Joint Channel and Power Allocation in Wireless Mesh Networks: A Game Theoretical Perspective

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Abstract—This paper addresses the throughput maximization problem in wireless mesh networks. For the case of cooperative access points, we present a negotiation-based throughput maximization algorithm which adjusts the operating channel and power level among access points automatically, from a game-theoretical perspective. We show that this algorithm converges to the optimal channel and power assignment which yields the maximum overall throughput with arbitrarily high probability. Moreover, we analyze the scenario where access points belong to different regulation entities and hence non-cooperative. The long-term behavior and corresponding performance are investigated and the analytical results are verified by simulations.

Index Terms—Wireless mesh networks, potential games, equilibrium efficiency, pricing

I. INTRODUCTION

METROPOLITAN wireless mesh networks gain enormous popularity recently [1]. The deployment of wireless mesh networks not only facilitates the data communication by removing cumbersome wires and cables, but also provides a means of Internet access scheme, which is a further step towards the goal of “communicating anywhere anytime”. No matter where the location is or the purpose that the wireless mesh network is deployed, the same conceptual layered architecture is utilized. Figure 1 illustrates the hierarchical structure of wireless mesh networks. The peripheral nodes are the access points (AP) which provide wireless access for the end users, or clients. Each AP is associated with a mesh router. They can be manufactured in a single device with two separate functional radios [2] [3], or simply connected with Ethernet cables [4]. The mesh routers are capable of communicating with each other via the wireless backbone. The central node is a gateway mesh router which functions as an information exchange between the wireless mesh access network and other networks such as the Internet. Both the routing algorithmic design and channel assignment for backbone mesh routers are interesting issues and attract tremendous attention [5]–[7].

In this paper, we investigate an important issue which needs to be solved in wireless mesh networks. As in Figure 1, the AP and its associated clients form a regular WLAN cell, which operates with the de facto IEEE 802.11 standards. The throughput of one cell depends on the signal-to-interference-plus-noise ratio (SINR) experienced at the receiver where the interference mainly comes from the other operating cells. For example, if each of the cells operates with IEEE 802.11b standard, we can utilize a different frequency band such as IEEE 802.11a or WiMAX [8], for the inter-cell communication among mesh routers and hence causes no interference to intra-cell transmissions. However, the co-channel interference from other operating cells is inevitable due to the limitation of available transmission channels, e.g., 3 non-overlapping channels in our example. Most current off-the-shelf APs are capable of adjusting the transmission rate according to the measured channel condition which is indicated by transmission bit error rate (BER). Given a particular modulation scheme, BER is uniquely determined by the SINR experienced by the receiver of the link. Generally speaking, higher SINR value yields lower BER and higher data rate. Therefore, the mutual interference dramatically degrades the transmission rate of each cell and the aggregated throughput of the whole network [9]. Each AP attempts to tune the physical parameters such as operating frequency1 and transmission power in order to maximize the SINR and hence the throughput. In our work, we investigate the issue of maximizing the overall throughput of the network, defined as the summation of throughput of all cells, by finding the optimal frequency and transmission power allocation strategy. Also, due to the concern of scalability and computational complexity, we prefer a decentralized solution to the throughput maximization problem.

Unfortunately, the throughput maximization problem is challenging. For example, the frequency and power selected by one AP affects the SINR of other APs, and vice versa. Worse yet, if the APs belong to different regulation entities, the non-cooperative APs may only want to maximize their own

1 We will use frequency and channel interchangeably.
cell’s throughput rather than the overall one. Therefore, the throughput maximization problem is coupled and finding the optimum solution is not straightforward. Moreover, traditional site-planning methods in cellular networks are not feasible either. For example, the network administrator may want to add more APs when more users are joining the network or disable some APs where the associated users fail to pay the bill. The network topology is not static, although the changes take place slowly. Therefore, the demand for adaptability and light computation burden requires a decentralized solution for the throughput maximization problem.

In this paper, we analyze the throughput maximization problem for both cooperative and non-cooperative scenarios. In the cooperative case, we model the interaction among all APs as an identical interest game and present a decentralized negotiation-based throughput maximizing algorithm for the joint frequency and power assignment. We show that this algorithm converges to the optimal frequency and power assignment strategy, which maximizes the overall throughput of the wireless mesh access network, with arbitrarily high probability. In the cases of non-cooperative APs, we prove the existence of Nash equilibria and show that the overall throughput performance is usually inferior to the cooperative cases. To bridge the performance gap, we propose a linear pricing scheme to combat with the selfish behaviors of non-cooperative APs.

