Capacity Bounds of Three-Dimensional Wireless Ad Hoc Networks

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***Abstract*—Network capacity investigation has been intensive in the past few years. A large body of work on wireless network capacity has appeared in the literature. However, so far most of the effort has been made on two-dimensional wireless networks only. With the great development of wireless technologies, wireless networks are envisioned to extend from two-dimensional space to three-dimensional space. In this paper, we investigate the throughput capacity of 3D regular ad hoc networks (RANETs) and of 3D nonhomogeneous ad hoc networks (NANETs), respectively, by employing a generalized physical model. In 3D RANETs, we assume that the nodes are regularly placed, while in 3D NANETs, we consider that the nodes are distributed according to a general Nonhomogeneous Poisson Process (NPP). We find both lower and upper bounds in both types of networks in a broad power propagation regime, i.e., when the path loss exponent is no less than 2.**

***Index Terms*—Three-dimensional wireless networks, regular ad hoc networks, nonhomogeneous ad hoc networks, throughput capacity.**

I. INTRODUCTION

Network capacity investigation has been intensive in the past few years. A big chunk of work exploring the capacity of wireless networks has appeared in the literature. When we ask ourselves why we should engage in this pursuit, two reasons should be obvious. First, network capacity unveils the asymptotic property of network performance. In face of the emerging large-scale networks of a large number of connected objects, asymptotic capacity is no longer a cliché and becomes even more critical. Second, network capacity predicts network performance limits as a function of the number of nodes in the network, regardless of detailed protocol design. In contrast, as an alternative way to evaluate the network performance, simulation or numerical results can only be obtained for a certain number of nodes and are hence deterministic. Besides, these results can only be made available after we design all the network protocols considering every detail, and may also require a lot of computing resources and time for large-scale networks. Therefore, capacity investigation is interesting and important in wireless networks. However, it is also a very challenging task on the other hand.

Gupta and Kumar [14] initiate the study on the capacity of wireless networks and show that the per-node throughput capacity (with unit bits per second) is $\Theta(1/\sqrt{n \log n})$ in random ad hoc networks and the per-node transport capacity (with unit bit-meters per second) is $\Theta(1/\sqrt{n})$ in arbitrary ad hoc networks, where n is the number of nodes in the network. A large body of work (e.g., [1], [3], [5], [7]–[9], [24], [35]–[37]) continues to study the capacity of static ad hoc networks with different network settings, while a tremendous amount of effort (e.g., [4], [13], [17], [25], [31], [41]) is also made on the capacity of mobile ad hoc networks, showing that mobility can significantly improve network capacity. In addition, the bulk of work on the capacity of hybrid wireless networks, such as [18], [23], [26], [29], [32], [43]–[45], proposes to place base stations in wireless networks and finds that network capacity can be boosted as well.

However, all the aforementioned work is conducted on two-dimensional networks only. With the great development of wireless technologies, wireless networks are envisioned to extend from two-dimensional space to three-dimensional space, connecting all kinds of objects such as computers, sensors, actuators, mobile phones, cars, planes, spacecrafts, ships, and submarines. The future three-dimensional (3D) wireless networks will be a fusion of the digital world and the physical world and bring together everything from individuals to objects, from data to services, etc. For example, in modern battlefields, 3D wireless networks need to be deployed to connect various military units together, like aircrafts, troops, and fleets. In the cases of natural disasters or terrorist attacks, we can set up 3D wireless networks to aid in the rescue affairs, which can enable the communications between rescuers in the air, e.g., unmanned aerial vehicles (UAVs) and helicopters, and those on the ground. To give another example, 3D wireless networks are also indispensable in space communications for the purposes of space or planet explorations. Unfortunately, in the literature, only a couple of papers like [15] tentatively study the capacity of 3D wireless networks. In particular, [15] explores the transport capacity in 3D arbitrary ad hoc networks and the throughput capacity in 3D random ad hoc networks, using both Protocol Model and Physical Model. In contrast, in this paper we investigate the throughput capacity of 3D regular ad hoc networks and of 3D nonhomogeneous ad hoc networks, respectively, by employing a generalized physical model.

More specifically, we consider a network with n nodes

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distributed in a three-dimensional cube $\mathcal{A}$ with edge L, where $L = n^{\alpha/3}$ ($0 \leq \alpha \leq 1$), and the network volume $|\mathcal{A}| = L^3$. In 3D regular ad hoc networks (RANETs), assuming that the n nodes are regularly placed, we find that the throughput capacity is lower bounded by $n^{(\gamma-4)/3}$ when $2 \leq \gamma < 3$, by $n^{-\frac{1}{3}}/\ln n$ when $\gamma = 3$, and by $n^{-\frac{1}{3}}$ when $\gamma > 3$, where γ is the path loss exponent, and is upper bounded by $\frac{P_{max}}{P_{min}} n^{-\frac{1}{3}}$ when the transmission power of the nodes can be tuned between P_{min} and P_{max} with $0 < P_{min} \leq P_{max}$. In 3D nonhomogeneous ad hoc networks (NANETs), we assume that the n nodes are distributed according to a general Nonhomogeneous Poisson Process (NPP), with the local intensity at point ξ in the network denoted by $\Psi(\xi)$, and $\int_{\mathcal{A}} \Psi(\xi) \mathrm{d}\xi = n$. The minimum and the maximum of $\Psi(\xi)$ are denoted by $\underline{\Psi}$ and $\overline{\Psi}$, respectively, which both scale with n. We also consider that the nodes have transmission powers which may range from P_{min} to P_{max}. We show that the throughput capacity is lower bounded by $\frac{\underline{\Psi}^2}{\overline{\Psi}^2 \ln n} (\frac{\ln n}{\underline{\Psi} n^{\alpha}})^{\frac{4-\gamma}{3}}$ when $2 \leq \gamma < 3$, by $\frac{\underline{\Psi}^2}{\overline{\Psi}^2 (\ln n) \ln(\frac{\underline{\Psi} n^{\alpha}}{\ln n})} (\frac{\ln n}{\underline{\Psi} n^{\alpha}})^{\frac{1}{3}}$ when $\gamma = 3$, and by $\frac{\underline{\Psi}^2}{\overline{\Psi}^2 \ln n} (\frac{\ln n}{\underline{\Psi} n^{\alpha}})^{\frac{1}{3}}$ when $\gamma > 3$, and is upper bounded by $\min\big\{\frac{P_{max}}{P_{min}}, \frac{P_{max}}{P_{min}} (\overline{\Psi}^2 n^{2\alpha-3})^{\frac{1}{3}}\big\}$.

The rest of this paper is organized as follows. Section II introduces some related work. In Section III, we introduce some notations, definitions, and models we will use throughout this paper. Then, Section IV and Section V present capacity bounds of 3D regular ad hoc networks and of 3D nonhomogeneous ad hoc networks, respectively. We finally conclude this paper in Section VI.

II. Related Work

Gupta and Kumar [14] initiate the study of capacity on wireless networks. They show that the per-node throughput capacity is $\Theta(1/\sqrt{n \log n})$ bits/sec in random ad hoc networks and the per-node transport capacity is $\Theta(1/\sqrt{n})$ bit-meters/sec in arbitrary ad hoc networks, where n is the number of nodes in the networks. Later, Franceschetti et al. [9] prove by percolation theory that the same $1/\sqrt{n}$ per-node throughput can also be achieved in random ad hoc networks. Buragohain et al. [5] study the throughput capacity in grid networks and find that the $\Omega(1/d)$ per-node throughput can be achieved, where $1 \leq d \leq \sqrt{n}$ is the average source-destination distance. By allowing an arbitrary small fraction of the nodes to be disconnected, Dousse et al. [7] show that the throughput cannot be improved much. Some other works such as [1], [3], [8], [35] further extend the capacity of ad hoc networks to cases with different network settings.

