Optimal Threshold Policy for In-home Smart Grid with Renewable Generation Integration

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Abstract

In-home Smart Grid (SG), the integration of Renewable Power Systems (RPSs) with Conventional Power Systems (CPSs), calls for cost-effective management for the electricity usages of end users’ household appliances. In this paper, by taking the charging process of RPSs and multiple types of household appliances into consideration, we have developed analytical models to characterize the electricity cost in the in-home smart grid. Based on these models, we formulate the electricity cost minimization problem as a finite-horizon continuous-time Markov decision process (CTMDP), from which we obtain a threshold policy to minimize the cost. Numerical results show that the threshold policy can manage the electricity usage very effectively.

Index Terms

Continuous-Time Markov Decision Process; Conventional Power System; In-home Smart Grid; Renewable Power System

1 INTRODUCTION

Smart Grid (SG) [1], [2] consists of both conventional power systems (CPSs) and renewable power systems (RPSs) such as solar and wind energy systems. By utilizing the communication technologies, SG offers many functionalities, such as realtime monitoring and realtime pricing (RTP) for electricity usages of end users’ household appliances. In particular, the in-home SG allows end users to manage the energy usage of their household appliances in a cost-efficient way. Fig. 1 depicts a general system architecture of the in-home SG adopted in many existing works or products (see, e.g., [1], [3], [4]). The in-home SG is composed of three major subsystems:
Fig. 1. The general architecture of an In-home SG

the RPS (Fig. 1(a)), the CPS (Fig. 1(b)), and the in-home Energy Management System (in-home EMS; Fig. 1(c)).

There are three major components in the RPS: the solar panel (Fig. 1(1)), the battery (Fig. 1(2)), and the DC/AC inverter (Fig. 1(3)). The solar panel gathers the solar power and stores it in the battery. The DC/AC inverter converts the direct current (DC) provided by the battery to the alternating current (AC) to serve the household appliances. More details of the RPS can be found in [3].

In the CPS, electrical power is generated by power plants and delivered to households through power lines. A smart meter (Fig. 1(5)) performs the appliance power metering task and generates the report on the electricity usage, and provides the RTP information to end users. The details of the CPS can be found in [5].

Based on the RTP information and the battery status of the RPS, the in-home EMS determines whether the power demand from an appliance should be served by either the RPS or the CPS (Fig. 1(4)). Depending on the availability of renewable energy sources and the RTP information, different control policies in the in-home EMS will incur different electricity costs, and the power supplied by the RPS and CPS may differ significantly. As pointed out in [1], [4], [6], the CPS has sufficient energy to supply power demands from the end users consistently, whereas the power supplied by the RPS is intermittent, depending on the availability of renewable energy sources (e.g., solar or wind energy). As described in [7] and [8], the cost of using one unit (i.e., kWh)
energy from the CPS is a function, say \( f(\cdot) \), of the current total power (unit: kW) supplied by the CPS. \( f(\cdot) \) will be elaborated in Section 3.1.

In contrast to the RTP, the cost to use one kWh from the RPS is a constant that depends on the cost of the RPS hardware facilities. If the hardware facilities are damaged due to aging, the cost of replacing the damaged facilities can be significant. Based on statistical data [9], the cost to use per kWh from the RPS is around 0.1884 USD.

Most existing in-home EMSs (e.g., in [3]) use the following policy: When a power demand arrives, it is served by the RPS as long as there exists energy in the battery; otherwise (i.e., no energy in the RPS), the power demand is served by the CPS. For ease of reference, we call this policy as default policy. The default policy may not minimize the electricity cost in an in-home SG. To address the optimality, we propose a threshold policy to minimize the cost of the electricity usage. In the threshold policy, in response to power demands of household appliances with different types of energy usage profiles, we use different thresholds to determine whether a power demand should be met by the CPS or the RPS. The threshold is dynamically adjusted based on the current state of the in-home SG, the power demand arrival time, and the RTP information.

In this paper, we first propose analytical models to characterize the cost for energy usage in an in-home SG when the default policy is applied. Because the renewable energy sources exhibit greater variability across timescales, the energy that can be supplied by the RPS is hard to predict. To smooth out the variability in the RPS, batteries will be deployed. To simplify the presentation, we term the battery charging/discharging as the “charging process” for the RPS. In our analytical models, we use a stochastic process to model the charging process for the RPS. We also consider multiple types of household appliances. The behavior of each type is characterized by its usage frequency, power requirement, and time duration for its electricity usage. Because of the uncertainty of the power demands from household appliances, we propose a threshold policy to minimize the electricity cost by applying finite-horizon Continuous-Time Markov Decision Processes (CTMDP) [10]. We conduct numerical study to compare the performance of the threshold policy against the default policy.

The main contributions of this work are summarized as follows:
• Our work is of the first few that consider both the charging process on the RPS and multiple types of household appliances in an in-home SG environment with both the RPS and CPS.
• Our proposed threshold policy is a multi-dimensional policy, i.e., for power demands of different appliances, different thresholds are used to determine whether the power demands should be served by the CPS or RPS.
• Our work is the first one that uses the finite-horizon CTMDP to deal with the cost minimization problem in an in-home SG. Compared to the previous works based on infinite-horizon optimization, our model is more amenable to be applied to a real system.

The rest of this paper is organized as follows. Section 2 surveys the related works. In Section 3, we develop analytical models for the default policy. Based on the analytical models, we propose our threshold policy in Section 4. Numerical studies have been carried out in Section 5 to demonstrate the effectiveness of our proposed scheme. Section 6 concludes our work.

2 Related Work

In this section, we survey the previous works mainly on minimizing the cost of electricity usage in the SG.

The previous works such as [4], [11–13] adopted the offline approach to minimizing the electricity cost. They mostly assumed that the power demand arrivals and the amount of electricity that the RPS can generate are known in advance. Based on these assumptions, the researchers were able to determine the amount of electricity to purchase from the CPS, and then schedule to serve the power demands. However, because the user behavior of using appliances and the charging process of the RPS cannot be precisely predicted, the amount of electricity (to be purchased from the CPS) and the scheduling algorithms may not meet users’ actual needs, degrading users’ quality of experience. How to manage unpredictable power demands is what we focus on in this paper.

To enhance the offline scheduling, the works [1], [6], [14], [15] adopted the online approach to minimize the electricity cost. The in-home EMS references the RTP information to decide whether the power demands, either new or currently being served, should be postponed or interrupted for a certain period.
More specifically, in [6], Koutsopoulos et al. used a CTMDP to minimize the electricity cost. When a new power demand arrives, if the RTP exceeds a threshold (obtained by CTMDP), the new power demand is postponed and put into a queue. Otherwise (i.e., the RTP is below the threshold), the new power demand is served immediately. In [14], Kim et al. classified the power demands into two categories: noninterruptible and interruptible. When the RTP exceeds a threshold (obtained by discrete-time MDP), the services for interruptible power demands are postponed. Based on the deadline constraint for power demands and the RTP information, they proposed a policy to decide when to resume the service of an interruptible power demand and when to serve a new power demand. However, the works [6] and [14] did not consider the RPS and its charging process.

In [15], Alizadeh et al. proposed a policy to determine how much energy to purchase from the CPS by a day-ahead scheduling, which relies on the prediction of the amount of electricity that can be produced by the RPS next day. Because the availability of the renewable energy sources is uncertain, the policy may not precisely determine the right amount of energy to purchase.

In [1], [16], Guo et al. considered the similar issue in [15] with different formulations. Based on the Lyapunov optimization technique, they obtained online scheduling algorithms to determine the amount of electricity to purchase from the CPS. In [17], Guo et al. shows that the Lyapunov optimization technique can also be used to better utilize the green energy and help cloud service providers reduce the carbon footprint. However, their solution targets at the infinite-horizon optimization, which may not work well for short-term optimization.

Note that our threshold policy is for a global minimization for the power usage within a day given that the power demands are unknown, and power demands from the end users can be served immediately.

