Available Bandwidth in Multirate and Multihop Wireless Ad Hoc Networks

Feng Chen, Hongqiang Zhai, and Yuguang Fang, *Fellow, IEEE,*

***Abstract*—The task of estimating path available bandwidth is difficult but paramount for QoS routing in supporting bandwidth-demanding traffic in multirate and multihop wireless ad hoc networks. The multirate capability and the impact of background traffic has not been carefully studied for the problem of estimating path available bandwidth in prior works. In this paper, we develop a theoretical model for estimating the available bandwidth of a path by considering interference from both background traffic and traffic along the path. We show that the clique constraint widely used to construct upper bounds does not hold any more when links are allowed to use different rates at different time. In our proposed model, traditional clique is coupled with rate vector to more properly characterize the conflicting relationships among links in wireless ad hoc networks where time-varying link adaption is used. Based on this model, we also investigate the problem of joint optimization of QoS routing and link scheduling. Several routing metrics and a heuristic algorithm are proposed. The newly proposed conservative clique constraint performs the best among the studied metrics in estimating available bandwidth of flows with background traffic.**

***Index Terms*—Available bandwidth, QoS routing, multirate, multihop, wireless ad hoc networks**

I. Introduction

In RECENT years, supporting multimedia traffic such as video in wireless ad hoc networks attracts lots of attention. This is mainly because of the increasing popularity of wireless video applications such as video phone, mobile TV, video-on-demand, and video gaming in wireless networks. However, the tough environment of wireless networks makes it difficult to satisfy the quality of service (QoS) requirements of multimedia traffic. Due to limited channel bandwidth, transmission rates are much smaller compared to those in wired networks. The broadcast nature of wireless transmissions makes it very difficult to estimate and control interference among links and flows. Furthermore, wireless networks often support multiple discrete channel rates, which have different transmission distance and requirements on the signal to noise ratio (SNR) and receiver sensitivities. How to fully utilize the multirate capability to support bandwidth-demanding traffic is an important research topic.

Mechanisms such as admission control, QoS routing and flow control are used to help support multimedia traffic. One of the important metrics for QoS routing, admission control and flow control is the available bandwidth. In this paper, we focus on the study of finding available bandwidth problem. Before admitting a multimedia flow, it is important to know whether a path can provide enough bandwidth for the flow. In our previous work [2], we have developed a theoretical model for calculating path capacity without considering background traffic. However, this problem becomes more difficult when there are some background traffic because the interference between a new flow and existing traffic is hard to estimate and control. In addition, many link adaptation schemes have been proposed to take advantage of the time-varying wireless signal and interference level in wireless multirate networks to improve the network capacity, aiming to support bandwidth-demanding traffic. However, modeling and evaluating the performance in wireless networks where each link could choose different link rate at different time has not been well studied so far.

Many Prior works have focused on estimating nodes' available bandwidth and extended results to estimate links' and paths' available bandwidth in QoS routing, admission control and flow control ([3]–[9]). A widely used approach is to measure the channel idle time and accordingly calculate a node's available bandwidth. Previous works have reasoned that since nodes in the same neighborhood or in each other's interference range share the same wireless channel, the total throughput of links interfering with each other along a path cannot exceed the channel bandwidth or the local available bandwidth. Based on this, they extend the channel idle time to path available bandwidth estimation.

There are also many works using flow contention graph and clique constraints to construct necessary and sufficient conditions or derive lower and upper bounds of paths' throughput to facilitate resource allocation, QoS routing, and flow control ([10]–[12]). In these works, a clique is referred to as a set of links satisfying that every two of them interfere with each other. The clique constraint is that the sum of all frequencies of the links in a clique is not larger than one, where a frequency for a link is defined as the link's throughput divided by the channel bandwidth. The clique constraint has been widely used to derive upper bounds for the path available bandwidth.

In this paper, we study the path available bandwidth estimation problem with background traffic in multirate and multihop wireless ad hoc networks. We assume that there exists a global

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Feng Chen and Yuguang Fang are with the Department of Electrical & Computer Engineering, University of Florida, Gainesville, Florida 32611, USA. Yuguang Fang is also a Changjiang Scholar Chair Professor with National Key Laboratory of Integrated Services Networks, Xidian University, Xi'an 710071, China (e-mail: chenf@ece.ufl.edu, fang@ece.ufl.edu).

Hongqiang Zhai is with Philips Research North America, Briarcliff Manor, New York 10510 (e-mail: hong.zhai@philips.com).

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optimal link scheduling and then we calculate the maximum available bandwidth of paths for any given background traffic. We formulate the path available bandwidth problem as a linear programming by extending the concepts of independent sets and cliques to take advantage of link adaptation. An independent set and a clique are not only specified by a set of links but also specified by the link rates. We analyze the upper bounds derived from cliques showing that the clique constraint becomes invalid for certain feasible link throughput vector. A counterexample for this clique constraint in multirate networks with link adaptation is shown in Section III-C.

We also extend the path available bandwidth estimation problem into a joint design of QoS routing and link scheduling to find paths with high available bandwidth. Due to the computational complexity of the joint design problem, we propose a heuristic algorithm to solve it by using the proposed metrics to estimate the paths' available bandwidth and QoS routing metrics to quickly find paths with high available bandwidth.

Most of the previous works do not consider the multiple discrete data rates and link adaptation capability. In this paper, our proposed concepts of independent sets and cliques consider the interference requirement imposed by discrete data rates and can be applied to networks with link adaptation scheme, while the previous models cannot be easily modified to achieve the same goal. Since different links may use different rates, resulting in non-unique channel bandwidth, nodes are sensitive to interference from those at different hop-distance away [13]. This makes it difficult to estimate interference by hop distance in order to calculate the path available bandwidth. Our proposed metrics for estimating the path available bandwidth consider both the interference from background traffic and that along the path, which find high available bandwidth paths when they are used as QoS routing metrics.

The rest of this paper is organized as follows. In Section II, we introduce the concepts of independent sets and cliques with discrete rates and compare them with previous models. We then use those concepts to formulate the problem of finding available path bandwidth estimation problem in multirate wireless ad hoc networks and study the capacity bounds in Section III. In Section IV, we show how the proposed model can be used to study the joint design of QoS routing and link scheduling. Several routing metrics have been proposed together with a heuristic algorithm to solve the joint optimization. We evaluate the performance of different QoS routing metrics and available bandwidth estimation metrics in Section V. Finally, Section VI concludes this paper.

