Mesh Networks—A Stochastic Optimization Approach

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Abstract In this paper, we investigate the routing optimization problem in wireless mesh networks. While existing works usually assume static and known traffic demand, we emphasize that the actual traffic is time-varying and difficult to measure. In light of this, we alternatively pursue a stochastic optimization framework where the expected network utility is maximized. For multi-path routing scenario, we propose a stochastic programming approach which requires no priori knowledge on the probabilistic distribution of the traffic. For the single-path routing counterpart, we develop a learning-based algorithm which provably converges to the global optimum solution asymptotically.

Keywords wireless mesh networks · network utility maximization · routing

1 Introduction

Wireless mesh networks provide last mile broadband Internet access with low cost yet high bandwidth. The mesh routers communicate with each other via wireless links. Meanwhile, several edge mesh routers provide wireless access to the client users where all the traffic is directed to the gateway

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node. Due to the multi-hop nature, the routing in wireless mesh networks is an important and interesting topic and has attracted significant attention from the community [1].

However, the existing routing schemes in wireless mesh networks usually assume that the traffic demands aggregated at edge routers are static and known. In practice, apparently, the instantaneous traffic demand may fluctuate dramatically due to the client users' behavior and the mobility of users. Therefore, the uncertainty of the traffic demands at edge routers is not negligible and needs to be considered meticulously in wireless mesh networks.

Traffic estimation is proposed as a feasible solution to harness the traffic uncertainty. Several prediction models are proposed in the literature [2–4]. However, we emphasize that the acquisition of accurate traffic information is by no means trivial and usually computationally expensive. Even worse yet, the estimation techniques yield poor performance in a highly dynamic environment. Therefore, routing design in wireless mesh networks with uncertain traffic demand is remarkably challenging.

To circumvent the burden of route recalculation, oblivious routing schemes are suggested [5–9]. A traffic-oblivious routing protocol requires limited traffic information and achieves worst-case performance guarantee. In other words, by sticking to a fixed routing strategy, the worst-case performance is optimized. Therefore, the instability and prohibitive overhead of rerouting are avoided and the performance is acceptable if the traffic pattern varies within a certain range. However, one noticeable drawback of oblivious routing schemes is the computational complexity [9]. In addition, the optimization on the worst-case performance is usually over-conservative. Also, limited knowledge is attained when the traffic pattern varies drastically and exceeds the tolerance bound. In light of these concerns, we propose a stochastic optimum routing strategy in wireless mesh networks where

the *expected* network utility is optimized, rather than the *worst-case* performance. Our schemes differ from previous work in several aspects. First, our algorithm does not require a priori knowledge of the traffic distribution. Therefore, the computational overhead of traffic estimation can be avoided. Secondly, our scheme does not require that the traffic pattern varies within a certain range. Thirdly, our proposed algorithms are amenable to decentralized implementations.

The remainder of this paper is organized as follows. Section 2 briefly outlines the related work. The system model used in this paper is introduced in Section 3. The routing optimization problems with multi-path and single-path routing constraints are investigated in Section 4 and Section 5, respectively. A numerical example is provided in Section 6 and Section 7 concludes this paper.

2 Related work

Routing plays a critical role in wireless mesh networks. In the literature, the routing problem is usually formulated as an optimization problem where heuristic or decomposition-based solutions are proposed. The existing works usually assume that the traffic input from each edge router is static and known. Meanwhile, the empirical studies based on real traffic traces reveal that the instantaneous traffic fluctuates from time to time and is difficult to predict. To address the uncertainty of traffic demand in mesh networks, two lines of research effort have been discussed. The first category, which is in analogy of *robust optimization* techniques, achieves optimal worst-case performance. Oblivious routing schemes fall into this category. The second category, which is in analogy of the general *stochastic optimization* techniques, alternatively pursues the expected utility maximization [10–14]. Our work falls into this category. In [13], the power scheduling problem with time-varying channel is investigated. In [11], the impact of noisy feedback is analyzed, where the traffic demand is assumed to be sufficiently large and a rate control algorithm, based on the dual decomposition approach, is implemented to adjust the ingress traffic. Nevertheless, in our work, the traffic demand is dynamic and may even appear zero sometime. Another related work is [10] where the throughput maximization routing is considered. The traffic dynamic in wireless mesh networks is well addressed. However, the authors assume the traffic statistics are known, i.e., the probability distribution of random traffic demand is assumed to be a priori. In our scheme, such information is not required, as will be clarified in following sections.