3 Analytical Model for the Default Policy

3.1 Problem Formulation

As shown in Fig. 1, we consider an in-home SG that consists of the RPS and CPS. In the default policy, new power demands are served by the RPS whenever there is electricity in the RPS. When the electricity stored in the RPS runs out, the in-home EMS switches the power demands from the RPS to the CPS.
Suppose that power demand arrivals to the in-home EMS form a Poisson process with rate \( \lambda_J \), and there are \( n \) types of household appliances among these arrivals. With probability \( p_k \), a power demand arrival is of type \( k \) appliance, where \( k \in \{1, 2, \ldots, n\} \) and \( \sum_{k=1}^{n} p_k = 1 \).

For a type \( k \) power demand, it requests \( w_k \) (\( \in \mathbb{R}_+ \)) kW for a time period that is assumed to have an exponential distribution with mean \( 1/\mu_k \) hrs. To simplify the notation, we denote the vector \( \mathbf{w} = (w_1, w_2, \ldots, w_n) \), \( \mathbf{u} = (\mu_1, \mu_2, \ldots, \mu_n) \), \( \mathbf{0} = (0, \ldots, 0) \) and \( \mathbf{e}_k = (0, \ldots, 0, 1, 0, \ldots, 0) \), where \( 1 \leq k \leq n \).

The charging process for the battery in the RPS is modeled as follows. The number of charges during the time period \([s, t]\), where \( t > s > 0 \), is assumed to have a Poisson distribution with mean \( \lambda_B(t - s) \). We assume that all charges are i.i.d. Let \( X \) denote the amount of kWh from one charge, which is assumed to be with a general probability distribution function \( F_C(\cdot) \). Let \( B_{\text{max}} \) (units: kWh) be the maximum amount of electricity that can be stored in the battery.

Note that with the exponential assumptions on inter-arrival times for power demand arrivals, the service time for power demands, and inter-arrival times for charges, our analytical models can provide mean-value analysis. These assumptions will be released through our simulation study.

Let \( S = (S_C, S_R, S_L) \) denote the system state, where \( S_C = (S_{C,1}, \ldots, S_{C,n}) \), \( S_R = (S_{R,1}, \ldots, S_{R,n}) \) and \( S_L \in \mathbb{R}_+ \). For \( k \in \{1, \ldots, n\} \), \( S_{C,k} \) (resp. \( S_{R,k} \)) denotes the number of type \( k \) appliances that are served by the CPS (resp. the RPS). \( S_L \) (unit: kWh) denotes the current energy level of the battery.

Let \( b \) (unit: USD/kWh) denote the cost for using per kWh from the RPS. As discussed in Section 1, on average, \( b \) is equal to 0.1884. The price for the usage of the CPS energy should depend on the aggregated demand from a number of consumers. In other words, if we assume that all consumers are i.i.d., the dynamic pricing function can be expressed as

\[
f(\cdot) = a \times \left( \sum_{i=1}^{Z} S_{C,i} \mathbf{w}^T \right)^2.
\]

In the above equation, \( Z \) is a random number to model the number of end users. \( S_{C,i} \) represents the \( S_C \) for end user \( i \), where \( 1 \leq i \leq Z \). In this study, to simplify our discussion, we consider
the case for $Z = 1$ and follow the previous works [7], [8] to assume $f(S)$ as follows:

$$f(S) = a \times (SCwT)^2 \text{ (unit: USD/kWh)},$$

(1)

where $SCwT$ represents the total instantaneous power demand supplied by the CPS and $a$ is a scaling factor with units $\frac{\text{USD}}{(\text{kW})^2 \text{kWh}}$, which measures the rate of price change response to the change of $(SCwT)^2$. Following the previous works [1], [4], [6], for smart grids, we assume that the CPS has sufficient energy to supply power demands from the end users.

We define three types of events that may occur in the in-home SG:

- The event $J_{\text{arrival}}$ represents that a power demand arrives at the system.
- The event $J_{\text{departure}}$ represents that a power demand departs from the system.
- The event $\text{charge}$ represents a charge on the RPS.

The random processes for the three types of events are independent Poisson processes with rates $\lambda_J, u$, and $\lambda_B$, respectively.

### 3.2 Electricity Cost for the Default Policy

In this section, we propose an analytical model to characterize the expected electricity cost when the default policy is applied to the in-home EMS.

Suppose the system state is $S^{(0)} = (SC, SR, SL)$ at $t_0$. Let $t_i$ (where $i \in \mathbb{N}$) denote the time when the $i$th event (that can be the event $J_{\text{arrival}}$, the event $J_{\text{departure}}$, or the event $\text{charge}$) occurs after $t_0$. Let $S^{(i+)} = (SC^{(i+)}, SR^{(i+)}, SL^{(i+)})$ and $S^{(i-)} = (SC^{(i-)}, SR^{(i-)}, SL^{(i-)})$ denote the system states at $lim_{\Delta \to 0^+} t_i + \Delta$ and $lim_{\Delta \to 0^+} t_i - \Delta$, respectively. By default, we let $S^{(0-)} = S^{(0)}$. Let $V_j(S^{(0)})$ denote the expected electricity cost from $t_0$ to $t_j$ subject to the initial system state $S^{(0)}$ at $t_0$. We obtain $V_j(S^{(0)})$, where $j \in \mathbb{N}$, by using the following recursive algorithm.

#### 3.2.1 The First Step

The first step of the recursive algorithm is to obtain $V_1(S^{(0)})$. Because the three types of events are independent Poisson processes, $t_1 - t_0$ is an exponential random variable with mean $\gamma^{-1}$, where $\gamma = \lambda_J + \lambda_B + (SC + SR)uT$. Given the initial state $S^{(0)}$, the service time of the appliances served by the RPS is unpredictable, so the electricity in the battery of the RPS may run out before $t_1$. Denote $\Phi = SL / (SRwT)$. For any $s, t \in [t_0, t_1]$ and $s < t$, let $C(s, t)$ denote the electricity cost...
during the time period \([s, t]\). We obtain \(V_1(S^{(0)})\) by considering the two cases: \(\{t_1 - t_0 > \Phi\}\) and \(\{t_1 - t_0 \leq \Phi\}\), i.e.,

\[
V_1(S^{(0)}) = \Pr[t_1 - t_0 > \Phi]E[C(t_0, t_1)|t_1 - t_0 > \Phi] + \Pr[t_1 - t_0 \leq \Phi]E[C(t_0, t_1)|t_1 - t_0 \leq \Phi],
\]

(2)

where \(\Pr[t_1 - t_0 > \Phi] = \exp(-\gamma \Phi)\), and \(E[C(t_0, t_1)|t_1 - t_0 > \Phi]\) and \(E[C(t_0, t_1)|t_1 - t_0 \leq \Phi]\) are obtained below:

**Case I:** \(t_1 - t_0 > \Phi\). In this case, the electricity of the battery in the RPS runs out before the first event occurs. Since \(C(t_0, t_1) = C(t_0, t_0 + \Phi) + C(t_0 + \Phi, t_1)\),

\[
E[C(t_0, t_1)|t_1 - t_0 > \Phi] = E[C(t_0, t_0 + \Phi)|t_1 - t_0 > \Phi] + E[C(t_0 + \Phi, t_1)|t_1 - t_0 > \Phi].
\]

(3)

For the first term on the right hand side of (3),

\[
E[C(t_0, t_0 + \Phi)|t_1 - t_0 > \Phi] = bS_Rw^T\Phi + f(S^{(0)})S_Cw^T\Phi.
\]

(4)

The second term on the right hand side of (3) is derived as follows:

Denote \(S' = (S_C + S_R, 0, 0)\). Because the electricity in the battery of the RPS runs out at \(t_0 + \Phi\), all power demands being served by the RPS should be switched to the CPS at time \(t_0 + \Phi\). Furthermore, because \(t_1 - t_0\) has an exponential distribution, from the memoryless property, \(t_1 - t_0 - \Phi\) has the exponential distribution with rate \(\gamma\). Hence,

\[
E[C(t_0 + \Phi, t_1)|t_1 - t_0 > \Phi] = f(S')(S_C + S_R)w^T\int_{t_0+\Phi}^{\infty} (t - t_0 - \Phi)\gamma e^{-\gamma(t-t_0-\Phi)}dt = \frac{1}{\gamma} f(S')(S_C + S_R)w^T.
\]

(5)

**Case II:** \(t_1 - t_0 \leq \Phi\). In this case, the electricity in the battery of the RPS does not run out before the first event occurs. We obtain

\[
E[C(t_0, t_1)|t_1 - t_0 \leq \Phi] = \int_{t_0}^{t_0+\Phi} C(t_0, t)\frac{\gamma e^{-\gamma(t-t_0)}}{\Pr[t_1 - t_0 \leq \Phi]}dt.
\]

(6)
Since for any $t \in (t_0, t_0 + \Phi)$,
\[ C(t_0, t) = bS_RW^T (t - t_0) + f(S^{(0)})S_CW^T (t - t_0), \]
(6) can be rewritten as
\[ \mathbb{E} \left[ C(t_0, t_1) \bigg| t_1 - t_0 \leq \Phi \right] \]
\[ = \frac{1 - \Phi e^{-\gamma \Phi} - \frac{1}{2} e^{-\gamma \Phi}}{Pr [t_1 - t_0 \leq \Phi]} \left( bS_R + f(S^{(0)})S_C \right) w^T. \] (7)

From (4), (5) and (7), (2) is rewritten as
\[ V_1(S^{(0)}) = \frac{1}{\gamma} e^{-\gamma \Phi} f(S') (S_C + S_R) w^T \]
\[ + \left( \frac{1}{\gamma} - \frac{1}{2} e^{-\gamma \Phi} \right) \left( bS_R + f(S^{(0)}) S_C \right) w^T. \] (8)

### 3.2.2 The $(j + 1)$th Step (for $j \geq 1$)

We obtain the expected cost function $V_{j+1}$ by using
\[ V_{j+1}(S^{(0)}) = V_1(S^{(0)}) + \mathbb{E}[C(t_1, t_{j+1})], \] (9)
where $V_1(S^{(0)})$ is the result of the first step. We derive $\mathbb{E}[C(t_1, t_{j+1})]$ as follows:

In the default policy, for a power demand arrival at $t_i$ (where $i \geq 1$), the power demand is served by the RPS if the energy in the battery is not empty (i.e., $S_L^{(i-)} > 0$). Otherwise (i.e., $S_L^{(i-)} = 0$), this power demand is served by the CPS. Therefore, we consider two cases \{t_1 - t_0 \geq \Phi\} and \{t_1 - t_0 < \Phi\} to obtain $\mathbb{E}[C(t_1, t_{j+1})]$.

**Case I:** $t_1 - t_0 \geq \Phi$: This case implies that the electricity of the RPS has been run out at time $t_0 + \Phi$. At time $t_0 + \Phi$, all power demands being served by the RPS should be switched to the CPS, i.e., at time $t_0 + \Phi$, the system state changes to $S' = (S_C + S_R, 0, 0)$.

Consider the system state $S^{(1-)}$ at $t_1^- = \lim_{\Delta \to 0^+} (t_1 - \Delta)$. Because the event arriving at $t_1$ can be an event_arrival, an event_departure, or an event_charge, we consider the following three cases to obtain $\mathbb{E} \left[ C(t_1, t_{j+1}) \bigg| t_1 - t_0 \geq \Phi \right]$.

**Case I-1.** The event arrival at $t_1$ is an event_arrival of a type $k$ power demand with probability $\lambda J_p k / \gamma$. In Case I, because the power demand will be served by the CPS, we have $S^{(1+)} =$
\((S_C + S_R + e_k, 0, 0)\). Therefore
\[
\mathbb{E} \left[ C(t_1, t_{j+1}) \mathbb{1}_{\text{Case I-1}} \mid t_1 - t_0 \geq \Phi \right] = \sum_{k=1}^{n} \frac{\lambda_k p_k}{\gamma} V_j((S_C + S_R + e_k, 0, 0)).
\] (10)

**Case I-2.** The event arrival at \(t_1\) is an event departure of a type \(k\) power demand with the probability \(\mu_k(S_{C,k} + S_{R,k})/\gamma\). At \(t_1^-\), the system state is \((S_C + S_R, 0, 0)\). Because a type \(k\) power demand leaves from the CPS at \(t_1\), at \(t_1^+\), the system state changes to \(S^{(I+)} = ((S_C + S_R - e_k)^+, 0, 0)\), where \((S_C + S_R - e_k)^+ = (S_{C,1} + S_{R,1}, \ldots, S_{C,k-1} + S_{R,k-1}, (S_{C,k} + S_{R,k} - 1)^+, S_{C,k+1} + S_{R,k+1}, \ldots, S_{C,n} + S_{R,n})\). Consequently, we have
\[
\mathbb{E} \left[ C(t_1, t_{j+1}) \mathbb{1}_{\text{Case I-2}} \mid t_1 - t_0 \geq \Phi \right] = \sum_{k=1}^{n} \frac{\mu_k(S_{C,k} + S_{R,k})}{\gamma} V_j((S_C + S_R - e_k)^+, 0, 0). \] (11)

**Case I-3.** The event arrival at \(t_1\) is an event charge with the probability \(\lambda_B/\gamma\). For each charging, the RPS gets \(X\) kWh that has CDF \(F_C(x)\). In this case, at \(t_1^+\), the system state changes to \(S^{(I+)} = (S_C + S_R, 0, X \wedge B_{\text{max}})\). Hence, we have
\[
\mathbb{E} \left[ C(t_1, t_{j+1}) \mathbb{1}_{\text{Case I-3}} \mid t_1 - t_0 \geq \Phi \right] = \frac{\lambda_B}{\gamma} \int_{0}^{\infty} V_j(S_C + S_R, 0, x \wedge B_{\text{max}}) dF_C(x). \] (12)

From Cases I-1, I-2, and I-3, we have
\[
\Pr[t_1 - t_0 \geq \Phi] \mathbb{E}[C(t_1, t_{j+1}) \mid t_1 - t_0 \geq \Phi] = e^{-\gamma \Phi} \{(10) + (11) + (12)\}. \] (13)

**Case II:** \(t_1 - t_0 < \Phi\): In this case, there is electricity in the RPS at \(t_1\), i.e., \(S_L(t_1) > 0\), where
\[
S_L(t) = \left( S_L - (t - t_0)S_R w_T \right)^+ \quad \text{for} \quad t_0 \leq t \leq t_1.
\]
Similar to Case I, we consider the following three cases to obtain \(\mathbb{E} \left[ C(t_1, t_{j+1}) \mid t_1 - t_0 < \Phi \right] \):

**Case II-1.** The event arrival at \(t_1\) is an event arrival of a type \(k\) power demand. Since there is electricity in the RPS at \(t_1\), i.e., \(S_L(t_1) > 0\), the type \(k\) power demand must be served by the RPS
under the default policy and the system state changes to $S^{(1+)} = (S_C, S_R + e_k, S_L(t_1))$. Therefore,

$$\Pr[t_1 - t_0 < \Phi] \mathbb{E} \left[ C(t_1, t_{j+1}) \mid t_1 - t_0 < \Phi \right]$$

$$= \int_{t_0}^{t_0 + \Phi} \left\{ \sum_{k=1}^{n} \frac{\lambda_{jk} p_k}{\gamma} V_j(S_C, S_R + e_k, S_L(t)) \right\} \gamma e^{-\gamma(t-t_0)} dt. \tag{14}$$