In face of limited capacity in traditional ad hoc networks, some researchers explore advanced communication technologies to improve network capacity. Peraki and Servetto [37] show that random networks using directional antennas can achieve an increase of $\Theta(\log^2(n))$ in maximum stable throughput compared to those using omnidirectional antennas. Yi et al. [42] also study the same problem and show certain performance gain. In our previous work [28], we find that the capacity gain of random networks with directional antennas is in fact bounded by $\mathrm{O}(\log n)$ when using multi-hop relay schemes, and dependent on the side lobe gain of directional antennas when using one-hop delivery schemes. We also show that both random networks and arbitrary networks using directional antennas can have constant per-node capacity under certain conditions. In addition, Aeron et al. [1] and Ozgur et al. [36] show that by carrying out cooperative distributed MIMO (Multi-Input-Multi-Output) transmissions, the capacity of random ad hoc networks can also be increased significantly. Niesen et al. [34] develop a new cooperative scheme that works for arbitrary ad hoc networks. Furthermore, there is another body of work, such as [2], [18], [21], [23], [24], [26], [29], [32], [33], [40], [43], [45], places powerful nodes like base stations into ad hoc networks, which can greatly enhance network capacity, too. Mobility has been found as another effective way to improve network capacity. In particular, a big chunk of work like [4], [13], [17], [31] show that a constant per-node throughput is achievable in mobile ad hoc networks at the cost of large end-to-end delay on the order of n with possible logarithmic terms. Li et al. [25] then demonstrate smooth trade-offs between throughput and delay by controlling nodes' mobilities which fills the gap between the existing random mobile networks and static networks.

All the aforementioned research is performed on two-dimensional networks only. In the literature, only a couple of papers like [15] tentatively study the capacity of 3D wireless networks. In particular, [15] explores the transport capacity of 3D arbitrary ad hoc networks and the throughput capacity of 3D random ad hoc networks, using both Protocol Model and Physical Model, and find that the transport capacity is $\Theta(V^{\frac{1}{3}} n^{-\frac{1}{3}})$ bit-meters/sec where V is the network volumn and the throughput capacity is $\Theta(n^{-\frac{1}{3}} \ln^{-\frac{2}{3}} n)$ bits/sec. There are some other works, such as [10], [11], [16], [19], [30], which study the capacity of 2D arbitrary or random networks and then generalize their results to 3D arbitrary or random networks. Specifically, [19] and [30] still employ the Physical Model. Using a generalized physical model, [11] and [16] consider one-hop traffic flows, and [10] studies random networks based on their physical geometric structures and gives similar results to those in [15]. In contrast, in this paper we focus on the throughput capacity of 3D regular ad hoc networks and of 3D nonhomogeneous ad hoc networks, by employing a generalized physical model which is different from the Physical Model proposed in [15]. In particular, according to the Physical Model, the data rate of a transmission is equal to W bits/sec when the SINR (Signal-to-Interference plus Noise Ratio) at the receiver is above a certain threshold, and equal to 0 otherwise. Differently, in the generalized physical model we use the Shannon's capacity to model the data rate of a transmission, which can better characterize the channel capacity from an information-theoretic point of view. This different interference model necessitates different techniques to investigate network capacity and also leads to new capacity results as we will present later.

Besides, only a couple of papers like [3], have studied the capacity of inhomogeneous ad hoc networks. But, they explore two-dimensional networks and assume some specific inhomogeneous node distribution models. For example, in [3] nodes are assumed to be placed according to a shot-noise Cox process (SNCP), which is essentially a cluster-based power-law distribution model. In contrast, we consider very general nonhomogeneous node distributions in this study. Besides, we consider nodes may use different transmission power, while most previous works, including [3], [15], assume nodes have the same transmission power.

III. Notations, Definitions, and Models

In this section, we introduce the notations and definitions we will use, as well as the assumptions we make throughout this paper.

A. Notations

We use the following notations [20]:

- $f(n) = \mathrm{O}(g(n))$ means $f(n)$ is asymptotically upper bounded by $g(n)$, i.e., $\limsup_{n\to\infty} |\frac{f(n)}{g(n)}| < \infty$.
- $f(n) = \Omega(g(n))$ means $f(n)$ is asymptotically lower bounded by $g(n)$, i.e., $\liminf_{n\to\infty} |\frac{f(n)}{g(n)}| > 0$.
- $f(n) = \Theta(g(n))$ means $f(n)$ is asymptotically tight bounded by $g(n)$, i.e., $0 < \liminf_{n\to\infty} |\frac{f(n)}{g(n)}| \leq \limsup_{n\to\infty} |\frac{f(n)}{g(n)}| < \infty$.
- $f(n) = o(g(n))$ means $f(n)$ is asymptotically negligible with respect to $g(n)$, i.e., $\lim_{n\to\infty} |\frac{f(n)}{g(n)}| = 0$.
- $f(n) = \omega(g(n))$ means $f(n)$ is asymptotically dominant with respect to $g(n)$, i.e., $\lim_{n\to\infty} |\frac{f(n)}{g(n)}| = \infty$.

Besides, we denote by $|\mathcal{A}|$ the 3-dimensional Lebesgue measure of a measurable set $\mathcal{A} \subseteq \mathbb{R}^3$.

B. Definitions

Throughput: As defined in the usual way, the time average of the number of bits per second that can be transmitted by each node to its destination is called the *per-node throughput*. The sum of per-node throughput over all the nodes in a network is called the *throughput of the network*.

Feasible Throughput: We say that a per-node throughput, denoted by $\lambda(n)$, is *feasible* if there exists a spatial and temporal scheduling scheme that yields a per-node throughput of $\lambda(n)$ bits/sec. Let $\lambda_i(n)$ denote the throughput of node i. We say that a per-node throughput, denoted by $\lambda(n)$, is *feasible by all nodes* if there exists a spatial and temporal scheduling scheme such that $\lambda_i(n) \geq \lambda(n)$ for all $i \in [1, n]$, and is *feasible on average* if there exists a spatial and temporal scheduling scheme such that $\frac{1}{n}\sum_{i=1}^{n} \lambda_i(n) \geq \lambda(n)$. In this paper, we will derive a per-node throughput feasible on average unless otherwise specified, which we call "per-node throughput" for simplicity.

Per-node Throughput Capacity: We say that the per-node throughput capacity in the network is of order $\mathrm{O}(f(n))$ bits per second if there is a deterministic constant $0 < c_1 < +\infty$ such that

$$\liminf_{n\to+\infty} \mathrm{P}(\lambda(n) = c_1 f(n) \text{ is feasible}) < 1,$$

and is of order $\Theta(f(n))$ bits per second if there are deterministic constants $0 < c_2 < c_3 < +\infty$ such that

$$\liminf_{n\to+\infty} \mathrm{P}(\lambda(n) = c_2 f(n) \text{ is feasible}) = 1,$$
$$\liminf_{n\to+\infty} \mathrm{P}(\lambda(n) = c_3 f(n) \text{ is feasible}) < 1.$$

C. Network Model

We consider a network with n nodes distributed in a three-dimensional cube $\mathcal{A}$ with edge L, where $L = n^{\alpha/3}$ ($0 \leq \alpha \leq 1$), and the network volume $|\mathcal{A}| = L^3$. Thus, we can model all kinds of networks including *dense networks* ($\alpha = 0$) in [29], *extended networks* ($\alpha = 1$) in [23], and *semi-extended networks* ($0 < \alpha < 1$) in [8]. We also assume that the network nodes can have transmission powers ranging from P_{min} to P_{max} with $0 < P_{min} \leq P_{max}$.

3D Regular Ad Hoc Networks (RANETs): We first assume n nodes are regularly placed in the network. An example is shown in Fig. 1(a), in which there is one node at the center of each cubelet.

3D Nonhomogeneous Ad Hoc Networks (NANETs): We then extend our study to the case in which n nodes are distributed according to a general Nonhomogeneous Poisson Process (NPP), with the local intensity at point ξ in the network denoted by $\Psi(\xi)$, and $\int_{\mathcal{A}} \Psi(\xi)\mathrm{d}\xi = n$. The minimum and the maximum of $\Psi(\xi)$ are denoted by $\underline{\Psi}$ and $\overline{\Psi}$, respectively, which both scale with n. We also assume $\underline{\Psi}n^{\alpha} = \omega(\ln n)$. Fig. 1(b) shows one example for 3D NANETs.

Moreover, we follow the process in [13] to choose random sender-receiver pairs so that each node is a source node for one flow and a destination node for at most O(1) flows.