3 Model

We consider a wireless mesh network depicted in Fig. 1. The mesh network consists of several edge routers, intermediate
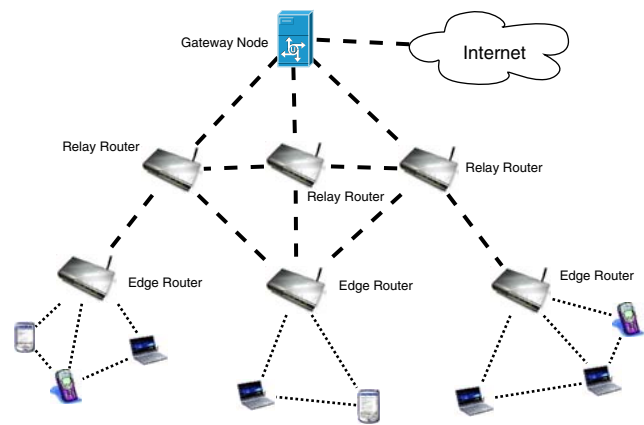

Fig. 1 Wireless mesh network topology

relay routers and a gateway node $\mathcal{W}$ which provides the connection to the Internet. Each edge router provides wireless access and is associated with a number of client users. Denote the set of edge routers as $\mathcal{M}$ and the aggregated traffic demand at the m-th edge router as z_m. Naturally, z_m is a random variable which is determined by the behaviors of client users. We assume that the instantaneous traffic demand at an edge router forms a stationary stochastic process. In this paper, we restrain ourselves to wireless mesh networks with a single gateway node. The extension to multiple gateways is left as future work.

There are two types of routing protocols in the literature, i.e., arc-routing and path-routing. In arc-routing protocols, the decision variable is the amount of traffic allocated on each link of the network, whereas the path-routing protocols adjust the fraction of traffic allocated on each distinct path. We follow the latter approach due to its practical merit [7]. For example, we can utilize the path diversity provided by OSPF/IS-IS and MPLS techniques. More specifically, OSPF divides the traffic evenly among all paths with the same cost whereas MPLS enables more flexible manipulation on the flows on all available paths. In our scenario, one edge router, say m, has $\mathcal{P}_m$ acyclic paths to the gateway node $\mathcal{W}$. The instantaneous traffic z_m is distributed among all $\mathcal{P}_m$ paths and each path possesses a fraction, denoted by r_m^k where[1] $m = 1, \cdots, \mathcal{M}$ and $k = 1, \cdots, \mathcal{P}_m$. We have

$$\sum_{k=1}^{\mathcal{P}_m} r_m^k \leq 1, \forall m \in \mathcal{M}. \tag{1}$$

Define $\mathbf{r}_m$ as the fraction vector and $\mathbf{r} = [\mathbf{r}_1, \cdots, \mathbf{r}_{\mathcal{M}}]$, which is the decision variable in our optimization framework

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[1] In this paper, we slightly abuse the notation by using the same symbol for a set and its cardinality for notation brevity.

throughout the paper. We define

$$\delta_m = \sum_{k=1}^{\mathcal{P}_m} r_m^k, \forall m \in \mathcal{M} \tag{2}$$

for notation succinctness.

We denote the set of wireless links in the mesh network as $\mathcal{E}$. We assume in this paper that a scheduling scheme is available for the medium access where each link possesses a time share of the channel access. For each link $e \in \mathcal{E}$, there is a link cost, denoted by l_e, which is a function of the instantaneous aggregated flow and the achievable data rate of the link, denoted by f_e and c_e, respectively. Note that c_e is given by

$$c_e = \tilde{c}_e \times \gamma_e \tag{3}$$

where $\tilde{c}_e$ denotes the nominal capacity of the link and γ_e is the fraction of time that link e is active following the scheduling. Without loss of generality, we assume the power of each link is fixed.

Each edge router has a utility function, denoted by $U(x_m)$, which reflects the degree of satisfaction by transmitting a flow of $x_m = z_m \times \delta_m$. In light of the link costs, the surplus of the m-th edge router, a.k.a., net reward, is given by

$$O_m = U_m(z_m \times \delta_m) - \sum_{k=1}^{\mathcal{P}_m} z_m r_m^k \sum_{e \in P_m^k} l_e(f_e) \tag{4}$$

where $e \in P_m^k$ represents all links along the k-th path of $\mathcal{P}_m$. We assume that the link cost function $l_e(f_e)$ is a non-decreasing and differentiable function of f_e. Two such well-known examples are

$$l_e(f_e) = \frac{1}{c_e - f_e} \tag{5}$$

which represents the average waiting time on link e, and

$$l_e(f_e) = \frac{f_e}{(c_e - f_e)c_e} \tag{6}$$

which reflects the average queueing time on link e, following a model of $M/M/1$ [15]. Note that the cost is considered as $+\infty$ if $f_e \geq c_e$.