II. Independent Sets and Cliques with Discrete Rates

In this section, we clarify the independent sets and cliques when discrete rates for wireless links are used.

A. Multiple Discrete Rates and Link Adaptation

In multirate wireless networks, each link may have several channel rates to choose from for each transmission. For example, the IEEE 802.11a protocol supports 6, 18, 36, and 54 Mbps. A higher rate signal travels shorter distance or has smaller transmission range than a lower rate one. This phenomena is captured by the receiver sensitivity. A successful transmission with the desired data rate requires the received signal power be higher than the receiver sensitivity for the rate. A higher data rate requires a higher receiver sensitivity. A successful transmission also requires the signal to interference plus noise ratio (SINR) at receiver be higher than a certain threshold. A higher data rate also requires a higher SINR. Let $RX_{se}(k)$ denote the receiver sensitivity of rate r_k. Let $SINR(k)$ denote the requirement of SINR of rate r_k. Therefore, a successful transmission with rate r_k satisfies the following two conditions:

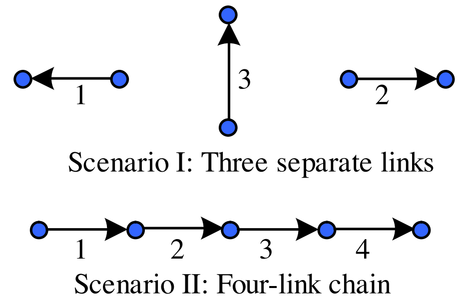

Fig. 1. Two simple topologies

$$Pr \geq RX_{se}(k) \text{ and } \frac{Pr}{P_{inf} + P_n} \geq SINR(k), \tag{1}$$

where P_{inf} is the interference power, P_n is the noise power and P_r is the received signal power.

When link adaptation scheme is used, each link can choose different rates at different time. In previous work [2], we study the multirate networks where each link can choose a fixed one from multiple rates and different link may choose different rates. In current wireless networks, link adaptation scheme is widely used to improve the network capacity and we need more tools to characterize the network capacity with link adaptation ability.

B. Independent Sets with Discrete Rates

The concept of independent set has been used to characterize a link set in which the links can be scheduled to transmit at the same time.

Let set $\{l_1, l_2, ...l_L\}$ denote all the links in consideration where L is the size of the set. Each link can choose from a set of data rates denoted by a rate set $\{r_k\}$, where $(1 \leq k \leq N_K)$, and N_K is the total number of rates. Let $E_i (1 \leq i \leq M)$ be the ith set of links. Let $\overrightarrow{R_i} = \{r_{i1}, r_{i2}, ..., r_{iL}\}$ be the rate vector for links in E_i and $r_{ij} \in \{r_k\}(1 \leq k \leq N_K)$.

An **independent set** with discrete rates is defined as: $(E_i, \overrightarrow{R_i})$ is an independent set if each link $L_j \in E_i$ can support rate $r_{ij}(> 0)$ indicated in $\overrightarrow{R_i}$ if all links in the set concurrently transmit.

A **maximal independent set** has to satisfy the following two conditions: First, each link in the independent set can support the required data rate when all the links in the set transmit at the same time. Second, inserting one more link into the set will decrease the link rate of at least one existing link in the set to a smaller value or even zero.

For example, in a four-link chain topology as shown in Fig. 1, we assume that all links can only support 36Mbps and 54Mbps if each of them transmits alone. We also assume that any two of links 1, 2, and 3 interfere with each other no matter whatever rates they use for transmission, and the same for links 2, 3, and 4. Links 1 and 4 interfere with each other if link 1 transmits with 54Mbps, but they do not interfere with each other if link 1 transmits with 36Mbps. Therefore, sets $\{L_1, 54\}$, $\{L_2, 54\}$, $\{L_3, 54\}$, and $\{(L_1, 36), (L_4, 54)\}$ are independent sets with the required data rates. $\{(L_1, 36), (L_4, 54)\}$ is a maximal independent set.

Notice that an independent set may not be an independent set any more if some links in the set transmit with a higher rate than that specified by $\overrightarrow{R}$.

C. Cliques with Discrete Rates

In previous works, a clique is defined as a set of links among which any two cannot transmit successfully at the same time. In multirate wireless networks where links are allowed to transmit with different rates at different times, a clique needs to be coupled with a rate vector, which is similar to the concept of independent sets with discrete rates discussed in the previous section.

In this paper, a **clique** C is defined as a set of multiple couples of a link and its transmission rate (L_i, r_i), and $C = \{(L_i, r_i)\}$. For any two links L_i and $L_j (i \neq j)$ in a clique C, not all transmissions will be successful if L_i transmits data with a rate r_i and L_j transmits data with a rate r_j at the same time. This phenomenon is also referred to as that L_i with rate r_i **interferes** with L_j with rate r_j.

A **maximal clique** C is defined as a clique satisfying that $C \cup \{(L_i, r_i)\}$ is not a clique for any couple (L_i, r_i), where $L_i \notin C$ and r_i is a positive rate achievable over L_i if it transmits alone.

We use the chain topology as shown in Fig. 1 to illustrate clique concept. $\{(L_1, 54), (L_2, 54), (L_3, 54)\}$ is a clique but not a maximal clique; $\{(L_1, 36), (L_2, 36), (L_3, 36)\}$, $\{(L_1, 54), (L_2, 54), (L_3, 54), (L_4, 54)\}$ and $\{(L_1, 36), (L_2, 54), (L_3, 54)\}$ are maximal cliques.

Apparently, if only links are considered, a maximal clique could be a subset of another maximal clique. This cannot happen in single-rate networks or in multirate networks where each link always uses a fixed rate.

D. Comparison with Other Models

Much prior work has used protocol model or physical model to construct conflict graph based on which the independent set or clique set is then constructed. For each link in these models, a fixed distance is used in the protocol model or a single SNR threshold is used in the physical model, which results in networks where either a single rate is used or each link uses a fixed rate. Some other work has also used Shannon-capacity rate model, i.e., $r_k = W \log_2 (1 + \frac{Pr}{P_{inf} + P_n})$ to calculate the maximal supportable rate on each link in the set, where any subset of links is an independent set. However, these models cannot be applied to the network where each link may choose a different rate at different time. In such scenarios, the construction of conflict graph without the pair of link and rate could not be proceeded as the required interference level depends on the chosen data rate.