**Case II-2.** The event arrival at $t_1$ is an event departure of a type $k$ power demand from the CPS (or RPS). If a type $k$ power demand leaves from the CPS (or RPS) at $t_1$, the system state changes to $S^{(1+)} = ((S_C - e_k)^+, S_R, S_L(t))$ (or $S^{(1+)} = (S_C, (S_R - e_k)^+, S_L(t))$) at $t_1^+$. Hence,

$$\Pr[t_1 - t_0 < \Phi] \mathbb{E} \left[ C(t_1, t_{j+1}) \mid t_1 - t_0 < \Phi \right]$$

$$= \int_{t_0}^{t_0 + \Phi} \left\{ \sum_{k=1}^{n} \frac{\mu_{kS_C,k} S_C}{\gamma} V_j((S_C - e_k)^+, S_R, S_L(t)) + \sum_{k=1}^{n} \frac{\mu_{kS_R,k}}{\gamma} V_j(S_C, (S_R - e_k)^+, S_L(t)) \right\} \gamma e^{-\gamma(t-t_0)} dt. \tag{15}$$

**Case II-3.** The event arrival at $t_1$ is an event charge. If the RPS gets $X$ kWh from the event charge, the system state changes to $S^{(1+)} = (S_C, S_R, X \wedge B_{\text{max}})$ at $t_1^+$. Hence,

$$\Pr[t_1 - t_0 < \Phi] \mathbb{E} \left[ C(t_1, t_{j+1}) \mid t_1 - t_0 < \Phi \right]$$

$$= \int_{t_0}^{t_0 + \Phi} \left\{ \frac{\lambda_B}{\gamma} \int_{0}^{\infty} V_j(S_C, S_R, (S_L(t) + x) \wedge B_{\text{max}}) dF_C(x) \right\} \gamma e^{-\gamma(t-t_0)} dt. \tag{16}$$

From Cases II-1, II-2, and II-3, we have

$$\Pr[t_1 - t_0 < \Phi] \mathbb{E} \left[ C(t_1, t_{j+1}) \mid t_1 - t_0 < \Phi \right]$$

$$= (14) + (15) + (16). \tag{17}$$

Therefore, from Case I and Case II, if the default policy is applied to the EMS, the expected electricity cost $V_{j+1}(S^{(0)})$ from $t_0$ to $t_{j+1}$ is equal to $V_1(S^{(0)}) + (13) + (17)$.

### 4 Threshold Policy

In this section, based on our analytical model for the default policy, we propose a threshold policy to reduce the electricity cost. An optimal threshold settings is derived based on the exponential assumptions on the inter-arrival times for power demand arrivals, the service times for power demands, and inter-arrival times for charges. In the real situation, the inter-arrival times and
service times may not be exponentially distributed. We will investigate the effectiveness of our optimal threshold settings through the simulation experiments by relaxing these assumptions.

The threshold policy is multi-dimensional, where we provide different thresholds for different types of power demands. For each type of appliance, we derive a set of thresholds at time points \( t_1, t_2, ..., t_{N-1} \), where \( t_0 < t_1 < t_2 < ... < t_{N-1} < t_N \). The time period \([t_0, t_N]\) is a finite time period during which we expect to obtain the minimum electricity cost. Our threshold policy is obtained based on the finite-horizon CTMDP approach. As pointed out in [20], the finite-horizon CTMDP approach requires large storage for the state space. Due to page limitation, we do not include the complexity analysis for storage space, and it can be found in [20].

Consider a power demand of type \( k \) appliance arriving at \( t_{N-j} \), where \( 1 \leq j \leq N-1 \). Let \( \tau_{N-j,k} \) be the policy at \( t_{N-j} \) applied to determine whether the power demand of type \( k \) appliance is served by the RPS or CPS. The value of \( \tau_{N-j,k} \) can be 1 or 0, which indicates that the power demand is served by the RPS or CPS. Denote \( \pi_{N-j} = (\tau_{N-j,1},...,\tau_{N-j,n}) \) with the following definition: For \( 1 \leq k \leq n \) and \( 1 \leq j \leq N-1 \),

\[
\tau_{N-j,k} = \begin{cases} 
1, & \text{if } S^{((N-j)-)}_{C,k} \geq g_k(j, S^{((N-j)-)}_{-k}); \\
0, & \text{if } S^{((N-j)-)}_{C,k} < g_k(j, S^{((N-j)-)}_{-k}).
\end{cases} 
\tag{18}
\]

In (18), \( g_k(\cdot, \cdot) \) is a function of the number of the remaining time steps, i.e., \( j \), and the current system state \( S^{((N-j)-)}_{-(k)} \) excluding \( S^{((N-j)-)}_{C,k} \), which is expressed as

\[
S^{((N-j)-)}_{-k} = (S^{((N-j)-)}_{C,k} - S^{((N-j)-)}_{C,k} e_k, S^{((N-j)-)}_{R}, S^{((N-j)-)}_{L}).
\]

Let \( U_j(S^{((N-j)+)}; \pi_{N-j+1},...,\pi_{N-1}) \) be the expected electricity cost accumulated from \( t_{N-j} \) to \( t_N \) by applying the threshold policies \( \pi_{N-j+1},...,\pi_{N-1} \). Our objective is to find an optimal threshold function \( g^*_k(\cdot, \cdot) \) for type \( k \) appliance so that the expected electricity cost

\[
U_j(S^{((N-j)+)}; \pi_{N-j+1},...,\pi_{N-1})
\]

is minimized. In other words, with function \( g^*_k(\cdot, \cdot) \), we obtain the corresponding threshold policy

\[
\pi^*_{N-j} = (\tau^*_{N-j,1},...,\tau^*_{N-j,n}),
\]

where

\[
\tau^*_{N-j,k} = \begin{cases} 
1, & S^{((N-j)-)}_{C,k} \geq g^*_k(j, S^{((N-j)-)}_{-k}); \\
0, & S^{((N-j)-)}_{C,k} < g^*_k(j, S^{((N-j)-)}_{-k}).
\end{cases} 
\tag{19}
\]
such that

$$U_j(S^{(N-j)^+}); \pi_{N-j+1}, \ldots, \pi_{N-1})$$

$$\geq U_j(S^{(N-j)^+}); \pi^*_j, \pi^*_j, \ldots, \pi^*_{N-1})$$

holds for arbitrary threshold policies $\pi_{N-j+1}, \ldots, \pi_{N-1}$.

To simplify the notation, we use $U^*_j(S^{(N-j)^+})$ to denote the minimum expected electricity cost from $t_{N-j}$ to $t_N$, i.e.,

$$U^*_j(S^{(N-j)^+}) \triangleq U_j(S^{(N-j)^+}); \pi^*_j, \pi^*_j, \ldots, \pi^*_{N-1})$$

$$= \min_{\pi_{N-j+1}, \ldots, \pi_{N-1}} U_j(S^{(N-j)^+}); \pi_{N-j+1}, \ldots, \pi_{N-1}).$$

In Theorem 1, we propose a recursive algorithm to obtain $U^*_j(S^{(N-j)^+})$ by using the CTMDP technique, (8) and the following functions

$$W(p, S) = \lambda_j \sum_{k=1}^{n} p_k U^*_{j-1}(S_C + e_k(1-p), S_R + e_k p, S_L), \quad (20)$$

where $p = 0, 1$ and $S = (S_C, S_R, S_L)$, and

$$X_1(S) = \sum_{k=1}^{n} \mu_k S_{C,k} U^*_{j-1}((S_C - e_k)^+, S_R, S_L),$$

$$X_2(S) = \sum_{k=1}^{n} \mu_k S_{R,k} U^*_{j-1}((S_C - e_k)^+, S_R, S_L),$$

$$X_3(S) = \lambda_B \int_{0}^{\infty} U^*_{j-1}(S_C, S_R, (S_L + x) \wedge B_{max}) dF_C(x). \quad (21)$$

Note that in (20), $p = 0$ means that a type $k$ power demand is served by the CPS. On the other hand, when $S_L > 0$, $p = 1$ means that the power demand is served by the RPS. More elaboration on (20) and (21) can be found in the proof of Theorem 1.