Define a topology matrix H_m for the m-th edge router where the element $H_{e,k}^m = 1$ represents that link e is on the k-th path of $\mathcal{P}_m$ and zero otherwise. Therefore, we have

$$f_e = \sum_{m=1}^{\mathcal{M}} \sum_{k=1}^{\mathcal{P}_m} H_{e,k}^m z_m r_m^k. \tag{7}$$

Note that in wireless mesh networks, the edge routers and relay routers are usually static. Therefore, the topology matrix, i.e., H_m, is a fixed binary matrix and can be acquired easily. However, the values of f_e, z_m and l_e are random due the uncertain traffic demand, which makes the routing problem more challenging. In Section 4, we formulate the multi-path routing optimization problem of wireless mesh networks in a stochastic programming framework where a distributed algorithmic solution is derived. The single-path routing scenarios are investigated in Section 5, where a learning-based algorithm is introduced to achieve the global optimum solution asymptotically.

4 Multi-path routing scenario

In this section, we consider the cases where multi-path routing is allowed for fault tolerance and load balancing purpose, i.e., the aggregated flow of each edge router is distributed among several available paths.

In the network, each edge router has a surplus given by Eq. 4. From the network's perspective, our objective is to maximize the overall surplus of the network, i.e., $\sum_{m=1}^{\mathcal{M}} O_m$. However, due to the randomness induced by the traffic demand uncertainty, the overall surplus itself is a random variable. Therefore, we alternatively pursue a stochastic optimum solution of $\mathbf{r}^*$, which maximizes the *expected overall surplus* of the network. Mathematically, the routing problem in the wireless mesh network is formulated as

$$\max_{\mathbf{r}} \mathbb{E} \left(\sum_{m=1}^{\mathcal{M}} O_m \right)$$

s.t.

$$\mathbb{E}(f_e) \leq c_e \ \forall e \in \mathcal{E} \tag{8}$$

$$\sum_{k=1}^{\mathcal{P}_m} r_m^k \leq 1 \ \forall m \in \mathcal{M} \tag{9}$$

$$f_e = \sum_{m=1}^{\mathcal{M}} \sum_{k=1}^{\mathcal{P}_m} H_{e,k}^m z_m r_m^k \ \forall e \in \mathcal{E} \tag{10}$$

$$O_m = U_m(z_m \times \delta_m) - \sum_{k=1}^{\mathcal{P}_m} z_m r_m^k \sum_{e \in P_m^k} l_e(f_e) \ \forall m \in \mathcal{M} \tag{11}$$

where $\mathbb{E}(.)$ is the expectation operator. We assume that the random traffic demands of all edge routers can be discretized into arbitrarily many yet finite states, where each state is represented by $s \in \mathcal{S}$. The time is slotted and within one slot, the traffic demands on all edge routers correspond to one of the states and remain unchanged within the current slot. Note that O_m, f_e, l_e and z_m are all state-dependent variables and hence we will add a superscript s in the following formulations. Denote the stationary probability distribution

of state s as π_s. We can rewrite the routing optimization problem as

$$\max_{\mathbf{r}} \sum_{s \in S} \pi_s \left(\sum_{m=1}^{\mathcal{M}} O_m^s \right)$$

s.t.

$$\sum_{s \in S} \pi_s (f_e^s) \le c_e \ \forall e \in \mathcal{E} \tag{12}$$

$$\sum_{k=1}^{\mathcal{P}_m} r_m^k \le 1 \ \forall m \in \mathcal{M} \tag{13}$$

$$f_e^s = \sum_{m=1}^{\mathcal{M}} \sum_{k=1}^{\mathcal{P}_m} H_{e,k}^m z_m^s r_m^k \ \forall e \in \mathcal{E} \tag{14}$$

$$O_m^s = U_m(z_m^s \times \delta_m) - \sum_{k=1}^{\mathcal{P}_m} z_m^s r_m^k \sum_{e \in P_m^k} l_e^s(f_e^s) \ \forall m \in \mathcal{M} \tag{15}$$

We can verify that if the stationary probability distribution π_s is known as a priori, the optimization problem is a deterministic convex optimization problem and the Slater's condition is satisfied. However, the actual value of π_s is either difficult to measure in practice, or needs significant computational overhead to estimate. Next, we propose a distributed solution which converges to the global optimum solution yet requires no information about the underlying probabilistic distribution.

We first obtain the Lagrangian as

$$
\begin{aligned}
L(\mathbf{r}, \lambda, \mu) &= \sum_{s \in S} \pi_s \left\{ \sum_{m \in \mathcal{M}} U_m(z_m^s \times \delta_m) \right. \\
&\quad \left. - \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{P}_m} z_m^s r_m^k \left(\sum_{e \in P_m^k} l_e^s(f_e^s) \right) \right\} \\
&\quad + \sum_{m \in \mathcal{M}} \lambda_m \left(1 - \sum_{k \in \mathcal{P}_m} r_m^k \right) \\
&\quad + \sum_{e \in \mathcal{E}} \mu_e \left(c_e - \sum_{s \in S} \pi_s f_e^s \right) \\
&= \sum_{s \in S} \pi_s \left\{ \sum_{m \in \mathcal{M}} (U_m(z_m^s \times \delta_m) + \lambda_m) \right. \\
&\quad + \sum_{e \in \mathcal{E}} \mu_e c_e - \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{P}_m} \\
&\quad \left. \times \left(z_m^s r_m^k \left(\sum_{e \in P_m^k} (l_e^s(f_e^s) + \mu_e) + \lambda_m r_m^k \right) \right) \right\}
\end{aligned}
\tag{16}
$$