A special case of our proposed model that each link chooses the maximal supportable rate as its desired data rate gives the same result as the previous work. Another special case of our proposed model that every link chooses the same data rate describes the single-rate network scenario. As it comes to characterize the network capacity, the maximal supportable data rate on each link should be considered. This is one of the reasons for models in [2] [11]. In this paper, we are able to characterize the network capacity when discrete data rates and link adaptation schemes are used. We are also able to study the effect of local link data selection and find the path available bandwidth given the desired data rates, while the previous model cannot easily be modified to achieve it.

III. Estimating Path Available Bandwidth in Multirate Wireless Networks

In this section, we present our method in estimating available bandwidth over a path in multirate wireless ad hoc networks.

A. Path Available Bandwidth Problem

In order to support QoS in multirate wireless ad hoc networks, one of the important functions that flow admission control scheme has to perform is to estimate the maximum path available bandwidth and admit new flows without affecting the existing flows. In this subsection, we show how our proposed concepts are used to formulate the path available bandwidth given traffic demand and background traffic. Let K denote the number of existing paths, $x_i (1 \leq i \leq K)$ denote the demand traffic load of the ith path, and $P_i (1 \leq i \leq K)$ denote the ith path. P_i also denotes the set of links on the ith path.

Given a new path P_{K+1}, we want to find out how much more traffic that the network could support over P_{K+1}. Let f_i denote the throughput over path P_i. The problem becomes maximizing throughput f_{K+1} over path P_{K+1} while guaranteeing the delivery of throughput $x_i (1 \leq i \leq K)$ over path $P_i (1 \leq i \leq K)$, respectively. Let $P = \bigcup_i P_i$. In P, we first find all maximal independent sets $\{E_\alpha, \overrightarrow{R_\alpha}\}$ $(1 \leq \alpha \leq \widehat{M})$ where $\widehat{M}$ is the total number of all maximal independent sets. Let $\lambda\tau$ be the time share scheduled for links in $\{E_\alpha, \overrightarrow{R_\alpha}\}$ to transmit. Let $I(P_k)$ be a row indicator vector in $\Re^{|P|}$, and

$$I_e(P_k) = \begin{cases} 1, & e \in P_k \\ 0, & e \notin P_k, e \in P \end{cases} \tag{2}$$

The problem to find the maximum throughput over path P_{K+1} can be formulated as

$$\begin{aligned} &\text{Maximize } f_{K+1} \\ &\text{Subject to:} \\ &\sum_{\alpha=1}^{\widehat{M}} \lambda_\alpha \leq 1,\ \lambda_\alpha \geq 0\ (1 \leq \alpha \leq \widehat{M}),\ f_{K+1} \geq 0 \\ &\sum_{\alpha=1}^{\widehat{M}} \lambda_\alpha R_\alpha - \sum_{k=1}^{K} x_k I(P_k) - f_{K+1} I(P_{K+1}) \geq 0 \end{aligned} \tag{3}$$

which can be solved by the standard linear programming. If the solution of this optimization problem f_{K+1} is larger than or equal to the flow's demand x_{K+1}, the demand for this new flow can be supported over the path P_{K+1} without affecting the bandwidth requirements of background traffic.

If there are more than one flow, say, V flows, with demands $x_k(K+1 \le k \le K+V)$ over paths $P_k(K+1 \le k \le K+V)$, respectively, requesting to join the network simultaneously, the problem can be formulated as

$$\begin{array}{l} \text{Maximize } \delta \sum\limits_{k=K+1}^{K+V} x_k I(P_k) \\ \text{Subject to:} \\ \sum\limits_{\alpha=1}^{\widehat{M}} \lambda_\alpha \le 1,\ \lambda_\alpha \ge 0\ (1 \le \alpha \le \widehat{M}),\ \delta \ge 0 \\ \sum\limits_{\alpha=1}^{\widehat{M}} \lambda_\alpha R_\alpha - \sum\limits_{k=1}^{K} x_k I(P_k) - \delta \sum\limits_{k=K+1}^{K+V} x_k I(P_k) \ge 0 \end{array} \tag{4}$$

where δ is a scaling factor. If there is an optimal solution with $\delta \ge 1$, then the bandwidth demands for all these V flows can be supported over their individual paths, respectively, without affecting the bandwidth requirements of background traffic.

We can use this optimization problem to evaluate the effects of different local data rate selection schemes on path bandwidth. In a local adaptation scheme, each link makes decision based on their observed local interference level. The problem of finding the path available bandwidth can be formulated similarly to equation(3). The difference is that the number of maximal independent sets here is less than or equal to $\widehat{M}$ and the maximal independent sets used here is only a subset of $\{E_\lambda, R_\lambda\}$.

Unfortunately, the problem of finding all the maximal independent sets involved in the above problem formulation is NP-hard. Instead of finding the exact solution, many researchers attempt to find the upper and lower bounds for the problems. However, we will soon point out that the clique constraint widely used for upper bounding the maximum supportable throughput is not valid any more in multirate wireless ad hoc networks where links are allowed to use different transmission rates at different time.

B. Clique Constraint in Multirate Wireless Networks

In this subsection, we discuss how to obtain an upper bound of a feasible link demand vector $\overrightarrow{Y} = \{y_1, y_2, ..., y_L\}$ and the upper bound of the available bandwidth f_{K+1} of a new path P_{K+1}, where $y_i(1 \le i \le L)$ is the throughput over link L_i.

In a single-rate wireless network, several research works (such as [11], [12]) have shown that the total time share for successful transmissions over all links in a clique cannot exceed one or the maximum available time share. Thus,

$$\sum_{L_i \in C} \frac{y_i}{r} = \frac{\sum_{L_i \in C} y_i}{r} \le 1.$$

Given the requirement that all links deliver the same throughput, each link's throughput is upper bounded by

$$s \le \frac{r}{N},$$

where r is the link rate and N is the size of the clique. To obtain a tighter bound, the maximum clique is widely used to construct an upper bound.