Theorem 1. $U^*_j(S^{(N-j)^+})$ is the minimum if the following two conditions hold: (i) $U^*_1(S^{(N-1)^+}) = V_1(S^{(N-1)^+})$;

(ii) $U^*_j(S^{(N-j)^+}) = V_1(S^{(N-j)^+})$ 

$$+ \frac{1}{\gamma} \mathbb{E} \left[ \sum_{m=1}^{3} X_m(S^{(N-j+1)^-}) + \min_{p=0,1} W(p, S^{(N-j+1)^-}) \right]$$

for $2 \leq j \leq N$. 

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Proof: Consider time point $\lim_{\Delta \to 0^+} (t_N - j - \Delta)$ (for $j \in \{1, \ldots, N - 1\}$), and the current system state is given by $S^{(N-j)-}$. Let $\widehat{U}_j(S^{(N-j)-}; \pi_{N-j}, \ldots, \pi_{N-1})$ denote the expected cost accumulated on the time interval $(t_N - j, t_N)$ subject to the system state $S^{(N-j)-}$ at time $\lim_{\Delta \to 0^+} (t_N - j - \Delta)$, and the threshold policies $(\pi_{N-j}, \ldots, \pi_{N-1})$ applied at the time points $t_N - j, \ldots, t_N - 1$. We define the minimum expected electricity cost $\widehat{U}^*_j(S^{(N-j)-})$ from $t_N - j$ to $t_N$ as:

$$\widehat{U}^*_j(S^{(N-j)-}) \triangleq \min_{\pi_{N-j}, \ldots, \pi_{N-1}} \widehat{U}_j(S^{(N-j)-}; \pi_{N-j}, \ldots, \pi_{N-1}),$$

where $j$ is the number of the remaining time steps during the time interval $(t_N - j, t_N)$.

Consider the time point $\lim_{\Delta \to 0^+} (t_{N-1} - \Delta)$, where the system state is $S^{(N-2)-}$. The decision $\pi_{N-2}$ made at time $t_{N-2}$ affects not only the expected electricity cost $\widehat{U}_1(S^{(N-1)-}; \pi_{N-2})$ during the time interval $(t_{N-2}, t_{N-1})$ but also the system state $S^{(N-1)-}$ at $\lim_{\Delta \to 0^+} (t_{N-1} - \Delta)$. Therefore, the optimal threshold policy $\pi^*_{N-2}$ at time $t_{N-2}$ should satisfy

$$\widehat{U}^*_1(S^{(N-1)-}) = \widehat{U}_1(S^{(N-1)-}; \pi^*_{N-1}) = \min_{\pi_{N-1}} \widehat{U}_1(S^{(N-1)-}; \pi_{N-1}). \quad (22)$$

Consider the time point $\lim_{\Delta \to 0^+} (t_{N-1} - \Delta)$, where the system state is $S^{(N-3)-}$. The decision $\pi_{N-3}$ made at time $t_{N-3}$ affects not only the expected electricity cost $\widehat{U}_2(S^{(N-2)-}; \pi_{N-3})$ during the time interval $(t_{N-3}, t_{N-2})$ but also the system state $S^{(N-2)-}$ at $\lim_{\Delta \to 0^+} (t_{N-2} - \Delta)$. Therefore, the optimal threshold policy $\pi^*_{N-3}$ at time $t_{N-3}$ should satisfy

$$\widehat{U}^*_2(S^{(N-2)-}) = \min_{\pi_{N-3}} \left\{ \widehat{U}_2(S^{(N-2)-}; \pi_{N-3}) + \mathbb{E} \left[ \widehat{U}_1(S^{(N-1)-}; \pi^*_{N-1}) \mid S^{(N-2)-} \right] \right\}. \quad (23)$$

Repeating the same procedures (i.e., (22) and (23)), we obtain

$$\widehat{U}^*_{N-1}(S^{(1)-}) = \min_{\pi_1} \left\{ \widehat{U}_1(S^{(1)-}; \pi_1) + \mathbb{E} \left[ \widehat{U}_{N-2}(S^{(2)-}; \pi_2, \ldots, \pi^*_{N-1}) \mid \pi_1, S^{(1)-} \right] \right\}. \quad (24)$$

By substituting (24) into

$$\widehat{U}^*_N(S^{(0)}) = V_1(S^{(0)}) + \mathbb{E} \left[ \widehat{U}^*_{N-1}(S^{(1)-}) \right], \quad (25)$$
we have

\[
\hat{U}_N^* (\mathcal{S}(0)) = V_1(\mathcal{S}(0)) + \mathbb{E} \left[ \min_{\pi_1} \left\{ \hat{U}_1(\mathcal{S}(1^-); \pi_1) \right. \right.
+ \left. \mathbb{E} \left[ \hat{U}_{N-2}(\mathcal{S}(2^-); \pi^*_{2}, \ldots, \pi^*_{N-1}) \bigg| \pi_1, \mathcal{S}(1^-) \right] \right] \right],
\]

(26)

where

\[
\hat{U}_1(\mathcal{S}(1^-); \pi_1) = \sum_{k=1}^{n} \frac{\lambda_{kp}}{\gamma} V_1(\mathcal{S}^{(1^-)}_C + e_k(1 - \pi_1), \mathcal{S}^{(1^-)}_R + e_k \pi_1, \mathcal{S}^{(1^-)}_L)
+ \sum_{k=1}^{n} \mu_k S^{(1^-)}_{C,k} V_1((\mathcal{S}^{(1^-)}_C - e_k)^+, \mathcal{S}^{(1^-)}_R, \mathcal{S}^{(1^-)}_L)
+ \sum_{k=1}^{n} \mu_k S^{(1^-)}_{R,k} V_1(\mathcal{S}^{(1^-)}_C, (\mathcal{S}^{(1^-)}_R - e_k)^+, \mathcal{S}^{(1^-)}_L)
+ \frac{\lambda_B}{\gamma} \int_0^{\infty} V_1(\mathcal{S}^{(1^-)}_C, \mathcal{S}^{(1^-)}_R, \mathcal{S}^{(1^-)}_L + x) dF_C(x),
\]

(27)

and

\[
\mathbb{E} \left[ \hat{U}_{N-2}(\mathcal{S}(2^-); \pi^*_{2}, \ldots, \pi^*_{N-1}) \bigg| \pi_1, \mathcal{S}(1^-) \right]
= \sum_{k=1}^{n} \frac{\lambda_{kp}}{\gamma} \mathbb{E} \left[ \hat{U}_{N-2}(\mathcal{S}(2^-); \pi^*_{2}, \ldots, \pi^*_{N-1}) \bigg| \mathcal{S}(1^-) \right]
= (\mathcal{S}^{(1^-)}_C + e_k(1 - \pi_1), \mathcal{S}^{(1^-)}_R + e_k \pi_1, \mathcal{S}^{(1^-)}_L)
+ \sum_{k=1}^{n} \frac{\mu_k}{\gamma} \mathbb{E} \left[ \hat{U}_{N-2}(\mathcal{S}(2^-); \pi^*_{2}, \ldots, \pi^*_{N-1}) \bigg| \mathcal{S}(1^-) = (\mathcal{S}^{(1^-)}_C - e_k)^+, \mathcal{S}^{(1^-)}_R, \mathcal{S}^{(1^-)}_L) \right]
+ \sum_{k=1}^{n} \frac{\mu_k}{\gamma} \mathbb{E} \left[ \hat{U}_{N-2}(\mathcal{S}(2^-); \pi^*_{2}, \ldots, \pi^*_{N-1}) \bigg| \mathcal{S}(1^-) = (\mathcal{S}^{(1^-)}_C, (\mathcal{S}^{(1^-)}_R - e_k)^+, \mathcal{S}^{(1^-)}_L) \right]
+ \frac{\lambda_B}{\gamma} \int_0^{\infty} \mathbb{E} \left[ \hat{U}_{N-2}(\mathcal{S}(2^-); \pi^*_{2}, \ldots, \pi^*_{N-1}) \bigg| \mathcal{S}(1^-) \right] dF_C(x) \bigg] .
\]