Note that μ is the link congestion price similar as [16–18]. The dual variables of λ ensure that the summation of fraction variables is less or equal to unity. Define

$$
\begin{aligned}
Q^s = \sup_{\mathbf{r}} &\left\{ \sum_{m \in \mathcal{M}} (U_m(z_m^s \times \delta_m) + \lambda_m) + \sum_{e \in \mathcal{E}} \mu_e c_e \right. \\
&\left. - \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{P}_m} \left(z_m^s r_m^k \left(\sum_{e \in P_m^k} (l_e^s(f_e^s) + \mu_e) + \lambda_m r_m^k \right) \right) \right\}
\end{aligned}
\tag{17}
$$

Therefore, the dual function is given by

$$g(\lambda, \mu) = \sum_{s \in S} \pi_s Q^s \text{ where } \lambda \ge 0, \mu \ge 0. \tag{18}$$

To achieve the minimum value of Eq. 18, or equivalently, to obtain the stochastic optimum value of $\mathbf{r}^*$, we utilize the stochastic primal-dual approach [17]. The dual variables, i.e., λ and μ are updated as

$$\lambda_m(n+1) = [\lambda_m(n) - \alpha_m(n)\zeta_m(n)]^+ \ \forall m \in \mathcal{M} \tag{19}$$

$$\mu_e(n+1) = [\mu_e(n) - \alpha_e(n)\xi_e(n)]^+ \ \forall e \in \mathcal{E} \tag{20}$$

where $[x]^+$ denotes $\max(x, 0)$. Similarly, the primal variable, i.e., r_m^k, is updated as

$$r_m^k(n+1) = [r_m^k(n) + \alpha_{m,k}(n)\eta_{m,k}(n)]_0^1 \tag{21}$$

where $[x]_a^b$ denotes $\max(\min(b, x), a)$ and we use symbol $\alpha(n)$ to represent the corresponding stepsizes, i.e., $\alpha_m(n)$, $\alpha_e(n)$ and $\alpha_{m,k}(n)$, generally. Note that the instantaneous link flow is considered as fixed when the fraction variable r_m^k is updated.

The stochastic subgradients, i.e., $\zeta_m(n)$, $\xi_e(n)$ and $\eta_{m,k}(n)$, can be attained by the Danskin's theorem [19], following a similar approach as in [13, 14]

$$\zeta_m(n) = 1 - \sum_{k \in \mathcal{P}_m} r_m^k \ \forall m \in \mathcal{M} \tag{22}$$

$$\xi_e(n) = c_e - \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{P}_m} z_m^s r_m^k H_{e,k}^m \ \forall e \in \mathcal{E} \tag{23}$$

$$\eta_{m,k}(n) = - \left[\lambda_m + z_m^s \left(\sum_{e \in P_m^k} (l_e^s + \mu_e) \right) \right] \ \forall m \in \mathcal{M}. \tag{24}$$

The distributed implementation of the algorithm is summarized as follows.

Algorithm:

Repeat:

– Each link measures

$$\xi_e(n) = c_e - \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{P}_m} z_m^s r_m^k H_{e,k}^m. \quad (25)$$

– Each link updates the link congestion price as

$$\mu_e(n+1) = [\mu_e(n) - \alpha_e(n)\xi_e(n)]^+. \quad (26)$$

– Each edge router measures

$$\zeta_m(n) = 1 - \sum_{k \in \mathcal{P}_m} r_m^k \quad (27)$$

and

$$\eta_{m,k}(n) = - \left[\lambda_m + z_m^s \left(\sum_{e \in P_m^k} (l_e^s + \mu_e) \right) \right]. \quad (28)$$

– Each edge router updates the fraction variable as

$$r_m^k(n+1) = [r_m^k(n) + \alpha_{m,k}(n)\eta_{m,k}(n)]_0^1. \quad (29)$$

– Each edge router updates the dual variable λ as

$$\lambda_m(n+1) = [\lambda_m(n) - \alpha_m(n)\zeta_m(n)]^+. \quad (30)$$

Until:

– The difference between successive iterations, i.e.,

$$\epsilon = |r_m^k(n+1) - r_m^k(n)| \quad (31)$$

is within a predefined convergence threshold ϵ'.

End

Note that the information needed in the algorithm is either locally attainable or acquirable by the feedback along the paths. Therefore, the algorithm is amenable to distributed implementation. The convergence of the algorithm follows the results of [11] and thus we provide the following theorem without proof.