In [2], we showed a similar result for multirate wireless networks where each link selects a fixed rate from multiple choices and uses the clique transmission time to obtain an upper bound of the path throughput. Let $r_i(1 \le i \le N)$ denote the link rate over link L_i in a clique with N links.

$$\sum_{L_i \in C} \frac{y_i}{r_i} \le 1.$$

Given the requirement that all links deliver the same throughput, each link's throughput is upper bounded by

$$s \le \frac{1}{\sum_{i=1}^{N} \frac{1}{r_i}} = \frac{1}{\hat{T}}, \tag{5}$$

where $\hat{T}$ is defined as the clique transmission time for one unit of traffic [2]. Similarly, as using the maximum clique above, the maximum clique transmission time is needed to derive the upper bound.

Next, we proceed to construct the clique constraint for wireless networks where each link is allowed to use different data rates at different time. Let $C_{ij}(1 \le j \le M_i)$ be the jth clique given a rate vector $\overrightarrow{R_i} = \{r_{i1}, r_{i2}, ..., r_{iL}\}$, and M_i is the total number of different cliques for $\overrightarrow{R_i}$. Let $\overrightarrow{I}_{C_{ij}}$ be an indicator vector for clique C_{ij}, and

$$\overrightarrow{I}_{C_{ij}}(k) = \begin{cases} 1, L_k \in C_{ij} \\ 0, L_k \notin C_{ij}, L_k \in P \end{cases}$$

Let T_{ij} be the clique time share of C_{ij} given $\overrightarrow{R}_i$:

$$T_{ij} = \sum_{k=1}^{L} \frac{y_k}{r_{ik}} \overrightarrow{I}_{C_{ij}}(k).$$

Let $\overrightarrow{g_i} = \{g_{i1}, g_{i2}, ..., g_{iL}\}$ be a feasible throughput vector for all links given a rate vector $\overrightarrow{R}_i$. For any feasible $\overrightarrow{Y}$, we can find a set of γ_i and feasible $\overrightarrow{g_i}$ to satisfy

$$y_j = \sum_{i=1}^{\Omega} \gamma_i g_{ij} (1 \le j \le L), \text{ and } \sum_{i=1}^{\Omega} \gamma_i \le 1,$$

where γ_i is the time share when $\overrightarrow{R}_i$ is used, and Ω is the total number of possible values of $\overrightarrow{R}_i$. We know that if $\overrightarrow{g_i}$ is feasible,

$$T_{ij_g} = \sum_{k=1}^{L} \frac{g_{ik}}{r_{ik}} \overrightarrow{I}_{C_{ij}}(k) \le 1, \text{ for all } i, j.$$

That is to say, if a single $\overrightarrow{R}_i$ is used and $\overrightarrow{Y}$ is achievable over $\overrightarrow{R}_i$, then $\max_{1 \le j \le M_i} T_{ij} \le 1$. However, this may not be true if multiple $\overrightarrow{R}_i$ is used.

If all links require the same throughput, we will show that we can no longer argue that

$$s \le \max_i \frac{1}{\max_{1 \le j \le M_i} \sum_{k=1}^{L} \frac{1}{r_{ik}} \overrightarrow{I}_{C_{ij}}(k)}.$$

That is the upper bound derived by each given R_i, even the largest one, cannot be used to upper bound the maximal available path bandwidth, if $\overrightarrow{R}_i$ has multiple choices of link rates. An example for this is given below.

C. Exemplar Scenario where Clique Constraints Become Invalid

In this subsection, we study scenario II in Fig. 1. Suppose there is a multihop flow traveling through links L_1, L_2, L_3, and L_4, and requires the same throughput over these four links, i.e., $f = y_1 = y_2 = y_3 = y_4$, where f is the end-to-end throughput of the flow, and $y_i (1 \le i \le 4)$ is the throughput over link L_i.

The optimization problem (3) generates the following link scheduling S,

$$S = \begin{cases} (\lambda_1 = 0.1, E_1 = \{L_1, 54\}), \\ (\lambda_2 = 0.3, E_2 = \{L_2, 54\}), \\ (\lambda_3 = 0.3, E_3 = \{L_3, 54\}), \\ (\lambda_4 = 0.3, E_4 = \{(L_1, 36), (L_4, 54)\}) \end{cases}$$

The throughput f can be supported by the following two rate vectors $\overrightarrow{R_1}$ and $\overrightarrow{R_2}$, their corresponding supported throughput vectors $\overrightarrow{f_1}$ and $\overrightarrow{f_2}$, and their time shares γ_1 and γ_2:

$$\begin{aligned} &\overrightarrow{R_1} = \{54, 54, 54, 54\}, \gamma_1 = 0.1, \overrightarrow{f_1} = \{54, 0, 0, 0\} \\ &C_1 = \{(L_1, 54), (L_2, 54), (L_3, 54), (L_4, 54)\} \\ &\overrightarrow{R_2} = \{36, 54, 54, 54\}, \gamma_2 = 0.9, \overrightarrow{f_2} = \{12, 18, 18, 18\} \\ &C_2 = \{(L_1, 36), (L_2, 54), (L_3, 54)\} \\ &f = \gamma_1 f_{1i} + \gamma_2 f_{2i} = 16.2 \end{aligned}$$

It is not difficult to show that the cliques with the maximum clique transmission time share are the above C_1 and C_2 for R_1 and R_2, respectively, whose clique constraints are valid for individual throughput vector f_1 and f_2, respectively, but not for the maximum end-to-end throughput f:

$$\begin{aligned} &\sum_{L_i \in C_1} \tfrac{f_{1i}}{R_{1i}} = 1, \sum_{L_i \in C_2} \tfrac{f_{2i}}{R_{2i}} = 1. \\ &\sum_{L_i \in C_1} \tfrac{y_i}{R_{1i}} = 1.2 > 1, \sum_{L_i \in C_2} \tfrac{y_i}{R_{2i}} = 1.05 > 1. \end{aligned}$$

Notice that the upper bounds of end-to-end throughput provided by cliques for either $\overrightarrow{R_1}$ or $\overrightarrow{R_2}$ (refer to Equation (5)) is less than $f = 16.2$:

$$\begin{aligned} &\overrightarrow{R_1} : s_1 \le \frac{1}{\sum_{L_i \in C_1} \frac{1}{R_{1i}}} = \frac{1}{\frac{4}{54}} = 13.5 < 16.2 \\ &\overrightarrow{R_2} : s_2 \le \frac{1}{\sum_{L_i \in C_2} \frac{1}{R_{1i}}} = \frac{1}{\frac{1}{36} + \frac{2}{54}} = \frac{108}{7} \approx 15.43 < 16.2 \end{aligned}$$

It clearly shows that the maximum feasible throughput vector does not satisfy any clique constraint in this example, and hence the clique constraint cannot directly provide an upper bound any more.