(31)
Using $W(\cdot, \cdot)$ defined in (20), we have

$$
(27) + (31) = \sum_{k=1}^{n} \frac{\lambda_{jk}}{\gamma} U_{N-1}^* \left( S_{C}^{(1)} - e_k (1 - \pi_1), S_{R}^{(1)} - e_k \pi_1, S_{L}^{(1)} \right) \\
= W(\pi_1, S_{C}^{(1)}, S_{R}^{(1)}, S_{L}^{(1)}). \tag{35}
$$

In terms of $X_1(\cdot), X_2(\cdot)$ and $X_3(\cdot)$ defined in (21),

$$
(28) + (32) = \sum_{k=1}^{n} \frac{\mu_k}{\gamma} U_{N-1}^* \left( (S_{C}^{(1)} - e_k)^+, S_{R}^{(1)}, S_{L}^{(1)} \right) \\
= \frac{1}{\gamma} X_1(S_{C}^{(1)}, S_{R}^{(1)}, S_{L}^{(1)}), \tag{36}
$$

where $X_1(\cdot)/\gamma$ is the expected electricity cost during $(t_1, t_N)$ given that a power demand departs from the CPS at $t_1$. Similar to (36),

$$
(29) + (33) = \sum_{k=1}^{n} \frac{\mu_k}{\gamma} U_{N-1}^* \left( (S_{C}^{(1)} - e_k - e_k)^+, S_{R}^{(1)}, S_{L}^{(1)} \right) \\
= \frac{1}{\gamma} X_2(S_{C}^{(1)}, S_{R}^{(1)}, S_{L}^{(1)}), \tag{37}
$$

where $X_2(\cdot)/\gamma$ is the expected electricity cost during $(t_1, t_N)$ given that a power demand departs from the RPS at $t_1$, and

$$
(30) + (34) = \frac{\lambda_B}{\gamma} \int_{0}^{\infty} U_{N-1}^* \left( S_{C}^{(1)}, S_{R}^{(1)}, (S_{L}^{(1)} + x) \wedge B_{\text{max}} \right) dF_C(x) \\
= \frac{1}{\gamma} X_3(S_{C}^{(1)}, S_{R}^{(1)}, S_{L}^{(1)}), \tag{38}
$$

where $X_3(\cdot)/\gamma$ is the expected electricity cost during $(t_1, t_N)$ given that an event_charge occurs at $t_1$.

From (36), (37) and (38), (26) can be re-written as

$$
U_{N}^*(S^{(0)}) = V_1(S^{(0)}) + \frac{1}{\gamma} \mathbb{E} \left[ \sum_{m=1}^{3} X_m(S_{C}^{(1)}, S_{R}^{(1)}, S_{L}^{(1)}) \right] \\
+ \min_{p=0,1} W(p, S_{C}^{(1)}, S_{R}^{(1)}, S_{L}^{(1)}). \tag{39}
$$

Therefore, Theorem 1 holds when $j = N$. The proof for $1 < j < N$ can be obtained by shifting the index $N$ of $U_{N}^*(S^{(0)})$ to $j$ and replacing $S^{(0)}$ with $S^{((N-j)-)}$ in (39). \[\square\]
Fig. 2. Decision scenarios for threshold policy

In the following, we use $U^*_j(S^{(N-j)})$ obtained from Theorem 1 to find the optimal threshold function $g^*_k(\cdot, \cdot)$. The decision scenario is illustrated in Fig. 2. Suppose that a power demand of type $k$ appliance arriving at $t_{N-j}$, and at $\lim_{\Delta \to 0^+} t_{N-j} - \Delta$, the system state is $S^{(N-j)-}$ with $S^{(N-j)-}_L > 0$. If the power demand is dispatched to be served by the RPS at $t_{N-j}$ (i.e., the policy applied at $t_{N-j}$ is $\tau_{N-j,k} = 1$), the system state at $\lim_{\Delta \to 0^+} (t_{N-j} + \Delta)$ changes to


Otherwise (i.e., the policy applied at $t_{N-j}$ is $\tau_{N-j,k} = 0$), the system state at $\lim_{\Delta \to 0^+} (t_{N-j} + \Delta)$ changes to


The two possible states at $\lim_{\Delta \to 0^+} (t_{N-j} + \Delta)$ result in two minimum expected electricity costs for the remaining time period $(t_{N-j}, t_N)$:

$$U^*_j(S^{(N-j)-}_C, S^{(N-j)-}_R + e_k, S^{(N-j)-}_L)$$

and

$$U^*_j(S^{(N-j)-}_C + e_k, S^{(N-j)-}_R, S^{(N-j)-}_L).$$

If $S^{(N-j)-}_L = 0$ (i.e., no electricity in the RPS) or

$$U^*_j(S^{(N-j)-}_C + e_k, S^{(N-j)-}_R, S^{(N-j)-}_L)$$

$$\leq U^*_j(S^{(N-j)-}_C, S^{(N-j)-}_R + e_k, S^{(N-j)-}_L), \tag{40}$$
we prefer the power demand of type \( k \) appliance arriving at \( t_{N-j} \) to be served by the CPS. Otherwise, i.e., \( S_{L}^{((N-j)-)} > 0 \) and

\[
U_{j}^{*}(S_{C}^{((N-j)-)}, S_{R}^{((N-j)-)}, S_{L}^{((N-j)-)}) + e_{k}, S_{R}^{((N-j)-)} + e_{k}, S_{L}^{((N-j)-)}) > U_{j}^{*}(S_{C}^{((N-j)-)}, S_{R}^{((N-j)-)} + e_{k}, S_{L}^{((N-j)-)})
\]

the power demand of type \( k \) appliance arriving at \( t_{N-j} \) is served by the RPS.

**Theorem 2.** Let \( S_{C,-k} = S_{C} - S_{C,k}e_{k} \). If

\[
g_{k}^{*}(j, S_{-k}) = \min\{ l \in \mathbb{N} \cup \{0\} | U_{j}^{*}(S_{C,-k} + (1 + l)e_{k}, S_{R}, S_{L}) > U_{j}^{*}(S_{C,-k} + l e_{k} + S_{R} + e_{k}, S_{L}) \}
\]

then the minimum expected cost \( U_{j}^{*}(S_{C}^{((N-j)+)}) \) is achieved by substituting \( g_{k}^{*}(j, S_{-k}^{((N-j)-)}) \) into \( \tau_{N-j,k}^{*} \) in (19).

**Proof:** Denote the system state at time \( \lim_{\Delta \to 0^+}(t_{N-j} - \Delta) \) by \( (S_{C}, S_{R}, S_{L}) \). Consider a power demand of type \( k \) appliance arriving at time \( t_{N-j} \). For \( q \) (where \( q \in \mathbb{N} \cup \{0\} \)), we define a threshold policy based on \( q \) as:

\[
\tau_{N-j,k}^{(q)} = \begin{cases} 
1, & S_{C,k} \geq q, \\
0, & S_{C,k} < q.
\end{cases}
\]

In the following, we consider two cases to prove that when \( q \neq g_{k}^{*}(j, S_{-k}) \), the expected electricity cost \( Q \) from \( t_{N-j} \) to \( t_{N} \) (obtained from policy \( \tau_{N-j,k}^{(q)} \)) is larger than or equal to the expected electricity cost \( G \) from \( t_{N-j} \) to \( t_{N} \) (obtained from policy \( \tau_{N-j,k}^{*} \)).

**Case 1:** \( S_{C,k} \geq q > g_{k}^{*}(j, S_{-k}) \) or \( q > g_{k}^{*}(j, S_{-k}) > S_{C,k} \). In this case, the \( \tau_{N-j,k}^{(q)} \) obtained from (43) is equal to \( \tau_{N-j,k}^{*} \) obtained from (19). Therefore \( Q = G \).