Theorem 1 *The algorithm converges to the global optimum with probability one, provided that the following constraints are satisfied* [11, 20]: *$\alpha(n) > 0$, $\sum_{n=0}^{\infty} \alpha(n) = \infty$, and $\sum_{n=0}^{\infty} (\alpha(n))^2 < \infty$, $\forall m \in \mathcal{M}$ and $e \in \mathcal{E}$, where α generally represents all stepsize parameters in* (19), (20) *and* (21).

5 Single-path routing scenario

In this section, we consider a wireless mesh network where single-path routing strategy is adopted. Therefore, the fraction variable r_m^k is a binary number in this scenario. More specifically, $r_m^k = 1$ denotes that the m-th edge router selects the k-th path to deliver traffic and the fraction variables on other paths are zeros. The integral property of the fraction variables complicates the routing optimization problem. We first express the routing optimization problem, with single path routing constraint, as

$$\max_{\mathbf{r}} \sum_{s \in S} \pi_s \left(\sum_{m=1}^{\mathcal{M}} O_m^s \right)$$

s.t.

$$\sum_{s \in S} \pi_s(f_e^s) \le c_e \ \forall e \in \mathcal{E} \quad (32)$$

$$\sum_{k=1}^{\mathcal{P}_m} r_m^k \le 1 \ \forall m \in \mathcal{M} \quad (33)$$

$$f_e^s = \sum_{m=1}^{\mathcal{M}} \sum_{k=1}^{\mathcal{P}_m} H_{e,k}^m z_m^s r_m^k \ \forall e \in \mathcal{E} \quad (34)$$

$$O_m^s = U_m(z_m^s \times \delta_m) - \sum_{k=1}^{\mathcal{P}_m} z_m^s r_m^k \sum_{e \in P_m^k} l_e^s(f_e^s) \ \forall m \in \mathcal{M} \quad (35)$$

$$r_m^k \in \{0, 1\} \quad (36)$$

Apparently, this is a stochastic integer programming which is difficult and computationally demanding to solve. For a survey on the algorithmic solutions of stochastic integer programming problems, refer to [21]. Distinguishing from previous work, we next propose a learning-based algorithm, which asymptotically converges to the global optimum of the aforementioned stochastic integer programming problem. First, we briefly overview the learning automata technique, based on which our algorithm is proposed.

5.1 Learning automata

Learning automata techniques have been broadly investigated in the networking community [22–27]. As one of the stochastic learning schemes, learning automata was first introduced in the control community for stochastic systems. As depicted in Fig. 2, the basic single user scenario where learning automata techniques can be applied consists of a random environment, a set of finite actions and a rational decision maker. At a time instance, the decision maker selects one of the actions according to the *selection probability*
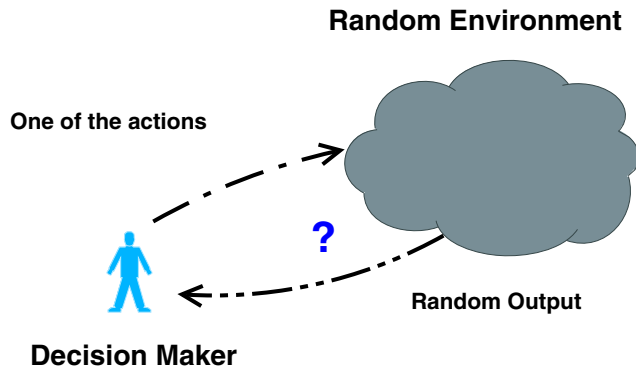
Random Environment

One of the actions

?

Random Output

Decision Maker

Fig. 2 Structure of a single user learning automata

vector **p**. The random environment responds with a stochastic output based on which the learning algorithm updates the selection probability vector and the iteration continues. In a stationary random environment, the standard learning automata algorithms asymptotically converge to an action which is stochastically optimal in the sense that the expected objective is maximized [28].

In our scenario, each edge router is an independent decision maker. The action space corresponds to the available paths of each edge router. Meanwhile, the random environment is the uncertain traffic demands at all edge routers jointly. To incorporate a multi-agent learning scenario, we form all $\mathcal{M}$ edge routers as a *learning team*, which targets to the stochastic optimum solution collectively. In addition, we utilize the gateway node $\mathcal{W}$ of the wireless mesh network as a *teacher*. After each decision maker selects an action, the gateway node multicasts a feedback signal, denoted by β, based on which each edge router updates the selection probability vector and the algorithm iterates until convergence.

5.2 Learning-based algorithm

Each edge router, say m, maintains a selection probability vector $\mathbf{p}_m$ and an *inner-state vector* $\mathbf{u}_m$. We use the notation (m, k) to indicate the k-th path of the m-th edge router, i.e., the k-th action from the action space of the decision maker m.

At the initialization phase, the value of $\mathbf{u}_m$ is randomly generated and $\mathbf{p}_m = [1/\mathcal{P}_m, \cdots, 1/\mathcal{P}_m]$. At time instance n, the algorithm executed at the m-th edge router is described as follows.