The rest of this paper is organized as follows. Section II outlines the system model we considered in the paper. The cooperative wireless mesh access networks and the non-cooperative counterpart are investigated in Section III and Section IV, respectively. An extension of our model is discussed in Section V and the performance evaluation is provided in Section VI. Finally, Section VII concludes this paper.

II. SYSTEM MODEL

In this paper, we consider a wireless mesh access network illustrated in Figure 1. Each AP and corresponding clients form a cell. Without loss of generality, we assume that all the cells operate with IEEE 802.11b standard and the interference exclusively comes from the cells with same frequency. Furthermore, the distance between cells are sufficiently large in the sense that the accumulated interference experienced at the receiver only affects the SINR value and not block the whole transmission. We assume that the channels are slow-varying additive white Gaussian noise (AWGN) channels. The channel gains of each pair of nodes are assumed to be constant over the time period of interest. As we are interested in the maximum achievable throughput, we consider the worst case where all APs are saturated. In other words, the APs always have packets to transmit and they can communicate with each other via the backbone mesh routers with negligible delay. Also, we assume that the APs are transmitters and clients are receivers due to the dominance of downlink traffic, as assumed\(^2\) in [11] [12] and [13]. We only focus on the joint frequency and power allocation where the contention behavior is less relevant and thus omitted. Therefore, we can simplify our model as that all the APs are transmitting data to the associated clients consistently. We assume that each AP is capable of adjusting the operating frequency and power as well as acquiring the SINR values measured at the client by short ACK messages.

Let us first consider the simplest case where there is only one cell in the wireless mesh access network, i.e., a single WLAN. In the following two sections, we assume that the APs have pre-determined and fixed modulation and coding schemes. In other words, upon receiving the SINR value\(^3\) measured by the client, denoted by $\gamma$, the AP tunes the physical parameters in order to maximize the throughput, which is defined as

$$R^*(\gamma) = \max_{R_i} R_i \times (1 - P_e(\gamma, R_i)) \quad (1)$$

where $R_i$ is the raw data rate specified by the IEEE 802.11 standard and $R^*$, i.e., the throughput of this cell, is a non-decreasing function of received SINR $\gamma$. $P_e$ is the error probability of the transmission channel, which is a function of SINR value providing the modulation and coding scheme [14]. Apparently, if there is only one cell in the mesh access network, the AP will boost the power as much as possible to increase the of $\gamma$ and thus the throughput is maximized.

We now consider the cases where $N$ cells coexist in the wireless mesh access network. Let $p_i$ and $f_i$ denote the power and frequency for the $i$-th AP, respectively. We use $p = [p_1, p_2, \ldots, p_N]$ and $f = [f_1, f_2, \ldots, f_N]$ to represent the power and frequency assignment vector for all $N$ APs. Therefore, for each cell $i$, the value of SINR\(^4\), i.e., $\gamma_i$, is a function of $(p, f)$. The throughput of one cell depends not only on the power level and frequency of itself, but also those of other APs in the network. Therefore, the throughput maximization problem is coupled and by no means straightforward.

In the following two sections, we will discuss the scenarios where the APs are cooperative and non-cooperative, respectively, under the assumption that the modulation and coding schemes of APs are pre-loaded and fixed. In Section V, we will extend our analysis by considering the scenario where the APs are capable of adaptive coding and modulation. The performance evaluation of all scenarios are provided by simulations in Section VI.

III. COOPERATIVE ACCESS NETWORKS

In this section, we consider the scenarios where all APs in the wireless mesh access network are cooperative. The transmission power of APs are quantized into discrete power levels for simplicity. From the system point of view, we want to find a joint frequency and power level assignment such that the overall throughput in the whole network is maximized. Our objective function can be written as

$$U_{\text{network}}(p, f) = \sum_{i=1}^{N} R^*_i(\gamma_i) = \sum_{i=1}^{N} R^*_i(p, f) \quad (2)$$

where $R^*_i$ is defined in (1).