**Case 2:** \( q > S_{C,k} \geq g_{k}^{*}(j, S_{-k}) \). In this case, from (43), \( \tau_{N-j,k}^{(q)} = 0 \). For the decision \( \tau_{N-j,k}^{(q)} = 0 \), the power demand of type \( k \) appliance is served by the CPS, the system state at \( t_{N-j} \) becomes \((S_{C,-k} + (S_{C,k} + 1)e_{k}, S_{R}, S_{L})\), and the minimum expected electricity cost for the future time period \((t_{N-j}, t_{N})\) is

\[
Q = U_{j}^{*}(S_{C,-k} + (S_{C,k} + 1)e_{k}, S_{R}, S_{L})
\]

(44)
From (19), we have $\tau^*_N - j, k = 1$. For the decision $\tau^*_N - j, k = 1$, the power demand is served by the RPS, and the system state at $t_N - j$ becomes $(S_{C, - k} + S_{C, k} e_k, S_R + e_k, S_L)$. Hence, we have

$$G = U^*_j (S_{C, - k} + S_{C, k} e_k, S_R + e_k, S_L).$$

(45)

In the following, we consider the following cases to compare $Q$ and $G$ for different $S_{C, k}$ (satisfying $q > S_{C, k} \geq g^*_k (j, - k)$):

**Case 2.1:** $q > S_{C, k} = g^*_k (j, - k)$. In this case, (44) and (45) are rewritten as follows.

$$Q = U^*_j (S_{C, - k} + (g^*_k (j, - k) + 1) e_k, S_R, S_L)$$

$$G = U^*_j (S_{C, - k} + g^*_k (j, - k) e_k, S_R + e_k, S_L)$$

By (42), we have $Q > G$.

**Case 2.2:** $q > S_{C, k} > g^*_k (j, - k)$. Let $S_{C, k} = g^*_k (j, - k) + a$ where $a \in \{1, 2, ..., q - g^*_k (j, - k) - 1\}$.

In this case, (44) and (45) are rewritten as

$$Q = U^*_j (S_{C, - k} + (g^*_k (j, - k) + 1) e_k + a e_k, S_R, S_L)$$

$$G = U^*_j (S_{C, - k} + g^*_k (j, - k) e_k + a e_k, S_R + e_k, S_L)$$

By the increasing property of $f(\cdot)$ and (42), we have $Q > G$.

**Case 3:** $q < g^*_k (j, - k)$. The proof for this case is similar to that for Cases 1 and 2, whose details are omitted.

Based on Theorem 2, we develop Algorithm 1 to compute $g^*_k (\cdot, \cdot)$, and obtain an Optimal Threshold Table (OTT) that can be implemented in the in-home EMS. When a power demand of type $k$ appliance arrives, the in-home EMS looks up the OTT to make the decision.

5 **Numerical Results**

In our performance study, we evaluate the performance of the default policy and the threshold policy in terms of the cost saving ratio

$$\alpha_j = \frac{(V_j - U^*_j)}{V_j} \times 100\%,$$

where $V_j$ and $U^*_j$ are the expected electricity costs saved by the default policy and the threshold policy, respectively. In the numerical results, we consider two scenarios for the parameter setups.
Algorithm 1: Optimal Threshold Functions

1. **Input:** \( N, n, \lambda_J, (p_1, \ldots, p_n), (w_1, \ldots, w_n), (\mu_1, \ldots, \mu_n), \lambda_B, F, f, b, B_{\text{max}}; \)

2. **Initialization:** \( U_1^*(S_C, S_R, S_L) = V_1(S_C, S_R, S_L); \)

3. **Loop:** for \( j \in \{2, 3, \ldots, N - 1\} \), using Theorem 1 to calculate \( U_j^*(S_C, S_R, S_L); \)

4. **foreach** \( k = 1, \ldots, n, j = 1, \ldots, N - 1 \) and \( S_{-k} \) do

5. \( \text{int } g_k(j, S_{-k}) = 0; \)

6. **for** \( l = 0; l + + \) do

7. if \( U_j^*(S_{C,-k} + (1 + l)e_k, S_R, S_L) > U_j^*(S_{C,-k} + le_k, S_R + e_k, S_L) \)

8. then \( g_k(j, S_{-k}) = l; \)

9. break;

10. end

11. end

12. return \( g_k(j, S_{-k}); \)

13. end

### TABLE 1

<table>
<thead>
<tr>
<th>Specifications of appliances</th>
<th>mean inter-arrival time (hrs)</th>
<th>power (kW)</th>
<th>mean service time (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer</td>
<td>8</td>
<td>0.2</td>
<td>2.67</td>
</tr>
<tr>
<td>Air conditioner</td>
<td>24</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Washing machine</td>
<td>48</td>
<td>0.31</td>
<td>2</td>
</tr>
</tbody>
</table>

**Scenario I:** We consider three types of household appliances with the specifications listed in Table 1. Initially, the system state at time \( t_0 \) is \( S_C^{(0)} = (1, 0, 0), S_R^{(0)} = (0, 1, 0), S_L^{(0)} = 1 \). We observe the behavior of expected electricity costs from \( t_0 \) to \( t_7 \). From Table 1, we set \( \lambda_J = 5.33 \) 1/hr, \( (p_1, p_2, p_3) = (0.67, 0.22, 0.11) \), \( u = (1/2.67, 1/6, 1/2) \) 1/hr, \( w = (0.2, 1, 0.31) \) kW. The capacity of the battery is \( B_{\text{max}} = 2 \) kWh. The charging rate is set to \( \lambda_B = 1/8 \) 1/hr. To simplify our discussion, we set \( X = 1 \) kWh for each charge and \( a = 1 \) in (1).

**Scenario II:** We use this scenario to investigate the effects of the power demand arrival rate. We set \( \lambda_J \) and \( u \) as follows: We consider only one type of appliance, i.e., \( n = 1 \), and \( \lambda_1 = \lambda_J = 0.1, 0.15, 0.2, 0.25, \ldots, 0.5 \) (units: 1/hr). By fixing \( \lambda_1 w_1/\mu_1 = 5 \) kW where \( w_1 = 2 \) kW (i.e., on average 5 kWh is consumed to serve the power demands for one hr), we have \( \mu_1 = 0.4\lambda_1 \) 1/hr. We observe the electricity cost for the time period \( [t_0, t_{10}] \). The setups for other parameters are the same as that in Scenario I.
TABLE 2
Analysis vs. simulation

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_j$ (analysis)</td>
<td>0.331</td>
<td>0.731</td>
<td>1.244</td>
<td>1.873</td>
<td>2.762</td>
</tr>
<tr>
<td>$V_j$ (simulation)</td>
<td>0.330</td>
<td>0.728</td>
<td>1.241</td>
<td>1.863</td>
<td>2.753</td>
</tr>
<tr>
<td>Error</td>
<td>0.324%</td>
<td>0.428%</td>
<td>0.217%</td>
<td>0.534%</td>
<td>0.326%</td>
</tr>
<tr>
<td>$U_j^*$ (analysis)</td>
<td>0.317</td>
<td>0.689</td>
<td>1.152</td>
<td>1.708</td>
<td>2.480</td>
</tr>
<tr>
<td>$U_j^*$ (simulation)</td>
<td>0.318</td>
<td>0.683</td>
<td>1.151</td>
<td>1.706</td>
<td>2.470</td>
</tr>
<tr>
<td>Error</td>
<td>0.32%</td>
<td>0.76%</td>
<td>0.10%</td>
<td>0.12%</td>
<td>0.42%</td>
</tr>
</tbody>
</table>

Fig. 3. The shape of a threshold surface

5.1 Simulation Validation

In this paper, we conduct simulation experiments to investigate the performance of the two policies. We implement the OTT obtained in Algorithm 1 in our simulation model for the threshold policy.