Algorithm:

Repeat:

– Selects a routing path from $\mathcal{P}_m$, say j, according to the selection probability vector $\mathbf{p}_m(n)$ and starts the transmission.

– After receiving the feedback signal from the gateway node $\mathcal{W}$, denoted by $\beta(n)$, the inner state vector $\mathbf{u}_m$ is updated as

$$u_{m,k}(n+1) = \left[u_{m,k}(n) + \theta(n)\beta(n) \right.$$
$$\times \left(1 - \frac{e^{u_{m,k}}}{\sum_{k \in \mathcal{P}_m} e^{u_{m,k}}} \right)$$
$$\left. + \sqrt{\theta(n)}\omega_{m,k}(n) \right]_0^{\mathcal{L}}, \text{ for } k = j \quad (37)$$

$$u_{m,k}(n+1) = \left[u_{m,k}(n) + \sqrt{\theta(n)}\omega_{m,k}(n) \right]_0^{\mathcal{L}}$$
$$\text{for } k \neq j \quad (38)$$

Recall that j denotes the selected action, i.e., the chosen path.

– The selection probability vector $\mathbf{p}_m$ is then updated, following

$$p_{m,k} = \frac{e^{u_{m,k}}}{\sum_{k \in \mathcal{P}_m} e^{u_{m,k}}} \ \forall k \in \mathcal{P}_m. \quad (39)$$

Until:

– $\max(\mathbf{p}_m(n)) > B$ where B is a predefined convergence threshold.

In the algorithm, $\theta(n)$ is the *learning parameter* of the algorithm satisfying $0 < \theta(n) < 1$. $\mathcal{L}$ is a positive number which keeps the inner state value bounded. The sequence of $\omega_{m,k}(n)$ is a set of random variables with zero mean and a variance of $\sigma^2(n)$. The global feedback signal $\beta(n)$ is calculated by the gateway node $\mathcal{W}$ as

$$\beta(n) = \frac{\sum_{m=1}^{\mathcal{M}} O_m^s}{\mathcal{J}} \quad (40)$$

where $\mathcal{J}$ is a sufficiently large number to normalize the output. In other words, the value of $\beta(n)$ is deliberately tuned within [0, 1]. We emphasize that the introduced noise parameter ω restrains the algorithm from being trapped in an inefficient equilibrium. Note that the value of $\beta(n)$ can be informed by efficient multicast algorithms, e.g., [29], initiated by the gateway node $\mathcal{W}$. The team learning is then executed in a decentralized fashion. The steady state behavior of the learning-based algorithm is given in the following theorem.

Theorem 2 *The proposed learning-based algorithm converges to the global optimum solution of the single-path routing optimization problem with probability one, if the following conditions are satisfied* [30]: *(1)* $\lim_{n \to \infty} \theta(n) = 0$ *and (2)* $\lim_{n \to \infty} \sigma(n) = 0$.

Proof The proof generalizes the results in [30]. First, we observe that the set of edge routers implicitly form an N-person game. Moreover, according to Eq. 40, all the players collectively maximize a common utility function following the global feedback from the gateway node. Therefore, the game can be viewed as an *identical interest game*. Note that the strategy space of each player is simply the routing paths, which are determined by the path selection probability vectors. Following Eq. 39, we hereby utilize the inner state vector $\mathbf{u}$ as the variable in our analysis.

We first define an indicator function $s_{m,k} = 1$ representing the *event* that the m-th edge router selects the k-th path in $\mathcal{P}_m$ and $s_{m,k} = 0$ otherwise. Note that

$$\frac{\partial \mathbb{E}(\beta|\mathbf{u})}{\partial u_{m,k}} = \frac{\partial \sum_k p_{m,k} \mathbb{E}(\beta|\mathbf{u}, s_{m,k})}{\partial u_{m,k}}$$
$$= \sum_k \frac{e^{u_{m,k}}}{\sum_k e^{u_{m,k}}} \left(1 - \frac{e^{u_{m,k}}}{\sum_k e^{u_{m,k}}}\right) \mathbb{E}(\beta|\mathbf{u}, s_{m,k}).$$

Also,

$$\mathbb{E}\left(\beta\left(1 - \frac{e^{u_{m,k}}}{\sum_k e^{u_{m,k}}}\right)|\mathbf{u}\right)$$
$$= \sum_k p_{m,k} \left(1 - \frac{e^{u_{m,k}}}{\sum_k e^{u_{m,k}}}\right) \mathbb{E}(\beta|\mathbf{u}, s_{m,k})$$
$$= \sum_k \frac{e^{u_{m,k}}}{\sum_k e^{u_{m,k}}} \left(1 - \frac{e^{u_{m,k}}}{\sum_k e^{u_{m,k}}}\right) \mathbb{E}(\beta|\mathbf{u}, s_{m,k})$$
$$= \frac{\partial \mathbb{E}(\beta|\mathbf{u})}{\partial u_{m,k}}.$$