Apparently, achieving the optimum end-to-end throughput $f = 16.2$ requires an appropriate link adaptation algorithm, which allows L_1 to transmit data with different data rates at different time in order to obtain higher end-to-end throughput than any fixed rate vectors.

IV. Joint Optimization of QoS Routing and Link Scheduling

In previous section, we show how to determine available bandwidth or the maximum end-to-end throughput of paths given the demands of background traffic and their paths. In this section, we focus on how to find available bandwidth from a source to its destination without known paths between them, but with known background traffic, which can be formulated as a joint design of QoS routing and link scheduling.

A. Joint Design of QoS Routing and Link Scheduling

We formulate this problem as a joint design of QoS routing and link scheduling as follows.

$$\begin{aligned} &\text{Maximize } f_{K+1} \\ &\text{Subject to:} \\ &\sum_{\{j:(i,j)\in E\}} y_{ij} - \sum_{\{j:(j,i)\in E\}} y_{ji} = \begin{cases} f_{K+1} & i = s, \\ 0 & i \in N \backslash \{s, t\} \\ -f_{K+1} & i = t \end{cases} \\ &y_{ij} \ge 0, \ (i,j) \in E \\ &\textstyle\sum_{1 \le \alpha \le M} \lambda_\alpha R_\alpha - \sum\limits_{1 \le k \le K} x_k I(P_k) - [y_{ij} | (i,j) \in E] \ge 0 \\ &\textstyle\sum_{1 \le \alpha \le M} \lambda_\alpha \le 1, \lambda_\alpha \ge 0, \ f_{K+1} \ge 0, \end{aligned} \tag{6}$$

where E is the link set of the network, s and t are the source and the destination of a new flow, i.e., the $(K+1)^{th}$ flow, respectively, y_{ij} is the link throughput from node i to node j over link L_{ij} delivered for the new flow, and $[y_{ij} | (i,j) \in E]$ represents the vector consisting of all link throughput values with the same ordering and dimension as R_α. The routing part of this problem formulation is the same as that in a max-flow problem, and the feasible condition of the link throughput comes from the link scheduling problem as shown in Equation (3). Normally, the solution of this problem will lead to multiple paths between the source s and its destination t.

In many cases, we are more interested in finding a single path to satisfy the bandwidth requirement of the new flow. In this paper, we focus on finding the path with the largest available bandwidth between the source s and its destination t with background traffic. The problem is formulated as follows,

$$\begin{aligned} &\text{Maximize } f_{K+1} \\ &\text{Subject to:} \\ &\sum_{\{j:(i,j)\in E\}} y_{ij} - \sum_{\{j:(j,i)\in E\}} y_{ji} = \begin{cases} f_{K+1} & i = s, \\ 0 & i \in N \backslash \{s, t\} \\ -f_{K+1} & i = t \end{cases} \\ &0 \le y_{ij} \le r_{ij}^{max} \cdot z_{ij}, \ (i,j) \in E \\ &\textstyle\sum_{\{j:(i,j)\in E\}} z_{ij} \le 1, z_{ij} \in \{0, 1\} \\ &\textstyle\sum_{1 \le \alpha \le M} \lambda_\alpha R_\alpha - \sum\limits_{1 \le k \le K} x_k I(P_k) - [y_{ij} | (i,j) \in E] \ge 0 \\ &\textstyle\sum_{1 \le \alpha \le M} \lambda_\alpha \le 1, \lambda_\alpha \ge 0, \ f_{K+1} \ge 0, \end{aligned} \tag{7}$$

where r_{ij}^{max} is the maximum achievable link rate over link L_{ij}. $z_{ij} = 1$ means that L_{ij} may have a nonzero throughput for the new flow. The third row means that there is at most one outgoing link from each node with a nonzero throughput for the new flow. The first three rows specify that there is only one path between the source and the destination. The links along that path have the same throughput and all other links have zero throughput for the new flow. This problem is a mixed integer programming.

Notice that the above two problem formulations both need to consider all links in the networks. However, the number of all links in a wireless network is not small in the broadcast environment. The computational complexity of finding all maximum independent sets increases dramatically with the number of links. In the following subsections, we will provide metrics in estimating the path available bandwidth and routing metrics used in the heuristic algorithm we propose to solve the joint design problem.

B. Estimating Available Bandwidth and QoS Routing Metrics

In distributed wireless networks, it is often not feasible to timely obtain the global link scheduling information and calculate accordingly the accurate available bandwidth of a new path. Therefore, it is important to develop a distributed algorithm to find a path and estimate the available bandwidth of that path with background traffic in mind.

To obtain background traffic information, each node can observe the channel utilization which can be obtained as a byproduct when carrier sensing mechanism is used. It calculates a channel idleness ratio $\lambda_{idle}(\leq 1)$, i.e., the ratio of the sensed idle time to the total sensing time. A link L_i assumes that it can transmit new traffic for a time share λ_i which is the smallest value of λ_{idle} for the transmitter and receiver of link L_i:

$$\lambda_i \leq \min\{\lambda_{idle,n_{it}}, \lambda_{idle,n_{ir}}\}, f \leq \lambda_i r_i \tag{8}$$

where n_{it} and n_{ir} are the transmitter and the receiver of link L_i, respectively, r_i is the effective data rate of link L_i and f is the available bandwidth of link L_i.