\(^2\)The dominance of the downlink traffic is verified by the experimental measurements in [10] as well.

\(^3\)Although there is no interference in this case, we adopt SINR instead of SNR for notation consistency.

\(^4\)Throughout the paper, the term SINR of the cell represents the average SINR among all the clients in the cell, which can be obtained by a moving average of the reported SINR value.
However, finding the optimal frequency and power assignment which maximizes (2) is non-trivial. The interdependency makes the problem coupled and difficult to solve by traditional optimization methods [15]. A combination of \((p, f)\) is named a profile and a naive approach to solve the problem is to investigate all profiles exhaustively. However, this is impossible in practice. For example, in a medium-size wireless mesh access network with 20 APs where each has 3 frequency channels and 10 power levels, the search space is \((3 \times 10)^{20}\) profiles! Obviously, the centralized algorithms are not favorable in the wireless mesh access network due to the scalability concern. Next, we will introduce a decentralized negotiation-based throughput maximization algorithm, from a game-theoretical perspective.

### A. Cooperative Throughput Maximization Game

The APs in the wireless mesh access networks are considered as players, i.e., decision makers of the game. We model the interaction among APs as a Cooperative Throughput Maximization Game (CTMG), where each player has an identical objective function \(U_i\), as

\[
U_i(p, f) = U_{network}(p, f) = \sum_{j=1}^{N} R_j^p(p, f), \forall i.
\]

For each player \(i\), all possible frequency and power level pairs form a strategy space \(\Phi_i\), which has a size of \(c \times l\), where \(c\) is the number of frequency channels available and \(l\) is the number of feasible power levels. Define

\[
\Omega = \Phi_1 \times \Phi_2 \times \cdots \times \Phi_N.
\]

Then, the \(N\) players autonomously negotiate about the joint frequency-power profile in \(\Omega\) in order to find the optimal profile which maximizes (3). However, due to the interdependency among \(N\) players caused by mutual interference, one question of interest is that whether this negotiation will eventually meet an agreement, a.k.a., a Nash equilibrium. The importance of Nash equilibria lies in that a possible steady state of the system is guaranteed. If the game has no Nash equilibrium, the negotiation process never stops and oscillates in an everlasting fashion. In addition, we are concerned about what the performance of the steady states would be, if exist, in an everlasting fashion. In addition, we are concerned about what the performance of the steady states would be, if exist, in an everlasting fashion.

**Lemma 1:** The CTMG is a potential game.

A potential game is defined as a game where there exists a potential function \(P\) such that

\[
P(a', a_{-i}) - P(a'', a_{-i}) = U_i(a', a_{-i}) - U_i(a'', a_{-i}) \forall i, a', a''
\]

where \(U_i\) is the utility function for player \(i\) and \(a', a''\) are two arbitrary strategies in \(\Phi_i\). More specifically, we have \(a' = [p'_1, f'_1]\) and \(a'' = [p''_1, f''_1]\). The notation of \(a_{-i}\) denotes the vector of choices made by all players other than \(i\). Potential games have been broadly applied in modeling the interactions in communication networks [16]. The popularity is on account of the nice properties of potential games, such as:

- Potential games have at least one Nash equilibrium.
- All Nash equilibria are the maximizers of the potential function, either locally or globally.
- There are several learning schemes available which are guaranteed to converge to a Nash equilibrium, such as better response and best response [17] [18].

For detailed description about potential games, readers are referred to [17] and [19], which investigate the potential game theory in engineering context.

We observe that in the cooperative case, each player has the same utility function as in (3), which is the overall throughput of the network. Apparently, one potential function of the game is the common utility function itself, i.e.,

\[
P = U_1 = U_2 = \cdots = U_N.
\]

In fact, the games where all players share the same utility function are called identical interest games [20], which is a special case of potential games and hence all the properties of potential games can be applied directly.