Therefore, from [31] (Chap.6), we conclude that the above dynamic weakly converges to the following SDE

$$d\mathbf{u} = \nabla\mathbb{E}(\beta|\mathbf{u}) + \sigma d\mathcal{Q} \qquad (41)$$

for a sufficiently small $\theta \to 0$ where $\mathcal{Q}$ is a standard Wiener Process. Note that the SDE (Eq. 41) falls into the category of Langevin Eq. [32] which is well-known that the probability measure concentrates on the global maximum solution of the objective function, i.e., $\mathbb{E}(\beta|\mathbf{u})$, for a sufficiently small $\sigma \to 0$ [30, 32]. Therefore, we conclude that in this identical interest game, the aforementioned learning-based algorithm will converge to the global optimum of the objective function asymptotically and thus completes the proof. □

6 Numerical example

We consider an illustrative wireless mesh network shown in Fig. 3. Among all available paths, we assign a set of paths
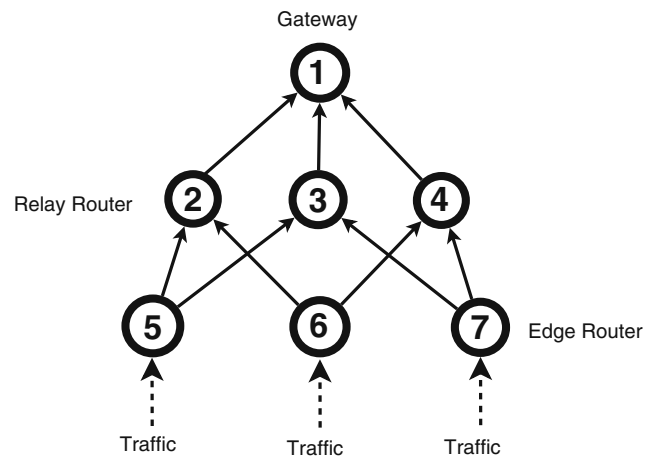

Fig. 3 Illustration of Available Paths

for each edge router and the corresponding links[2] are shown in Fig. 3.

The gateway node is indicated as node 1. Three edge routers are marked as node 5, 6 and 7, respectively, which consistently deliver the aggregated traffic to the gateway node. The available paths for each edge router are provided in Table 1.

Note that each edge router has two acyclic paths to reach the destination. The link cost function follows Eq. 5. The traffic is randomly generated at each edge router with Gaussian distribution. The mean is 5 and the variance is 1. In addition, we limit the traffic to be in the range of [0, 10]. We emphasize that these settings do not involve any loss of generality. By setting the traffic of each router with identical statistical characteristics, the stochastic optimum solution coincides with the load-balancing solution. The computational difficulty of calculating the global optimum of the stochastic integer programming problem can be avoided. Therefore, we can easily verify the efficacy of our proposed algorithms. In addition, the actual achievable data rate is a function of the transmission power and the scheduling algorithm. Without loss of generality, we assume that the achievable data rate of each link is 10. We utilize the same stepsize for the stochastic gradient approach in Eqs. 19, 20 and 21, which is inversely proportional to the number of iterations.

Table 1 Available paths for each edge router

Edge router 5	P_5^1:	$\{5 \to 2 \to 1\}$
	P_5^2:	$\{5 \to 3 \to 1\}$
Edge router 6	P_6^1:	$\{6 \to 2 \to 1\}$
	P_6^2:	$\{6 \to 4 \to 1\}$
Edge router 7	P_7^1:	$\{7 \to 3 \to 1\}$
	P_7^2:	$\{7 \to 4 \to 1\}$

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[2] Note that the actual network topology can be much larger.

We first investigate the multi-path routing scenario where each edge router can divide the incoming traffic on two available paths. Restated, it is straightforward to verify that, given the current settings of link achievable rates and statistical characteristics, the stochastic optimum solution coincides with the load-balancing solution. In other words, the solution of $r_5^1 = r_5^2 = r_6^1 = r_6^2 = r_7^1 = r_7^2 = \frac{1}{2}$ maximizes the expected overall network surplus and thus solves the routing optimization problem. This observation is verified in Fig. 4.

Figure 4 depicts the evolution of 6 fraction variables of edge routers where pair $m - k$ denotes r_m^k. At initialization, we manually set the fraction variables as $r_5^1 = 1, r_5^2 = 0$, $r_6^1 = 1, r_6^2 = 0$ and $r_7^1 = 0, r_7^2 = 1$ which is remarkably biased. Then, the distributed algorithm derived in Section 4 is executed. As pictorially shown by Fig. 4, all 6 fraction variables evolve with the iterations and converge to the stochastic optimum solution gradually.