To estimate the available bandwidth of a path, we also need to consider the interference among links on the path. We here define a local interference clique for a path. A **local interference clique** is a clique in which all links are in sequential order on the path. We follow the approach in [2] to find local interference cliques. For a clique $C = \{L_1, L_2, ..., L_{|C|}\}$, and the corresponding idle time ratio for these links, $\overrightarrow{\lambda} = \{\lambda_1, \lambda_2, ..., \lambda_{|C|}\}$, we have

$$\sum_{i=1}^{|C|} \frac{f}{r_i} \leq 1. \tag{9}$$

We could further have

$$f \leq \min\{\frac{1}{\sum_{i=1}^{|C|} \frac{1}{r_i}}\}, \lambda_i \times r_i (1 \leq i \leq |C|). \tag{10}$$

This actually provides an upper bound of available end-to-end bandwidth of a path P given the rate vector $\overrightarrow{R} = \{r_1, r_2, .., r_{|P|}\}$ and the link idleness vector $\overrightarrow{\lambda}$.

The above equation (10) gives an accurate estimation about the path available bandwidth if any two links' idle times are not overlapped. It may give a loose upper bound. A conservative estimation is to add another constraint by assuming that the time share λ_i of link L_i is shared by all links in a clique with their individual time share less than λ_i, which bounds the throughput for any k links in C by:

$$\sum_{i=1}^{k} \frac{f}{r_i} \leq \max_{1 \leq i \leq k} \lambda_i.$$

If λ_i is ordered in increasing order as $\{\lambda_1 \leq \lambda_2 \leq ... \leq \lambda_{|C|}\}$, then it is equivalent to

$$f \leq \min_{i:1 \leq i \leq |C|} \frac{\lambda_i}{\sum_{j=1}^{i} \frac{1}{r_j}}. \tag{11}$$

In traditional routing algorithms without considering background traffic, several works such as [2] have shown that both end-to-end transmission delay (E2ETD) and local clique transmission time (LCTT) are good routing metrics to find a path with high end-to-end capacity. Here, we design two routing metrics based on E2ETD and LCTT as follows to take the background traffic into consideration. Since link L_i's available throughput f_i is equal to $\lambda_i \times r_i$, the average delay for one unit of traffic is equal to $\frac{1}{\lambda_i \times r_i}$. Therefore, the average end-to-end delay T_{e2e} of path P and the maximum average clique transmission delay T_C^* satisfy

$$T_{e2e} = \sum_{L_i \in P} \frac{1}{\lambda_i r_i}, \text{ and } T_C^* = \max_{C:clique} \sum_{L_i \in C} \frac{1}{\lambda_i r_i}. \tag{12}$$

Similar to Equation (5) which uses clique transmission time to construct an upper bound, here we propose another estimation of the available bandwidth by considering both clique constraint and background traffic for given $\overrightarrow{R}$ and $\overrightarrow{\lambda}$, i.e.,

$$f \leq \frac{1}{T_C^*} = \frac{1}{\max_{C:clique} \sum_{L_i \in C} \frac{1}{\lambda_i r_i}}. \tag{13}$$

Equation (9), (10), (11, 12 or 13) can all be used for estimating the path available bandwidth. We can also use the estimated available bandwidth calculated by each of those equations to serve as a routing metric. Each intermediate node on a path estimates the available bandwidth from the source to itself on the path, and uses it in distributed routing algorithms as any other routing metric like hop count.

In this subsection, we propose several metrics to estimate path available bandwidth and propose to use them as routing metrics as well. Most of them rely on the measurement of channel idle time, which is straightforward and simple. However, we must notice that these metrics may not be able to provide upper bounds if $\overrightarrow{R}$ has multiple choices of link rates. Furthermore, the method using channel idle time is not always reliable either. If a global link scheduling is allowed or a contention based MAC protocol is used, the link scheduling and hence the channel idle time for background traffic may change after new flows join. For example, a three-link topology is shown in the Scenario I of Fig. 1. Suppose we want to find the maximum available path bandwidth along a one-hop path over link L_3. Suppose link L_1 and L_2 do not interfere with or hear transmission from each other, but link L_3 interferes with and hears both the transmissions over L_1 and L_2. The background traffic over L_1 and L_2 occupies the same time share λ, but their time shares do not overlap with each other. The optimal scheduling is to let L_3 transmit $1-\lambda$, and

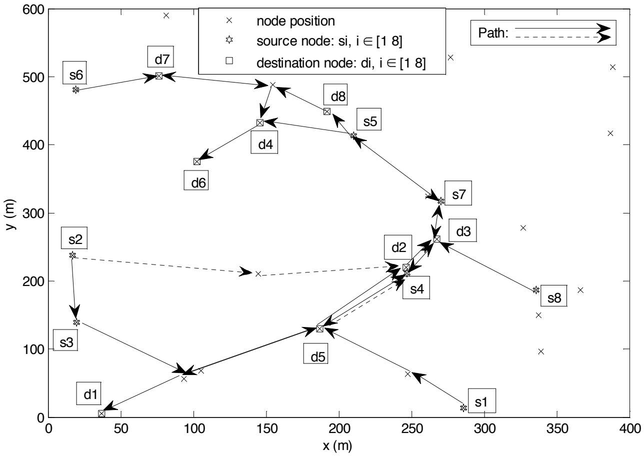

Fig. 2. Random topology

let L_1 and L_2 completely overlap with each other. However, using the mechanism of channel idle time to estimate available bandwidth, the flow over L_3 is only admitted if it occupies a time share not larger than $1 - 2\lambda$.

C. A Heuristic Algorithm to the Joint Optimization Problem

Since considering all links in a network dramatically increases the computational complexity in order to find all maximum independent sets in Problems (6) and (7), in this subsection, we develop a heuristic algorithm to solve the problem by using the proposed path available bandwidth metrics as QoS routing metrics.