D. Channel Capacity Model

Let d_{ij} denote the distance between a node i and another node j. The reception power at node j of the signal from node i, denoted by P_{ij}, follows the power propagation model described in [38], i.e.,

$$P_{ij} = C\frac{P_i}{d_{ij}^{\gamma}}, \tag{1}$$

where P_i is the transmission power of node i, γ is the path loss exponent, and C is a constant related to the antenna profiles of the transmitter and the receiver, wavelength, and so on. As a common assumption, we assume $\gamma \geq 2$ in outdoor environments [38].

We consider the Shannon Capacity as the channel capacity between two nodes. Specifically, a transmission from node i to node j can have channel capacity, R_{ij}, which is calculated as follows:

$$R_{ij} = B\log_2(1 + SINR_{ij}), \tag{2}$$

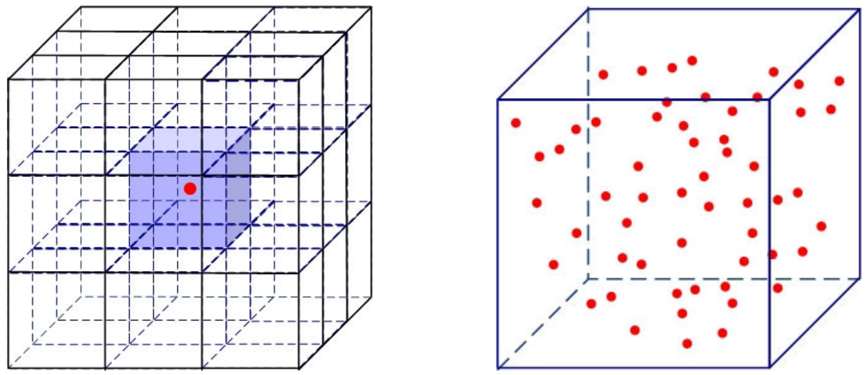
where B is the channel bandwidth, and

$$SINR_{ij} = \frac{C\frac{P_i}{d_{ij}^{\gamma}}}{N_0 + \sum_{k\neq i} C\frac{P_k}{d_{kj}^{\gamma}}}$$

is the SINR (Signal-to-Interference plus Noise Ratio) of the signal from node i to node j, with N_0 being the ambient noise power. In this study, we consider the channel bandwidth B to be a constant.

IV. Capacity of Three-Dimensional Regular Ad Hoc Networks

In this section, we investigate the capacity of three-dimensional regular ad hoc networks (RANETs). Both a lower



(a) A Regular Ad Hoc Network.

(b) A Nonhomogeneous Ad Hoc Network.

Fig. 1. Examples for 3D ad hoc networks.

bound and an upper bound on the capacity will be presented, respectively.

A. A Lower Bound on Capacity

We first find a lower bound on capacity by obtaining an achievable throughput. As mentioned in Section III-C, a 3D RANET can be divided into cubelets with an edge length of $l = n^{(\alpha-1)/3}$ and a node at the center. We let all nodes employ the same transmission power so that the transmission range is l, the distance between two neighboring nodes[1].

We divide a 3D RANET into groups each of which contains twenty-seven cubelets, as shown in Fig. 1(a). The twenty-seven cubelets in each group are numbered from 1 to 27 in the same way. We further divide time into sequences of successive slots, denoted by t ($t = 0, 1, 2, 3, ...$). During a time slot t, all nodes in cubelets that are numbered $(t \mod 27) + 1$ are allowed to transmit packets.

Consider a time slot when the node T_i in cubelet C_i is allowed to transmit to another node R_i. Denote by P_t the nodes' transmission power. The reception power level at R_i, denoted by $P_{r,i}$, is thus

$$P_{r,i} = \frac{CP_t}{l^\gamma}.$$

Besides, those nodes that may interfere with the transmission of T_i are located on the sides of concentric cubelets centered at C_i. The nodes on the sides of the first (or smallest) concentric cubelet are said to be at "tier 1", and so on and so forth. Note that the total number of interfering nodes at tiers 1 to j is equal to $(2j+1)^3 - 1$. Thus, at the jth tier, the number of interfering nodes, denoted by N_j, is

$$\begin{aligned} N_j &= (2j+1)^3 - (2j-1)^3 \\ &= 2 \times (12j^2 + 1) \\ &\le 26j^2, \end{aligned}$$

and the distance from an interfering transmitter to R_i, denoted by l_j, satisfies

$$l_j \ge (3j - 1) \cdot l.$$

[1]Due to physical limitations, a receiver is able to successfully receive a signal only if the received signal strength is above a certain threshold, which is usually defined as receiver sensitivity. The transmission range is thus defined as the range within which the receivers can successfully receive signals.

Thus, according to the power propagation model in (1), the cumulative interference at R_i, denoted by I_i, can be calculated as follows:

$$\begin{aligned} I_i &\le \sum_{j=1}^{N_{max}} 26j^2 \times \frac{CP_t}{[(3j-1)l]^\gamma} \\ &\le \frac{26CP_t}{l^\gamma}\left[1 + \sum_{j=2}^{N_{max}} (3j-1)^{2-\gamma}\right] \\ &= \frac{26CP_t}{l^\gamma}\left[1 + \sum_{j=1}^{N_{max}-1} (3j+2)^{2-\gamma}\right] \end{aligned}$$

where N_{max} is the maximum number of tiers. Obviously, we have

$$N_{max} = n^{\frac{\alpha}{3}} / n^{\frac{\alpha-1}{3}} = n^{\frac{1}{3}}.$$

Case I: $\gamma = 2$.

When $\gamma = 2$, the cumulative interference is

$$\begin{aligned} I_i &\le \frac{26CP_t}{l^\gamma}[1 + \sum_{j=1}^{N_{max}-1} 1] \\ &= \frac{26CP_t}{l^\gamma} \cdot n^{\frac{1}{3}}. \end{aligned}$$

Like in [22] [40], we consider an interference dominated environment where noise can be ignored. Thus, the SINR at the receiver R_i, denoted by $SINR_i$, is

$$SINR_i \ge \frac{\frac{CP_t}{l^\gamma}}{\frac{26CP_t}{l^\gamma} \cdot n^{\frac{1}{3}}} = \frac{1}{26} n^{-\frac{1}{3}}.$$

According to the Shannon Capacity, the transmission rate from T_i to R_i, denoted by R_C^i, is

$$R_C^i \ge B \log_2\left(1 + \frac{1}{26} n^{-\frac{1}{3}}\right) \approx \frac{B}{26} n^{-\frac{1}{3}}.$$

Case II: $2 < \gamma < 3$.

In this case, we get

$$\begin{aligned} I_i &\leq \frac{26CP_t}{l^\gamma}\left[1+\sum_{j=1}^{N_{max}-1}(3j+2)^{2-\gamma}\right] \\ &\leq \frac{26CP_t}{l^\gamma}\left[1+\int_0^{N_{max}}(3j+2)^{2-\gamma}\mathbf{d}j\right] \\ &= \frac{26CP_t}{l^\gamma}\left[1+\frac{(3j+2)^{3-\gamma}}{3(3-\gamma)}\Big|_0^{N_{max}}\right] \\ &\leq \frac{26CP_t}{l^\gamma}\left[1+\frac{(4N_{max})^{3-\gamma}}{3(3-\gamma)}\right] \\ &\leq \frac{52\cdot 4^{3-\gamma}}{3(3-\gamma)}\cdot\frac{CP_tN_{max}^{3-\gamma}}{l^\gamma}, \end{aligned}$$

and hence

$$SINR_i \geq \frac{\frac{CP_t}{l^\gamma}}{\frac{52\cdot 4^{3-\gamma}}{3(3-\gamma)}\cdot\frac{CP_tN_{max}^{3-\gamma}}{l^\gamma}} = \frac{3(3-\gamma)}{52\cdot 4^{3-\gamma}}n^{\frac{1}{3}(\gamma-3)}.$$

Since $\gamma < 3$, we can obtain that

$$\begin{aligned} R_C^i &\geq B\log_2\left(1+\frac{3(3-\gamma)}{52\cdot 4^{3-\gamma}}n^{\frac{1}{3}(\gamma-3)}\right) \\ &\approx \frac{3(3-\gamma)B}{52\cdot 4^{3-\gamma}}n^{\frac{1}{3}(\gamma-3)}. \end{aligned}$$

Case III: $\gamma = 3$.