In the literature, both best response and better response are popular learning mechanisms that have been utilized in potential games [21]–[23]. At each step of the best response approach, one of the players investigates its strategy space and chooses the one with maximum utility value. This updating procedure is carried out sequentially. The primary drawback of the best response is the computational complexity, which grows linearly with the cardinality of the strategy space. An improvement of the best response is the so-called better response, where at each step, the player updates as long as the randomly selected strategy yields a better performance. The dramatically reduced computation is the tradeoff with the convergence speed. Both the best response and the better response dynamics are guaranteed to converge to a Nash equilibrium in potential games [16]. However, there may be multiple Nash equilibria in a potential game and the performance of different equilibria may vary dramatically. Therefore, although the best response and the better response could guarantee the convergence, they may reach an undesirable Nash equilibrium with inferior performance.

Let us consider an illustrative example in Figure 2. There are four labeled APs in the network. \(A\) and \(B\) are close to each other, and so are \(C\) and \(D\). Without loss of generality, we assume that the APs have the same power and only adjust the operating frequencies in an order of \(A \rightarrow B \rightarrow C \rightarrow D\) to avoid the interference. The adaptation continues with the best response mechanism until a Nash equilibrium is reached. Suppose that there are two frequency channels available, say 1 and 2. First, \(A\) randomly selects one channel, say 1. \(B\) will
pick 2. Next, C has the chance to update. Since C is closer to B than A, channel 1 will be selected. Finally, D will choose channel 2. By inspection, we claim that profile $1 - 2 - 2 - 1$ is a Nash equilibrium since no player is willing to update its strategy unilaterally. Meanwhile, we observe that another profile $1 - 2 - 1 - 2$ is also a Nash equilibrium. Obviously, the second Nash equilibrium generates much less interference than the first Nash equilibrium and hence yields superior performance in terms of overall throughput. However, the best response only leads to the less desirable Nash equilibrium.

In fact, the existence of multiple Nash equilibria is observed in [21] by simulations. However, the authors fail to specify which one would be the steady state of their game due to the limitation of the best response, even in a statistical fashion. Recall that the Nash equilibria are the maximizers of the potential function in potential games, converging to an inferior Nash equilibrium analogously indicates being trapped at a local optimum of the potential function. However, it is the global optimum, i.e., the optimal Nash equilibrium, that is the desirable steady state which we are yearning for.

Next, we introduce a negotiation-based throughput maximization algorithm (NETMA) which can converge to the optimal Nash equilibrium with arbitrarily high probability.

**B. NETMA: NEgotiation-based Throughput Maximization Algorithm**

We assume that the APs are homogeneous and each has a unique ID for routing purpose. Each AP maintains two variables $D_{pre}$ and $D_{cur}$. The AP has the knowledge of its current throughput and records it in $D_{cur}$. Whenever there is a change of throughput caused by external interference, the AP sets $D_{pre} = D_{cur}$ and resets $D_{cur}$ with the newly measured throughput. When the wireless mesh access network enters the negotiation phase\(^5\), NETMA is executed. The detailed procedure of NETMA is provided as follows.

**NETMA:**

- **Initialization:** For each AP, a pair of frequency and power level is randomly selected. Set $D_{pre} = D_{cur}$, the current throughput.
- **Repeat:**
  1. Randomly choose one of the AP, say $k$, as the updating one, i.e., each AP updates with probability $1/N$.
  2. For the updating AP $k$:
     a) Randomly chooses a pair of frequency and power level, say $f'$ and $p'$, from the strategy space $\Phi$. Then the AP computes the current throughput with $f'$ and $p'$ and records it into $D_{cur}$.
     b) Broadcasts a short notifying message which contains its unique ID $j$ to all the other APs in the mesh access network.

\(^5\)We assume the channel is slow-varying and the change of throughput for a single cell is due to the mutual interference only.
\(^6\)To reduce the negotiation overhead, a negotiation phase can be initiated by the network administrator after a new contracted user joins or a current user terminates the service, or on a daily basis.

3) For each AP other than $k$, say $j$:
   a) If the $\gamma_j$ value changes, records the previous throughput into $D_{pre}$ and the current throughput into $D_{cur}$. Remains unchanged otherwise.
   b) Upon receiving the notifying message, a three-value vector of $[D_{pre}, D_{cur}, ID_j]$ is sent back to the $k$-th AP.
4) After receiving all the three-value vectors by counting the identifiers $ID_j$, the $k$-th AP computes the sum throughput before and after $f'$ and $p'$ are selected, which are denoted by $P_{pre}$ and $P_{cur}$.
5) For a smoothing factor $\tau > 0$, the $k$-th AP keeps $f'$ and $p'$ with probability
   \[
   e^{P_{pre}/\tau} = \frac{1}{1 + e^{(P_{pre} - P_{cur})/\tau}} \tag{7}
   \]
6) The $k$-th AP broadcasts another short notifying message, which indicates the end of updating process and a specific number $\delta$, to all the other APs.