To ensure the stability, we run 100,000 simulation experiments to obtain the result. In the $i$th experiment ($1 \leq i \leq 100,000$), we calculate the electricity cost from time $t_0$ to $t_j$, which is denoted by $\hat{V}_{j,i}$ (for the default policy) and $\hat{U}_{j,i}^*$ (for the threshold policy). Then we obtain the expected cost $\bar{V}_j$ and $\bar{U}_j^*$ from the 100,000 simulation experiments by

$$\bar{V}_j = \frac{1}{10^5} \sum_{i=1}^{10^5} \hat{V}_{j,i} \quad \text{and} \quad \bar{U}_j^* = \frac{1}{10^5} \sum_{i=1}^{10^5} \hat{U}_{j,i}^*.$$

As shown in Table 2 (where we apply the parameter setup in Scenario I), the errors between the analytical and simulation results fall within one percent, demonstrating consistent findings from both our analytical models and simulation experiments.

5.2 Optimal Threshold Table

In this section, we discuss the application of the OTT in the threshold policy. In Fig. 3, we set $a = 1$, $j = 3$, $N = 8$ and plot $g_k^*(j = 3, \cdot)$ by (42) to obtain the optimal policy $\tau_{N-j=5}^* = (\tau_{N-j=5,k=1}^*)$ at $t_{N-j=5}$. Here, we suppose that the power demand has the arrival rate $\lambda_j = 1/3\ 1/\text{hr}$, the power...
Fig. 4. Optimal threshold function ($g_1(j, \cdot)$) for different remaining time steps ($j = 7, 5, 3, 1$)

![Graphs showing optimal threshold function for different remaining time steps.](image)

(a) Thresholds for different power requirements

(b) Effects of $\lambda_J$ on $\alpha_{10}$

Fig. 5. Effects of $w_1$ and $\lambda_J$ on $g^*(3, \cdot)$ and $\alpha_{10}$

requirement $w_1 = 2$ kW and the mean service time 2/3 hrs. The charging process on the RPS has rate $\lambda_B = 2$ 1/hr, and the amount of electricity for each charge is fixed to $X = 1$ kWh.

Consider that the power demand arrives at $t_5$ with the system state $(S_{C,1}, S_{R,1}, S_L)$. If $(S_{R,1}, S_L, S_{C,1})$ falls above the surface in Fig. 3, then this power demand will be served by the RPS (i.e., $\tau_5^* = 1$). Otherwise (i.e., $(S_{R,1}, S_L, S_{C,1})$ falls below the surface), this power demand will be served by the CPS (i.e., $\tau_5^* = 0$).

Using the same parameter setups for Fig. 3, in Fig. 4, we plot the surfaces for $g_1^*(j, \cdot)$, where $j = 7, 5, 3, 1$. Fig. 4 shows that for different $j$, the surface for $g_1^*(j, \cdot)$ has different shapes, which indicates that the threshold policy varies along with $j$.

In Fig. 5(a), we use the same parameter setups for Fig. 3, except that we change the power requirement for a power demand (i.e., $w_1 = 1, 2, ..., 7$). Fig. 5(a) shows that $g_1^*(j, \cdot)$ increases along with $w_1$. It implies that for a power demand with larger power requirement, the threshold policy tends to dispatch it to the CPS.
Fig. 6. Effects of $j$, variance $v_J$, $v_u$ and $v_B$ on $\alpha$

5.3 Performance Evaluation

Considering Scenarios I and II, we investigate the performance $\alpha_j$ of the threshold policy w.r.t. the default policy:

Effects of Arrival Rate $\lambda_j$: In Fig. 5(b), we study the effects of $\lambda_j$ by considering Scenario II, where we observe the system by fixing $\lambda_1 w_1/\mu_1 = 5$ kW As shown in this figure, when $\lambda_j$ increases from 0.1 to 0.5, $\alpha_j$ increases significantly. This phenomenon indicates that the threshold policy can save more costs when we use the same amount of electricity (from both the RPS and the CPS) to serve more power demands (i.e., power demands arrive more frequently). For example, when $\lambda_j = 0.5$, $\alpha_j$ is around 10%.

Effects of Observed Time Length $j$: In Fig. 6(a), we consider Scenario I and $v_J = 1/\lambda_j^2$, i.e., inter-arrival time of the power demands has the exponential distribution. Fig. 6(a) shows that $\alpha_j$ is an increasing function of $j$, which implies that the more power demands served by the in-home SG, the more electricity cost can be saved by the threshold policy. For example, when $j = 7$, $\alpha_j$ is around 10%.

Effects of Variance $v_J$, $v_u$ and $v_B$: In the real world, the inter-arrival times and service times of power demands and the inter-arrival times of charges may not be exponentially distributed. If we can obtain the data measured from utility companies, it can be more helpful to demonstrate the performance of the threshold policy. However, real data on power demands are often security sensitive and very hard to obtain from utility companies. As pointed out in [18], the gamma distribution has been widely used to approximate many other distributions. In Figs. 6(a)-6(c),

![Fig. 6](image-url)
TABLE 3
Load shifting capability

<table>
<thead>
<tr>
<th>j</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_C^{(j)}$ (default)</td>
<td>0.248</td>
<td>0.395</td>
<td>0.496</td>
<td>0.541</td>
<td>0.566</td>
</tr>
<tr>
<td>$L_C^{(j)}$ (threshold)</td>
<td>0.241</td>
<td>0.417</td>
<td>0.478</td>
<td>0.501</td>
<td>0.517</td>
</tr>
</tbody>
</table>

we assume that the inter-arrival times and service times of power demands and the inter-arrival times of charges have gamma distributions with means $1/\lambda_J, 1/\mu_1, 1/\mu_2, 1/\mu_3$, and $1/\lambda_B$ and variances $v_J = \alpha/\lambda_J^2$, $v_u = \alpha(1/\mu_1^2, 1/\mu_2^2, 1/\mu_3^2)$ and $v_B = \alpha/\lambda_B^2$, respectively, where $\alpha = 0.1, 1, 10$. Figs. 6(a) and 6(b) indicate that the threshold policy has better performance for larger variances $v_J$ and $v_u$. Fig. 6(c) shows that the effects of the variance $v_B$ on the tendency of $\alpha_j$ are minor.

Load Shifting Capability: In a classical study [19] about power systems, Berger et al. claimed that the RTP can effectively help the CPS shift the service load to the RPS. We study the load shifting capability $L_C^{(j)}$ of the threshold policy and the default policy under Scenario I, where $L_C^{(j)} = \frac{S_C^{(j)} w_T}{(S_C^{(j)} + S_R^{(j)}) w_T}$. A smaller $L_C^{(j)}$ indicates that more service loads of the CPS is shifted to the RPS. Table 3 shows that the threshold policy has a smaller $L_C^{(j)}$ than the default policy when $j \geq 6$. In other words, if the system serves enough number of power demands, compared to the default policy, the threshold policy can manage the in-home SG more effectively.

6 Conclusion

We studied how to reduce the electricity cost for the in-home SG by jointly considering the RPS and CPS. We first proposed analytical models for the electricity cost in which the default policy was applied to the in-home SG. In our model, the charging process on the RPS and multiple types of household appliances are taken into consideration. Based on the analytical model, we proposed the threshold policy to reduce the electricity cost by applying the finite-horizon CTMDP. It is easy to implement the threshold policy in the in-home SG once the OTT is established. Simulation experiments were conducted to validate the correctness of the analytical models and to study the performance of both two policies.

Our performance study showed that

- The threshold policy can dynamically adjust the threshold according to the system status in the in-home SG.
When more power demands are served by the in-home SG, the threshold policy can save more electricity cost than the default policy.

The threshold policy can save more cost when we use the same amount of electricity to serve more power demands.

The threshold policy has better load shifting capability than the default policy.

To conclude, compared to the default policy, the threshold policy can manage the in-home SG more effectively and more economically.

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REFERENCES


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