When $\gamma = 3$, we can get that

$$\begin{aligned} I_i &\leq \frac{26CP_t}{l^\gamma}\left[1+\int_0^{N_{max}}\frac{1}{3j+2}\mathbf{d}j\right] \\ &= \frac{26CP_t}{l^\gamma}\left[1+\frac{1}{3}\ln(3N_{max}+2)-\frac{1}{3}\ln 2\right] \\ &\leq \frac{26CP_t}{l^\gamma}\left[\frac{1}{3}\ln(4N_{max})+1-\frac{1}{3}\ln 2\right] \\ &= \frac{26CP_t}{l^\gamma}\left[\frac{1}{3}\ln N_{max}+1+\frac{1}{3}\ln 2\right] \\ &\leq \frac{26CP_t}{l^\gamma}\ln N_{max}. \end{aligned}$$

As a result, the SINR of the transmission from T_i to R_i is

$$SINR_i \geq \frac{\frac{CP_t}{l^\gamma}}{\frac{26CP_t}{l^\gamma}\ln N_{max}} = \frac{3}{26\ln n},$$

and

$$\begin{aligned} R_C^i &\geq B\log_2\left(1+\frac{3}{26\ln n}\right) \\ &\approx \frac{3B}{26\ln n}. \end{aligned}$$

Case IV: $\gamma > 3$.

In this case, the cumulative interference can be calculated as

$$\begin{aligned} I_i &\leq \frac{26CP_t}{l^\gamma}\left[1+\int_0^{N_{max}}(3j+2)^{2-\gamma}\mathbf{d}j\right] \\ &= \frac{26CP_t}{l^\gamma}\left[1+\frac{2^{3-\gamma}}{3(\gamma-3)}\right] \\ &< \frac{26CP_t}{l^\gamma}\left[1+\frac{1}{3(\gamma-3)}\right] \\ &= \frac{26CP_t}{l^\gamma}\cdot\frac{3\gamma-8}{3\gamma-9}. \end{aligned}$$

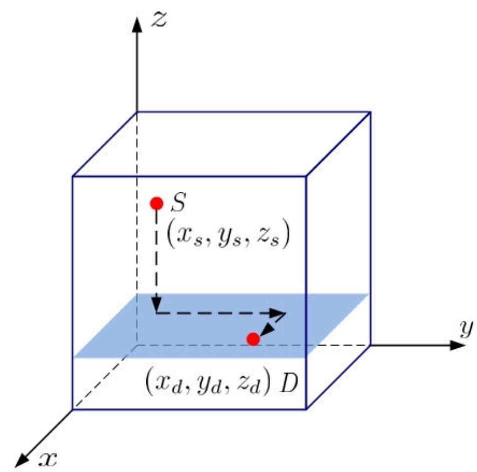


Fig. 2. The routing strategy used for packet delivery in 3D RANETs.

Thus, we can have that

$$SINR_i \geq \frac{\frac{CP_t}{l^\gamma}}{\frac{26CP_t}{l^\gamma}\cdot\frac{3\gamma-8}{3\gamma-9}} = \frac{3\gamma-9}{26(3\gamma-8)},$$

and

$$R_C^i \geq B\log_2\left(1+\frac{3\gamma-9}{26(3\gamma-8)}\right),$$

which can be lower bounded by a constant.

Note that every node transmits every 27 time slots. The following results follow subsequently.

Lemma 1: In 3D RANETs, the transmission rate of each node, denoted by R_C, is

$$R_C \geq \begin{cases} \frac{B}{c_1}n^{-\frac{1}{3}} & \text{if } \gamma = 2 \\ \frac{B}{c_2}n^{\frac{1}{3}(\gamma-3)} & \text{if } 2 < \gamma < 3 \\ \frac{B}{c_3\ln n} & \text{if } \gamma = 3 \\ \frac{B}{c_4} & \text{if } \gamma > 3 \end{cases} \tag{3}$$

where $c_1 = 702$, $c_2 = 468\cdot 4^{3-\gamma}/(3-\gamma)$, $c_3 = 234$, $c_4 = 27\log_2^{-1}\left(1+\frac{3\gamma-9}{26(3\gamma-8)}\right)$.

Besides, we use the following routing strategy to relay the packets. Specifically, as shown in Fig. 2, assume a source node S is located at (x_s, y_s, z_s) and its destination node D is located at (x_d, y_d, z_d). Packets from this source node are firstly relayed from (x_s, y_s, z_s) to (x_s, y_s, z_d), then to (x_s, y_d, z_d), and finally to (x_d, y_d, z_d).

Denote the average distance between the source-destination pairs of the information flows by $\bar{L}$. Then, we can easily get $\bar{L} \leq c_5 n^{\alpha/3}$ where $0 < c_5 < \sqrt{3}$ [6], and the average number of hops, denoted by $\bar{H}$, is

$$\begin{aligned} \bar{H} &= \frac{||z_s-z_d||}{l}+\frac{||y_s-y_d||}{l}+\frac{||x_s-x_d||}{l} \\ &\leq \frac{3\bar{L}}{l}. \end{aligned}$$

Thus, a per-node throughput, denoted by $\lambda(n)$, is feasible if

$$\lambda(n)\cdot\bar{H} \quad\leq\quad R_C,$$

i.e.,

$$\lambda(n) \leq \frac{1}{3c_5 n^{1/3}}R_C,$$

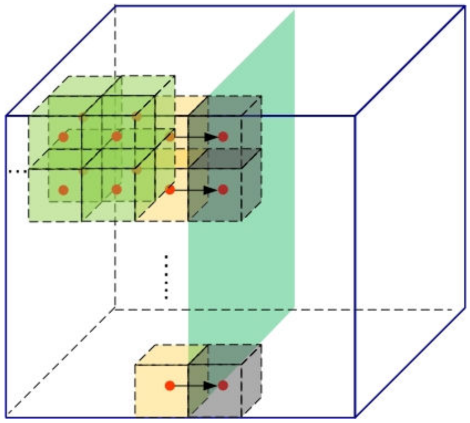


Fig. 3. Finding an upper bound on capacity using a cut plane.

which leads to the following theorem.

***Theorem** 1:* A lower bound on the capacity of 3D RANETs is

$$\lambda(n)=\Omega\begin{cases}\frac{B}{3c_1c_5}n^{-\frac{2}{3}} & \text{if } \gamma=2,\\ \frac{B}{3c_2c_5}n^{\frac{\gamma-4}{3}} & \text{if } 2<\gamma<3,\\ \frac{B}{3c_3c_5\ln n}n^{-\frac{1}{3}} & \text{if } \gamma=3,\\ \frac{B}{3c_4c_5}n^{-\frac{1}{3}} & \text{if } \gamma>3.\end{cases}$$

B. An Upper Bound on Capacity

Next, we investigate the upper bound on network capacity by finding the maximum amount of information that passes a cut plane in a 3D RANET.

As shown in Fig. 3, a cut plane is chosen such that the network space is divided into two parts each of which contains $n/2$ nodes. The transmitters are on the left side and the receivers are on the right side. We divide the left part of the network into tiers each of which are composed of *blocks* of different sizes. Specifically, at tier j ($j \geq 1$), the *blocks* have an edge length of $2^{j-1}\cdot l$. We now show that each *block* as a whole can have at most a constant transmission rate.

Assume the transmission power of each node falls into the interval $[P_{min}, P_{max}]$ with $P_{min} \leq P_{max}$. We use P_r^j, I^j, and $SINR^j$ to denote the reception power, interference suffered at receiver, and the SINR of transmissions originated from transmitters in tier j blocks, respectively. We also denote the total transmission rate of all nodes in a tier j block by R_C^j. Then, at tier 1, since the minimum distance between a transmitter and a receiver is l, we have

$$P_r^1 \leq \frac{P_{max}}{l^\gamma}.$$

Besides, the minimum interference is observed when there is only one other transmitter right next to the current one with the minimum transmission power P_{min}. Thus, we can get

$$I^1 \geq \frac{P_{min}}{(\sqrt{2}l)^\gamma}.$$

As a result, neglecting the noise, we can have

$$SINR^1 \leq 2^{\frac{\gamma}{2}}\frac{P_{max}}{P_{min}},$$

and hence

$$R_C^1 \leq B\log_2(1+SINR^1) \leq 2^{\frac{\gamma}{2}}\frac{P_{max}}{P_{min}}B$$

due to the fact that $\log_2(1+x) < x$ for $x > 0$.