The heuristic algorithm has four steps. First, squeeze the time share of independent sets for background traffic, and find a corresponding feasible link scheduling. This is done by solving the following optimization problem:

$$\begin{aligned}
&\text{Maximize } \delta \\
&\text{Subject to:} \\
&\sum_{\alpha=1}^{\widehat{M}} \lambda_\alpha \le 1 \\
&\sum_{\alpha=1}^{\widehat{M}} \lambda_\alpha R_\alpha - \sum_{k=1}^{K} \delta x_k I(P_k) \ge 0 \\
&\lambda_\alpha \ge 0 \ (1 \le \alpha \le \widehat{M}), \ \delta \ge 0.
\end{aligned} \tag{14}$$

Since the background traffic is feasible, the solution of this problem satisfies $\delta \ge 1$. A new scheduling $\{(E_i, \frac{\lambda_i}{\delta}, \overrightarrow{R_i})\}$ could also support background traffic $\{x_i(1 \le i \le K)\}$. The new scheduling satisfies:

$$\begin{aligned}
&\sum_{\alpha=1}^{M'} \lambda'_\alpha R_\alpha - \sum_{k=1}^{K} x_k I(P_k) = 0 \\
&\sum_{\alpha=1}^{\widehat{M}} \frac{\lambda_\alpha}{\delta} = \sum_{\alpha=1}^{M'} \lambda'_\alpha \le 1, M' \ge \widehat{M},
\end{aligned}$$

where each set $E_\alpha(1 \le \alpha \le M')$ is an independent set, but not necessarily a maximum independent set.

Second, calculate the time share for each link which could be scheduled without interfering with the background traffic or the existing link scheduling. Given a rate r_i at a link L_i, for each independent set E_α, if $L_i \notin E_\alpha$ and $E_\alpha \cup (L_i, r_i)$ is still an independent set, the time share λ_α is available for link L_i with transmission rate r_i. Therefore, the total available time share for link L_i with transmission rate r_i is given by:

$$\sum_{\substack{\alpha: E_\alpha \cup (L_i, r_i) \text{is an independent set} \\ L_i \notin E_\alpha}} \lambda_\alpha + (1 - \sum_{\alpha=1}^{M'} \lambda'_\alpha).$$

Third, use the available time share developed from step two to do the distributed QoS routing. Any routing metrics proposed in Section IV-B can be used. To increase the chance of finding the optimal path, we can find ℓ paths at a time by storing paths with the best ℓ values of the used routing metrics.

Finally, solve the path available bandwidth for new path(s). If multiple paths are found, choose the path with the largest available bandwidth.

We will use the above heuristic algorithm to find the path with the largest available bandwidth in Section V.

The problem of finding all the maximal independent sets is NP-hard problem. Interested readers are referred to previous efforts in dealing with such a problem [16] [17]. The heuristic algorithm first takes advantage of the fact that not all links carry consistent traffic in a typical non-saturated networks and try to reduce the number of links involved for computation. However, the link scheduling part cannot be fully implemented in a distributed fashion. Despite these limitations, we believe that the theoretical formulation for problems in wireless network with link adaptation should be able to provide meaningful guidance for protocol designs such as admission/flow control and inspire possible distributed solutions such as what we have proposed for distributed routing.

V. PERFORMANCE EVALUATION

In this section, we use the developed theoretical model and the heuristic algorithm to study path available bandwidth. We evaluate the QoS routing metrics we proposed and how well they perform in estimating the path available bandwidth. Part of the experiments we did for showing that the clique constraint is no longer valid in wireless networks with link adaptation scheme is presented in Section III-C for demonstration purpose.

A. Comparisons among Distributed QoS Routing Metrics

In this subsection, we use the heuristic algorithm proposed in Section IV-C to study the performance of different QoS routing metrics.

In the simulation, 30 nodes are randomly located in a 400m $\times$ 600m rectangle area as shown in Fig. 2. Four 802.11a rates are used, i.e., 54, 36, 18, and 6Mbps. The propagation exponent is set as 4. The transmission distances of these four rates are 59, 79, 119, 158m, respectively. Their SNR requirement are 24.56, 18.80, 10.79, 6.02dB, respectively ([15]). 8 sources and their destinations are randomly chosen and each flow's demand is 2Mbps. Due to space limitation, we only compare three routing metrics, hop count, end-to-end transmission delay (e2eTD), and average end-to-end delay (average-e2eD) (refer to Equation (12)).

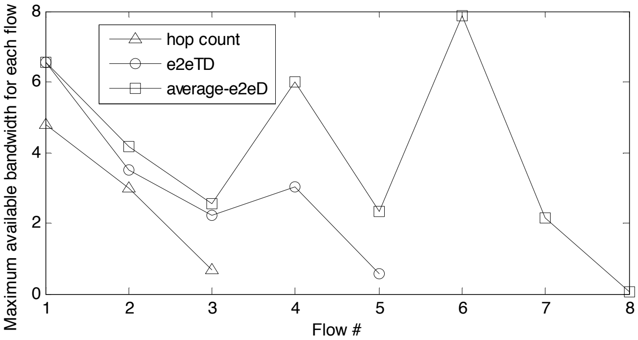

Fig. 3. Available bandwidth

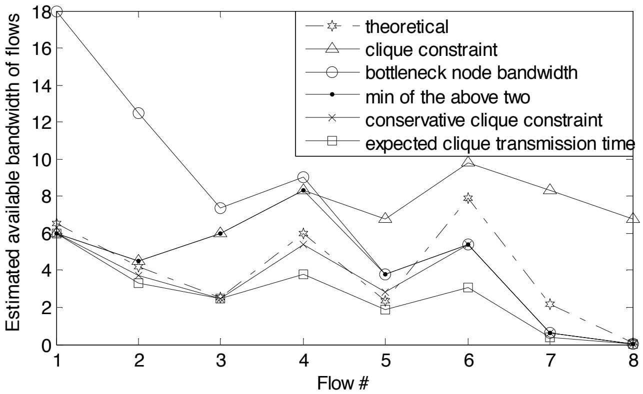

Fig. 4. Estimated available bandwidth

In the simulation, we assume that flows join the network one by one. The simulation stops when the demand of one flow is not satisfied. Fig. 2 also shows the paths found by the routing metric average-e2eD, which are illustrated by solid arrows. The e2eTD finds different paths for some flows, and the dotted arrows show some different links used by e2eTD. Fig. 3 shows the available bandwidth of each flow's path found by different routing metrics. Apparently, the average-e2eD can find paths with the largest available bandwidth among these three metrics, and it fails to find a path to satisfy the demand for the 8th flow. The e2eTD fails to find a path to satisfy the demand for the 5th flow, and the hop count metric fails at the 3rd flow.