Next, we investigate the single-path routing scenario. The network settings are the same as in multi-path scenarios except that each edge router can only select one of the available paths to transmit. The learning parameter is set to $\theta(n) = 1/n$ where n is the index of the current iteration. The noise parameters are zero mean Gaussian random variables with diminishing variances, e.g., $\sigma(n) = 1/n$. The bounding parameter $\mathcal{L}$ is 100 and $\mathcal{J}$ is 100. The convergence threshold B is 0.9999. The learning-based algorithm is executed until each edge router sticks to one of the available routing paths. The convergence behavior of the learning-based algorithm is demonstrated in Fig. 5.

In Fig. 5, we denote pair $m - k$ as the probability that the m-th edge router picks the k-th path for next iteration. For example, $5 - 1$ represents the probability that edge router 5 selects the first path, i.e., P_5^1. As illustrated in Fig. 5, $5 - 1$, $6 - 2$ and $7 - 1$ promptly approach to unity while others diminish to null. From Fig. 3, it is apparent that such a routing strategy is indeed a global optimum solution of
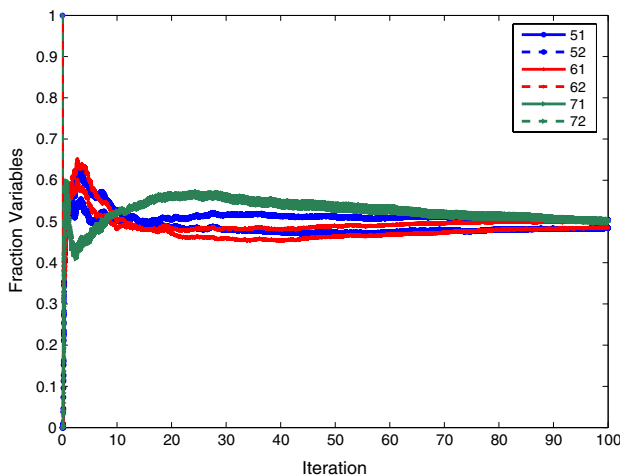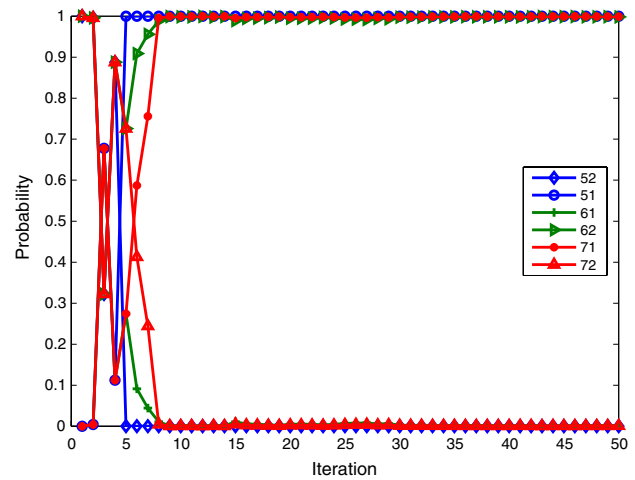

Fig. 5 Convergence of selection probability vectors

the routing optimization problem with single-path routing constraints. The learning-based algorithm finds this global optimum solution effectively. Note that this routing strategy is not the only global optimum solution. For instance, ($p_5^2 = 1$, $p_6^1 = 1$, $p_7^2 = 1$) is another global optimum solution due to the statistical symmetry of the network. By adjusting the initial conditions, the learning algorithm finds other global optimum routing strategies as well. Duplicate results are omitted.

7 Conclusions and future work

In this paper, we consider the routing optimization problem in wireless mesh networks under uncertain traffic demands. For multi-path routing scenario, we investigate the problem in a stochastic programming framework and a distributed algorithmic solution is derived. For the single-path routing scenario, the problem is formulated as a stochastic integer programming where a learning-based algorithm is proposed.

In our work, we consider a single gateway in the wireless mesh network. The extension to the multiple gateway scenarios remains as future research. In addition, the analysis for the performance of our schemes under non-stationary traffic demand is challenging and needs further investigation.

Fig. 4 Convergence of fraction variables

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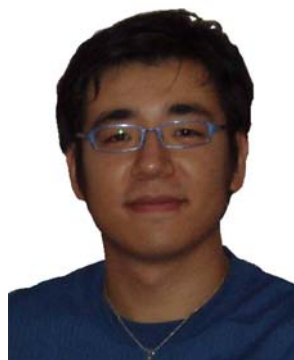
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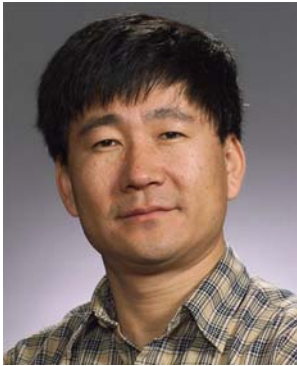
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