Similarly, at tier j, the minimum distance between a transmitter and a receiver, denoted by d_{min}^j, is

$$d_{min}^j = (1+2+...+2^{j-2}+1)\cdot l = 2^{j-1}l,$$

and

$$P_r^j \leq \frac{P_{max}}{\left(2^{j-1}l\right)^\gamma}.$$

Denote by d_{max}^j the maximum distance between another transmitter in the same block and the receiver. Then,

$$d_{max}^j = \sqrt{\left[2^{j-1}l+(2^{j-1}-1)l\right]^2+\left[(2^{j-1}-1)l\right]^2\times 2}.$$

Let n_j denote the number of nodes in a block at tier j. So we get

$$\begin{aligned} I^j &\geq n_j\frac{P_{min}}{(d_{max}^j)^\gamma}\\ &\geq \frac{n_jP_{min}}{\left(\sqrt{(2^jl)^2+2(2^{j-1}l)^2}\right)^\gamma}\\ &= \frac{n_jP_{min}}{\left(\sqrt{6}\cdot 2^{j-1}\cdot l\right)^\gamma}.\end{aligned}$$

As a result, we can obtain

$$SINR^j \leq \frac{6^{\frac{\gamma}{2}}}{n_j}\frac{P_{max}}{P_{min}},$$

and hence

$$\begin{aligned} R_C^j &\leq n_jB\log_2\left(1+\frac{6^{\frac{\gamma}{2}}}{n_j}\frac{P_{max}}{P_{min}}\right)\\ &\leq 6^{\frac{\gamma}{2}}\frac{P_{max}}{P_{min}}B.\end{aligned}$$

We further denote the number of blocks at tier j by X_j. Then, we have

$$X_j = \frac{L^2}{l^2}\cdot\left(\frac{1}{4}\right)^{j-1}.$$

Thus, the per-node throughput, i.e., $\lambda(n)$, satisfies

$$\begin{aligned}\lambda(n) &= \frac{\sum_j X_jR_C^j}{n/2}\\ &\leq \frac{\frac{L^2}{l^2}(1+\frac{1}{4}+\frac{1}{16}+...)6^{\frac{\gamma}{2}}\frac{P_{max}}{P_{min}}B}{n/2}\\ &= 6^{\frac{\gamma}{2}}\frac{8P_{max}}{3P_{min}}Bn^{-\frac{1}{3}}.\end{aligned}$$

We can thus have the following theorem.

***Theorem** 2:* An upper bound on the capacity of 3D RANETs is

$$\lambda(n) = \mathrm{O}\left(6^{\frac{\gamma}{2}}\frac{8P_{max}}{3P_{min}}Bn^{-\frac{1}{3}}\right).$$

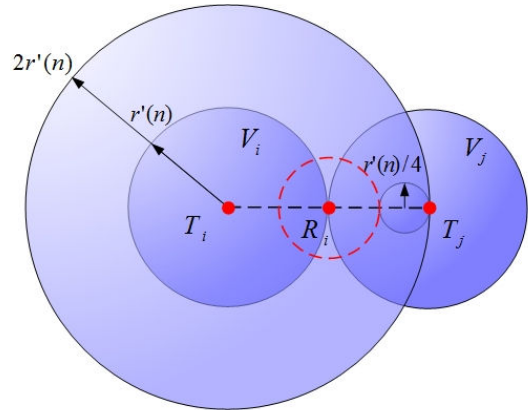


Fig. 4. Disjoint balls in a 3D NANET.

V. Capacity of Three-Dimensional Nonhomogeneous Ad Hoc Networks

In this section, we explore the capacity of three-dimensional nonhomogeneous ad hoc networks (NANETs), where the distribution of the n nodes follows a general NPP.

A. A Lower Bound on Capacity

We divide the network space into cubelets with an edge length of $l' = \left(\frac{c' \ln n}{\underline{\Psi}}\right)^{1/3}$, where $c'(c' > 2)$ is a constant. Since $\underline{\Psi} n^{\alpha} = \omega(\ln n)$, we have $l' = o(n^{\alpha/3}) = o(L)$. Then, we have the following lemma.

***Lemma** 2:* No cubelet is empty with high probability (w.h.p.).

Proof: For a cubelet C_b, the probability that there is no node in it, denoted by P_e, is

$$P_e = e^{-\int_{C_b} \Psi(\xi) d\xi} \leq e^{-\underline{\Psi}(l')^3} = \frac{1}{n^{c'}}.$$

So, $P_e \to 0$ as $n \to \infty$. Moreover, let n_c be the number of cubelets in the network. We have $n_c = n^{\alpha}/(l')^3$. Then, the probability that at least one cubelet has no node in it, denoted by P_E, is

$$P_E \leq n_c \cdot P_e \leq \frac{n^{\alpha}}{(l')^3} \cdot \frac{1}{n^{c'}} = \frac{\underline{\Psi} n^{\alpha}}{(c' \ln n) n^{c'}}.$$

Since $\underline{\Psi} n^{\alpha} \leq n$ and $c' > 2$, we can get that

$$P_E \leq \frac{1}{(c' \ln n) n^{c'-1}} \to 0,$$

i.e., no cubelet is empty w.h.p.. ■

We choose the transmission power P_t' so that the transmission range is $r'(n) = \sqrt{6} l'$. Thus, we can enable the transmissions between any two nodes located in two neighboring cubelets. We also choose the physical carrier sensing range to be $2r'(n)$. Then, as shown in Fig. 4, the balls centered at the transmitters transmitting at the same time with radius $r'(n)$ are disjoint. So are the inside balls with radius $r'(n)/4$ and a transmitter on the boundary.

Consider a transmission from a transmitter T_i to an arbitrary receiver R_i located at point $\xi_0 \in \mathbb{R}^3$. The reception power level at ξ_0, denoted by $P_r(\xi_0)$, is

$$P_r(\xi_0) \geq \frac{CP_t'}{\left(r'(n)\right)^{\gamma}}.$$

Let $\mathbb{T} = \{x_k\}$ denote the set of transmitters transmitting at the same time as T_i, where x_k also stands for the position of a node. We also denote the transmission volume by $|V_t|$. Then, the cumulative interference suffered at ξ_0, denoted by $I(\xi_0)$, is

$$\begin{aligned} I(\xi_0) &= \sum_{x_k \in \mathbb{T}} \frac{CP_t'}{||x_k - \xi_0||^{\gamma}} \\ &= CP_t' \sum_{x_k \in \mathbb{T}} \frac{1}{||x_k - \xi_0||^{\gamma}} \cdot \frac{1}{|V_t|/64} \cdot (|V_t|/64) \\ &\leq \frac{64CP_t'}{|V_t|} \int_0^{2\pi} \int_0^{\pi} \int_{r'(n)/2}^{\sqrt{3}L} \frac{1}{\rho^{\gamma}} \cdot \rho^2 \sin\phi \\ &\quad \mathbf{d}\rho \mathbf{d}\phi \mathbf{d}\theta \\ &= \frac{256\pi CP_t'}{|V_t|} \int_{r'(n)/2}^{\sqrt{3}L} \rho^{2-\gamma} \mathbf{d}\rho. \end{aligned}$$

Case I: $\gamma = 2$.