B. Estimation of Path Available Bandwidth

In this subsection, we evaluate metrics studied in Section IV-B including "clique constraint (Equation 9)", "bottleneck node bandwidth (Equation 8)", "min of the above two (Equation 10)", "conservative clique constraint(Equation 11)", "expected clique transmission time (Equation 13)". We apply these metrics to the paths found by the routing metric average-e2eD in the above subsection.

From Fig. 4, we can observe that "clique constraint" underestimates the available bandwidth when the background traffic is light due to the negligence of the advantages of link adaptation, and overestimates the available bandwidth when the background traffic is heavy because it does not take background traffic into consideration. "bottleneck node bandwidth" considers the effect of background traffic but

TABLE I
AVERAGE NUMBER OF FLOWS ADMITTED BEFORE ANY REJECTION OF NEW FLOWS

hop count	e2eTD	average-e2eD
1.7742	4.5484	4.7742

TABLE II
ESTIMATION OF PATH AVAILABLE BANDWIDTH

	clique constraint	bottleneck node bandwidth	min of the previous two	conservative clique constraint	expected clique transmission time
MSE	3.2773	47.9882	2.7686	0.7430	1.5014
ARD	24.77%	77.73%	21.05%	9.70%	14.96%

ignores the interference among traffic along the new path, and hence overestimates the available bandwidth especially when the background traffic is light. "conservative clique constraint" accounts for both clique constraints and background traffic, and performs the best among these metrics. "expected clique transmission time" obtains lower values of available bandwidth and performs a little worse than "conservative clique constraint". Furthermore, all metrics except "clique constraint" underestimate the available bandwidth when background traffic is heavy. This demonstrates the shortage of using channel idle time to estimate the available bandwidth and verifies the previous results in this paper.

C. Average Performance in Random Topologies

We use the same configuration to run the simulation for 30 random topologies. The average number of flows admitted before any rejection of new flows are listed in Table I. The mean squared error (MSE) and average ratio of difference (ARD) are used to quantify the errors introduced by the metrics in estimating the available bandwidth compared to the theoretical value. As above, we consider all the paths admitted by using the metric "average-e2eD" in the random topologies. Let

$$X = [x_1, x_2, ..., x_n]$$

be the estimation of available bandwidth, and

$$Y = [y_1, y_2, ..., y_n]$$

be the theoretical value of available bandwidth of these paths. The MSE of X compared to Y is equal to

$$MSE = \frac{\sum_{i=1}^{n} (x_i - y_i)^2}{n}$$

The ARD of X from Y is defined as

$$ARD = \frac{\sum_{i=1}^{n} \frac{|x_i - y_i|}{y_i}}{n}$$

The MSEs and ARDs of five metrics are listed in Table II.

Apparently, these results verify the observations in previous subsections: hop count is a very poor routing metric to find a good path; average-e2eD performs a little better than e2eTD in finding path with bandwidth requirements; the metric "conservative clique constraint" performs much better than all other four metrics in estimating the available bandwidth of paths.

VI. CONCLUSIONS

In this paper, we have developed a theoretical model for calculating path available bandwidth by extending independent sets and cliques to mulirate and mulithop networks where link adaptation is allowed. Upper and lower bounds have been studied using cliques and independent sets. The model has also been extended to a joint design of QoS routing and link scheduling, and a heuristic algorithm has been proposed to solve the joint design problem. Furthermore, several QoS routing metrics and metrics to estimate available path bandwidth have also been investigated and compared.

From the theoretical model and performance evaluation results, we further have the following key observations in multirate and multihop networks:

- The clique constraint, a widely used condition to construct upper and even lower bounds of throughput, becomes invalid in mulitrate networks where links are allowed to transmit with different rates at different time;
- Channel idle time, a widely used metric, is not always effective to estimate nodes', links', and path's available bandwidth;
- The proposed "conservative clique constraint" performs the best among several studied metrics in estimating path available bandwidth by considering both the impact of background traffic and interference among the traffic along the path.

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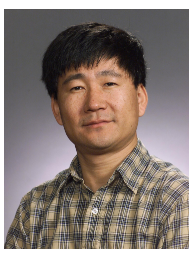
Feng Chen received the Ph.D degree in Electrical and Computer Engineering from University of Florida in August 2009 and the B.E. degrees in Electrical Engineering Department and Computer Science Department from Huazhong University of Science and Technology, Wuhan, in July 2005. Her research interests include wireless netwoks protocol designs, performance analysis and distributed computing algorithms.

Hongqiang Zhai received the Ph.D. degree in Electrical and Computer Engineering from University of Florida in August 2006 and the B.E. and M.E. degrees in Electrical Engineering from Tsinghua University, Beijing China, in July 1999 and January 2002, respectively. He is now a senior member of research staff in Wireless Communications and Networking Department of Philips Research North America. He is the recipient of the Best Paper Award at the 14th IEEE International Conference on Network Protocols (ICNP 2006). His research interests include performance analysis, medium access control, and cross-layer design in wireless networks.



Yuguang Fang (F'08) received a Ph.D. degree in Systems Engineering from Case Western Reserve University in 1994 and a Ph.D degree in Electrical Engineering from Boston University in 1997. He is a Professor in Department of Electrical and Computer Engineering at University of Florida. He holds a University of Florida Research Foundation (UFRF) Professorship from 2006 to 2009, a Changjiang Scholar Chair Professorship with Xidian University, Xi'an, China, from 2008 to 2011, and a Guest Chair Professorship with Tsinghua University, China, from 2009 to 2012. He has published over 250 papers in refereed professional journals and conferences. Dr. Fang received the US NSF Career Award, ONR Young Investigator Award, and is the recipient of the Best Paper Award in IEEE ICNP2006 Dr. Fang is also active in professional activities. He is currently serving as the Editor-in-Chief for IEEE Wireless Communications and serves/served on several editorial boards of technical journals including IEEE Transactions on Mobile Computing, IEEE Transactions on Communications, IEEE Transactions on Wireless Communications, IEEE Wireless Communications Magazine and ACM Wireless Networks. He has been actively participating in professional conference organizations such as serving as the Technical Program Vice-Chair for IEEE INFOCOM'2005 and a member of Technical Program Committees for IEEE INFOCOM (1998, 2000, 2003-2010) and ACM Mobihoc (2008-2009). He is a Fellow of the IEEE.