When $\gamma = 2$, the cumulative interference can be calculated as:

$$\begin{aligned} I(\xi_0) &\leq \frac{256\pi CP_t'}{|V_t|} \int_{r'(n)/2}^{\sqrt{3}L} 1 \mathbf{d}\rho \\ &\leq \frac{256\pi CP_t'}{\frac{4}{3}\pi \left(r'(n)\right)^3} \cdot \sqrt{3}L \\ &= \frac{192\sqrt{3}CP_t'L}{\left(r'(n)\right)^3}. \end{aligned}$$

As mentioned before, we consider an interference dominated environment. Thus, the SINR suffered by the receiver at ξ_0, denoted by $SINR(\xi_0)$, is

$$SINR(\xi_0) \geq \frac{CP_t'/\left(r'(n)\right)^2}{192\sqrt{3}CP_t'L/\left(r'(n)\right)^3}.$$

Recall that $r'(n) = \sqrt{6}\left(\frac{c' \ln n}{\underline{\Psi}}\right)^{1/3}$ and $L = n^{\alpha/3}$. We can get

$$SINR(\xi_0) \geq \frac{1}{96\sqrt{2}} \cdot \left(\frac{c' \ln n}{\underline{\Psi} n^{\alpha}}\right)^{\frac{1}{3}}.$$

Thus, the transmission rate from T_i to R_i, denoted by R_C^i, is

$$R_C^i \geq B \log_2 \left(1 + \frac{1}{96\sqrt{2}} \cdot \left(\frac{c' \ln n}{\underline{\Psi} n^{\alpha}}\right)^{\frac{1}{3}}\right).$$

Let $c_1' = \frac{1}{96\sqrt{2}}$. Since $\underline{\Psi} n^{\alpha} = \omega\left(\ln n\right)$, then $\ln n/(\underline{\Psi} n^{\alpha}) \to 0$, and hence

$$R_C^i \geq c_1' B \left(\frac{c' \ln n}{\underline{\Psi} n^{\alpha}}\right)^{\frac{1}{3}}.$$

Case II: $2 < \gamma < 3$.

When $2 < \gamma < 3$, we can get

$$\begin{aligned} I(\xi_0) &\leq \frac{256\pi CP_t'}{|V_t|} \int_{r'(n)/2}^{\sqrt{3}L} \rho^{2-\gamma} \mathbf{d}\rho \\ &\leq \frac{256\pi CP_t'}{\frac{4}{3}\pi \left(r'(n)\right)^3} \cdot \frac{(\sqrt{3}L)^{3-\gamma}}{3-\gamma} \\ &= \frac{192CP_t'(\sqrt{3}L)^{3-\gamma}}{(3-\gamma)\left(r'(n)\right)^3}, \end{aligned}$$

and hence

$$
\begin{aligned}
SINR(\xi_0) &\geq \frac{CP_t'/\big(r'(n)\big)^{\gamma}}{\frac{192CP_t'(\sqrt{3}L)^{3-\gamma}}{(3-\gamma)(r'(n))^3}} \\
&= \frac{(3-\gamma)2^{\frac{3-\gamma}{2}}}{192}\cdot\left(\frac{c'\ln n}{\underline{\Psi}n^{\alpha}}\right)^{\frac{3-\gamma}{3}}.
\end{aligned}
$$

Thus, the transmission rate from T_i to R_i, i.e., R_C^i, can be obtained by

$$
R_C^i \geq B\log_2\left(1+\frac{(3-\gamma)2^{\frac{3-\gamma}{2}}}{192}\cdot\left(\frac{c'\ln n}{\underline{\Psi}n^{\alpha}}\right)^{\frac{3-\gamma}{3}}\right).
$$

Let $c_2' = \frac{(3-\gamma)2^{\frac{3-\gamma}{2}}}{192}$. Then we can get

$$
R_C^i \geq c_2' B\left(\frac{c'\ln n}{\underline{\Psi}n^{\alpha}}\right)^{\frac{3-\gamma}{3}}.
$$

Case III: $\gamma = 3$.

When $\gamma = 3$, we can obtain that

$$
\begin{aligned}
I(\xi_0) &\leq \frac{256\pi CP_t'}{|V_t|}\int_{r'(n)/2}^{\sqrt{3}L}\rho^{-1}\mathbf{d}\rho \\
&= \frac{256\pi CP_t'}{\frac{4}{3}\pi\big(r'(n)\big)^3}\cdot\left[\ln\sqrt{3}L - \ln\frac{r'(n)}{2}\right] \\
&= \frac{192CP_t'}{\big(r'(n)\big)^3}\ln\frac{2\sqrt{3}L}{r'(n)},
\end{aligned}
$$

As a result, the SINR of the transmission from T_i to R_i located at ξ_0, is

$$
\begin{aligned}
SINR(\xi_0) &\geq \frac{CP_t'/\big(r'(n)\big)^3}{\frac{192CP_t'}{(r'(n))^3}\ln\frac{2\sqrt{3}L}{r'(n)}} \\
&= \frac{1}{192\ln\frac{2\sqrt{3}n^{\alpha/3}}{\sqrt{6}(\frac{c'\ln n}{\underline{\Psi}})^{1/3}}} \\
&= \frac{1}{192\big(\ln\sqrt{2}+\frac{1}{3}\ln\frac{\underline{\Psi}n^{\alpha}}{c'\ln n}\big)} \\
&\geq \frac{1}{256\ln\frac{\underline{\Psi}n^{\alpha}}{c'\ln n}}.
\end{aligned}
$$

The last step is due to the fact that $\ln\sqrt{2} < \ln\frac{\underline{\Psi}n^{\alpha}}{c'\ln n}$. Thus, the transmission rate from T_i to R_i, i.e., R_C^i, can be calculated as

$$
R_C^i \geq B\log_2\left(1+\frac{1}{256\ln\frac{\underline{\Psi}n^{\alpha}}{c'\ln n}}\right) \approx c_3' B\ln^{-1}\left(\frac{\underline{\Psi}n^{\alpha}}{c'\ln n}\right).
$$

where $c_3' = 1/256$.

Case IV: $\gamma > 3$.

When $\gamma > 3$, the cumulative interference can be calculated as

$$
\begin{aligned}
I(\xi_0) &\leq \frac{256\pi CP_t'}{|V_t|}\int_{r'(n)/2}^{\sqrt{3}L}\rho^{2-\gamma}\mathbf{d}\rho \\
&= \frac{256\pi CP_t'}{\frac{4}{3}\pi\big(r'(n)\big)^3}\cdot\frac{\rho^{3-\gamma}}{3-\gamma}\Bigg|_{r'(n)/2}^{\sqrt{3}L} \\
&\leq \frac{192CP_t'}{(\gamma-3)2^{3-\gamma}}\big(r'(n)\big)^{-\gamma}.
\end{aligned}
$$

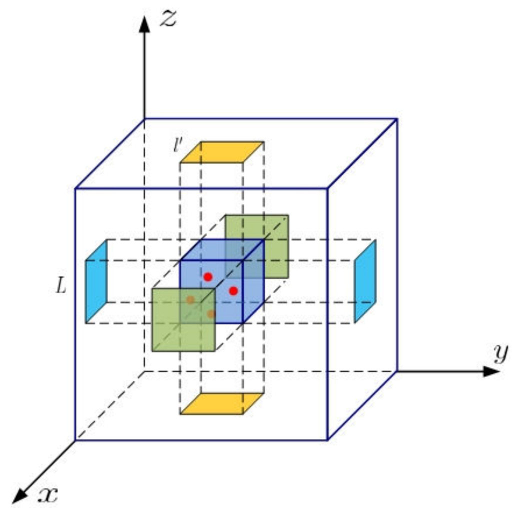


Fig. 5. The routing strategy used for packet delivery in 3D NANETs.

Thus, the SINR of the transmission from T_i to R_i, located at ξ_0, is

$$
\begin{aligned}
SINR(\xi_0) &\geq \frac{CP_t'/\big(r'(n)\big)^{\gamma}}{\frac{192CP_t'}{(\gamma-3)2^{3-\gamma}}\big(r'(n)\big)^{-\gamma}} \\
&= \frac{(\gamma-3)2^{3-\gamma}}{192},
\end{aligned}
$$

which can be lower bounded by a constant.

As a result, by letting $c_4' = \frac{(\gamma-3)2^{3-\gamma}}{192}$, the transmission rate from T_i to R_i is

$$
R_C^i \geq B\log_2\big(1+c_4'\big).
$$

Thus, we have the following lemma.

Lemma 3: In 3D NANETs, the data rate of each transmission, denoted by R_C', is:

$$
R_C' \geq \begin{cases}
c_1' B\big(\frac{c'\ln n}{\underline{\Psi}n^{\alpha}}\big)^{\frac{1}{3}} & \text{if } \gamma = 2, \\
c_2' B\big(\frac{c'\ln n}{\underline{\Psi}n^{\alpha}}\big)^{\frac{3-\gamma}{3}} & \text{if } 2 < \gamma < 3, \\
c_3' B\ln^{-1}\big(\frac{\underline{\Psi}n^{\alpha}}{c'\ln n}\big) & \text{if } \gamma = 3, \\
B\log_2(1+c_4') & \text{if } \gamma > 3.
\end{cases}
$$

Notice that nodes' transmission range $r'(n)$ is chosen to be

$$
r'(n) = \sqrt{6}l' = \sqrt{6}\left(\frac{c'\ln n}{\underline{\Psi}}\right)^{\frac{1}{3}}.
$$

So the maximum number of nodes that share the transmission rate R_C', denoted by N_n, is

$$
\begin{aligned}
N_n &= \overline{\Psi}\cdot 8|V_t| \\
&= \overline{\Psi}\cdot\frac{32}{3}\pi\big(r'(n)\big)^3 \\
&= 64\sqrt{6}c'\pi\frac{\overline{\Psi}\ln n}{\underline{\Psi}}.
\end{aligned}
$$

Besides, we employ a routing strategy similar to that in Section IV-A. As shown in Fig. 5, the maximum number of nodes that each cubelet relays packets for, denoted by N_r, can be obtained by

$$
N_r = 3(l')^2 L\cdot\overline{\Psi}.
$$

Since the minimum number of nodes in each cubelet, denoted by n_c, is

$$n_c = \underline{\Psi}(l')^3,$$

the maximum average traffic load for each node, denoted by $\bar{H}'$, is

$$\bar{H}' = \frac{N_r}{n_c} = \frac{3L\overline{\Psi}}{\underline{\Psi}l'} = \frac{3\overline{\Psi}}{\underline{\Psi}}\left(\frac{\underline{\Psi}n^\alpha}{c'\ln n}\right)^{\frac{1}{3}}.$$

As a result, a per-node throughput, denoted by $\lambda'(n)$, is feasible if the following holds:

$$\lambda'(n)\bar{H}' \le \frac{R'_C}{N_n},$$

i.e.,

$$\lambda'(n) \le \frac{1}{192\sqrt{6}\pi c'} \cdot \frac{\underline{\Psi}^2}{\overline{\Psi}^2 \ln n} \cdot \left(\frac{c'\ln n}{\underline{\Psi}n^\alpha}\right)^{\frac{1}{3}} \cdot R'_C.$$

We can thus have the following theorem.

***Theorem** 3:* A lower bound on the capacity of 3D NANETs is

$$\lambda'(n) = \Omega\begin{cases} \frac{c'_1 B}{192\sqrt{6}\pi c'}\frac{\underline{\Psi}^2}{\overline{\Psi}^2\ln n}\left(\frac{c'\ln n}{\underline{\Psi}n^\alpha}\right)^{\frac{2}{3}} & \text{if } \gamma = 2, \\ \frac{c'_2 B}{192\sqrt{6}\pi c'}\frac{\underline{\Psi}^2}{\overline{\Psi}^2\ln n}\left(\frac{c'\ln n}{\underline{\Psi}n^\alpha}\right)^{\frac{4-\gamma}{3}} & \text{if } 2 < \gamma < 3, \\ \frac{c'_3 B}{192\sqrt{6}\pi c'}\frac{\underline{\Psi}^2}{\overline{\Psi}^2(\ln n)\ln(\frac{\underline{\Psi}n^\alpha}{c'\ln n})}\left(\frac{c'\ln n}{\underline{\Psi}n^\alpha}\right)^{\frac{1}{3}} & \text{if } \gamma = 3, \\ \frac{B\log_2(1+c'_4)}{192\sqrt{6}\pi c'}\frac{\underline{\Psi}^2}{\overline{\Psi}^2\ln n}\left(\frac{c'\ln n}{\underline{\Psi}n^\alpha}\right)^{\frac{1}{3}} & \text{if } \gamma > 3. \end{cases}$$

Notice that in the network, we have $\underline{\Psi} = \mathrm{O}(\overline{\Psi})$. Assuming $\underline{\Psi}n^\alpha = n^x$ and $\overline{\Psi}n^\alpha = n^y$ where $x, y > 0$, we get

$$\underline{\Psi} = n^{x-\alpha}, \quad \overline{\Psi} = n^{y-\alpha}.$$

Since

$$\underline{\Psi}n^\alpha \le n \le \overline{\Psi}n^\alpha,$$

we can have that $0 < x \le 1$ and $y \ge 1$.

Thus, when $\underline{\Psi} = \Theta(\overline{\Psi})$, i.e., when $x = y = 1$, we get

$$\underline{\Psi}n^\alpha = \overline{\Psi}n^\alpha = \Theta(n),$$

and hence

$$\frac{\underline{\Psi}^2}{\overline{\Psi}^2\ln n}\left(\frac{\ln n}{\underline{\Psi}n^\alpha}\right)^z = \Theta\left(\frac{(\ln n)^{z-1}}{n^z}\right)$$

where $z \in \mathbb{R}$. When $\underline{\Psi} = o(\overline{\Psi})$, i.e., $0 < x < 1 < y$, we obtain

$$\frac{\underline{\Psi}^2}{\overline{\Psi}^2\ln n}\left(\frac{\ln n}{\underline{\Psi}n^\alpha}\right)^z = n^{2x-2y-zx}(\ln n)^{z-1}.$$

The following results thus follow.

***Corollary** 1:* Assume $\underline{\Psi} = n^{x-\alpha}$ and $\overline{\Psi} = n^{y-\alpha}$ where $0 < x \le 1 \le y$. An achievable per-node throughput in 3D NANETs, denoted by $\lambda'(n)$, is as follows:

1) When $\underline{\Psi} = \Theta(\overline{\Psi})$,

$$\lambda'(n) = \begin{cases} \Omega\left(n^{\frac{\gamma-4}{3}}\left(\ln n\right)^{\frac{1-\gamma}{3}}\right) & \text{if } 2 \le \gamma < 3, \\ \Omega\left(n^{-\frac{1}{3}}\left(\ln n\right)^{-\frac{2}{3}}\left(\ln\frac{n}{\ln n}\right)^{-1}\right) & \text{if } \gamma = 3, \\ \Omega\left(n^{-\frac{1}{3}}\left(\ln n\right)^{-\frac{2}{3}}\right) & \text{if } \gamma > 3. \end{cases}$$

2) When $\underline{\Psi} = o(\overline{\Psi})$,

$$\lambda'(n) = \begin{cases} \Omega\left(n^{\frac{\gamma+2}{3}x-2y}\left(\ln n\right)^{\frac{1-\gamma}{3}}\right) & \text{if } 2 \le \gamma < 3, \\ \Omega\left(n^{\frac{5}{3}x-2y}\left(\ln n\right)^{-\frac{2}{3}}\left(\ln\frac{n}{\ln n}\right)^{-1}\right) & \text{if } \gamma = 3, \\ \Omega\left(n^{\frac{5}{3}x-2y}\left(\ln n\right)^{-\frac{2}{3}}\right) & \text{if } \gamma > 3. \end{cases}$$

B. An Upper Bound on Capacity

We then use percolation theory [12], [39] to find an upper bound on the capacity of 3D nonhomogeneous ad hoc networks.

We divide the network space into cubelets with an edge length of l'_c. Then, the 3D network space can be decomposed into L/l'_c 2D *planar* areas each of which contains $(L/l'_c) \times (L/l'_c)$ cubelets. Thus, the capacity of the 3D network can be upper bounded by the sum of the capacity of each individual 2D *planar* network with sources and destinations on the same *plane*.

Let $0 < p < 1$ be a constant. Choosing

$$(l'_c)^3 = -\frac{\ln p}{\overline{\Psi}},$$

we know that the probability that a cubelet C_b is empty, denoted by P_e, is

$$P_e = e^{-\int_{C_b}\Psi(\xi)d\xi} \ge e^{-\overline{\Psi}(l'_c)^3} = p.$$

Since $p_c \approx 0.59$ is the critical probability of independent site percolation in a square lattice, choosing $p > p_c$ can make all L/l'_c 2D *planar* areas percolated. Moreover, in each 2D *planar* network, inside any *rectangle* of size $L \times (\kappa\log n)l'_c$ ($\kappa > 0$), there exists at least one path composed of $\Theta(L/l'_c)$ empty cublets connecting the top side with the bottom side of the network.

Consider one of the 2D *planar* networks as shown in Fig. 6. We choose a *rectangle* to the right of $y = y_0$ with width $h = (\kappa\log n)l'_c$ so that on the left side of $y = y_0$ there are $n/2$ nodes in the 3D cube. Then, inside this *rectangle* there is at least one crossing path composed of $\Theta(L/l_c)$ empty cubelets. Our objective is to find an upper bound on the amount of information $\mathcal{I}$ that can traverse from left to right through the cut planes in the 3D network. Since there are at least $n/2$ end-to-end data flows going through the cut planes, the per-node throughput can be upper bounded by $\mathcal{I}/(n/2)$.

We further divide the left part of the cutting plane (as shown in Fig. 6) into tiers of *blocks*. *Blocks* at different tiers are of different sizes. Specifically, the tier j ($j \ge 1$) blocks are of size $(2^{j-1}l'_c) \times (2^{j-1}l'_c) \times l'_c$. Besides, Tier 1 blocks are those right next to the rightmost empty cubelets along the crossing path (as shown in Fig. 6). Tier 2 blocks are next to Tier 1 blocks, and so on and so forth.

As a result, the minimum distance between a transmitter in a tier j block and its corresponding receiver, denoted by d^j_{min}, is

$$d^j_{min} = (1 + 1 + 2 + ... + 2^{j-2}) \cdot l'_c = 2^{j-1}l'_c,$$

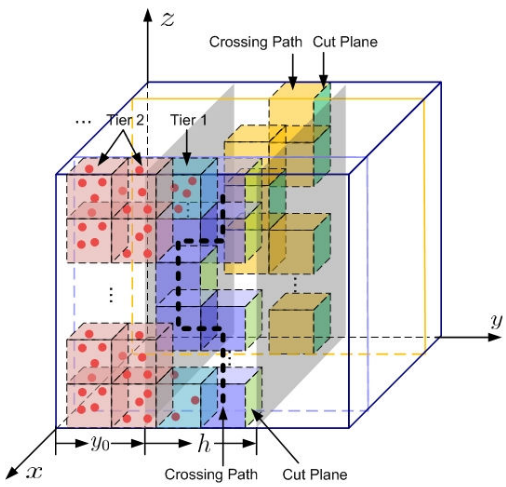


Fig. 6. Finding an upper bound by decomposing a 3D network into 2D networks.

and the reception power at the receiver, denoted by P_r^j, is

$$P_r^j \leq \frac{CP_{max}}{\left(2^{j-1}l_c'\right)^{\gamma}}.$$

Denote by d_{max}^j the maximum distance between another transmitter in the same block and the receiver. Then,

$$d_{max}^j = \sqrt{\left(2^{j-1}+2^{j-1}\right)^2 + \left(2^{j-1}\right)^2 + 1} \cdot l_c'.$$

Let n_j denote the number of nodes in a block at tier j. The cumulative interference suffered at the receiver, denoted by I^j, can be obtained as

$$I^j \geq n_j \frac{CP_{min}}{\left(d_{max}^j\right)^{\gamma}} \geq \frac{n_j CP_{min}}{\left(\sqrt{3}\cdot 2^j l_c'\right)^{\gamma}}.$$

As a result, we can get

$$SINR^j \leq \frac{(2\sqrt{3})^{\gamma}}{n_j} \frac{P_{max}}{P_{min}},$$

and hence

$$\begin{aligned} R_C^j &\leq n_j B \log_2\left(1 + \frac{(2\sqrt{3})^{\gamma}}{n_j}\frac{P_{max}}{P_{min}}\right) \\ &\leq (2\sqrt{3})^{\gamma}\frac{P_{max}}{P_{min}}B. \end{aligned}$$

We denote the number of non-empty tier j blocks in each 2D *planar* network by Y_j. Since as shown in Fig. 6 the crossing path might go through some of the blocks, some blocks may be empty. Thus, we have

$$Y_j \leq \frac{L}{l_c'} \cdot \left(\frac{1}{2}\right)^{j-1}.$$

So, the per-node throughput, i.e., $\lambda'(n)$, satisfies

$$\begin{aligned} \lambda'(n) &= \frac{(L/l_c')\cdot \sum_j Y_j R_C^j}{n/2} \\ &\leq \frac{\frac{L^2}{(l_c')^2}(1+\frac{1}{2}+\frac{1}{4}+...)(2\sqrt{3})^{\gamma}\frac{P_{max}}{P_{min}}B}{n/2} \\ &= (2\sqrt{3})^{\gamma}\frac{4P_{max}}{P_{min}}B\left(\frac{\overline{\Psi}^2 n^{2\alpha-3}}{\ln^2 p}\right)^{\frac{1}{3}}. \end{aligned}$$

Moreover, since in the network there are at most n non-empty blocks, we also have

$$\lambda'(n) \leq \frac{n\cdot R_C^j}{n/2} \leq \left(2\sqrt{3}\right)^{\gamma}\frac{2P_{max}}{P_{min}}B.$$

We can thus have the following theorem.

Theorem 4: An upper bound on the capacity of 3D NANETs is

$$\begin{aligned} \lambda'(n) &= \mathrm{O}\Bigg(\min\Bigg\{(2\sqrt{3})^{\gamma}\frac{2P_{max}}{P_{min}}B, \\ &\qquad \left(2\sqrt{3}\right)^{\gamma}\frac{2P_{max}}{P_{min}}B\left(\frac{\overline{\Psi}^2 n^{2\alpha-3}}{\ln^2 p}\right)^{\frac{1}{3}}\Bigg\}\Bigg). \end{aligned}$$

Considering a special case when $\overline{\Psi} = \Theta(\underline{\Psi})$, we get $\overline{\Psi}n^{\alpha} = \underline{\Psi}n^{\alpha} = \Theta(n)$. Thus, we can have the following result.

Corollary 2: Assume $\overline{\Psi} = n^{y-\alpha}$ where $y \geq 1$. An upper bound on the capacity of 3D NANETs is as follows:

1) When $\overline{\Psi} = \Theta(\underline{\Psi})$,

$$\lambda'(n) = \mathrm{O}\left(n^{-\frac{1}{3}}\right).$$

2) When $\overline{\Psi} = o(\underline{\Psi})$,

$$\lambda'(n) = \mathrm{O}\left(\min\left\{\frac{P_{max}}{P_{min}}, \frac{P_{max}}{P_{min}}n^{\frac{2y-3}{3}}\right\}\right).$$

VI. Conclusion

In this paper, we have explored the capacity of three-dimensional wireless ad hoc networks, including 3D regular ad hoc networks (RANETs) and 3D nonhomogeneous ad hoc networks (NANETs). Both lower and upper capacity bounds have been obtained under a generalized physical model for the two types of networks, respectively, when the path loss exponent is no less than 2. We find that lower capacity bounds are dependent on the power propagation environment, i.e., the path loss exponent, while upper capacity bounds are not. Moreover, note that the capacity of 2D random wireless networks is on the order of $(n\log n)^{-\frac{1}{2}}$, while Gupta and Kumar [15] show that the capacity of 3D random wireless networks is higher and on the order of $(n\log n)^{-\frac{1}{3}}$ under both Protocol Model and Physical Model. In contrast, our results reveal that 3D random wireless networks (and 3D regular wireless networks as well) may even have lower capacity than 2D wireless networks when $2 \leq \gamma < \frac{5}{2}$ under the generalized physical model. In our future work, we will investigate how to further bridge the gap between the lower bound and the upper bound on the capacity of 3D ad hoc networks.

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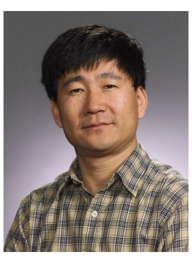

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