- **Until:** The stopping criteria $\Gamma$ is met.

Note that in step 6, the specific format of $\delta$ depends on the predefined stopping criterion $\Gamma$. For example,

- If the stopping criterion is the maximum number of negotiation steps, $\delta$ is a counter which adds one after each updating process.
- If the stopping criterion is that no AP has updated for a certain number of steps, $\delta$ is a binary number where 1 means updating.
- If the stopping criterion is that the difference between sum throughput obtained in consecutive steps are less than a predefined threshold $\epsilon$, $\delta$ is the calculated sum throughput after each updating process.

We can have other stopping criteria $\Gamma$s and corresponding formats of $\delta$ as well.

The NETMA algorithm is inspired by the work in [24], where a similar algorithm was first introduced in the context of stream control in MIMO interference networks. The distinguishing feature of this type of negotiation algorithms, from the better response and the best response, is the randomness deliberately introduced on the decision making in step 5. The rationale can be illustrated in Figure 2 intuitively. If there is no randomness in decision making, i.e., $\tau = 0$, the four APs may get trapped at a low efficiency Nash equilibrium $1 - 2 - 2 - 1$. However, with the randomness caused by nonzero $\tau$, they may reach an intermediate state $1 - 2 - 2 - 2$ and arrive at the optimum Nash equilibrium $1 - 2 - 1 - 2$ eventually. Moreover, the updating rule in step 5 also implies that if $f'$ and $p'$ yield a better performance, i.e., $P_{pre} - P_{cur} < 0$, the $k$-th AP will keep them with high probability. Otherwise, it will change with high probability.

The steady state behavior of NETMA is characterized in the following theorem.

**Theorem 1:** NETMA converges to the optimal Nash equilibrium in CTMG with arbitrarily high probability.

**Proof:** The proof of Theorem 1 follows similar lines of the proof in [24] and [25].

First, we observe that the joint frequency-power negotiation generates an $N$-dimensional Markovian chain. Figure 3 illus-
the stationary distribution is

$$P_r^*(\tilde{S}) = \mathcal{K}e^{P(\tilde{S})/\tau}$$  \hspace{1cm} (11)

where $\mathcal{K}$ is a constant. By applying the probability conservation law [27] [26], we obtain the stationary distribution for the Markovian chain as

$$P_r^*(\tilde{S}) = \frac{e^{P(\tilde{S})/\tau}}{\sum_{S_i \in \Omega} e^{P(S_i)/\tau}}$$  \hspace{1cm} (12)

for arbitrary state $\tilde{S} \in \Omega$.

In addition, we observe that the Markovian chain is irreducible and aperiodic. Therefore, the stationary distribution given in (12) is valid and unique.

Let $S^*$ be the optimal state which yields the maximum value of potential function $P$, i.e.,

$$S^* = \text{argmax}_{S_i \in \Omega} P(S_i).$$  \hspace{1cm} (13)

From (12), we have

$$\lim_{\tau \to 0} P_r^*(S^*) = 1$$  \hspace{1cm} (14)

which substantiates that NETMA converges to the optimal state in probability.

Finally, the analogous analysis can be straightforwardly extended to an N-dimensional Markovian chain and thus completes the proof.

In NETMA, there is no central computational unit required. The joint frequency-power assignment is achieved by negotiations among cooperative APs and the maximum overall throughput is achieved with arbitrarily high probability. The autonomous behavior and decentralized implementation make NETMA suitable for large scale wireless mesh access networks. Moreover, NETMA has fast adaptability for the topology change of the wireless mesh access networks. NETMA does not depend on any rate adaption algorithms, nor on any underlying MAC protocols. In our simulation in Section VI, we use IEEE 802.11b as the MAC layer protocol. However, it can be easily extended to arbitrary MAC protocol with multi-rate multi-channel capability, such as IEEE 802.11a. In addition, even with the existence of exterior interference source, such as coexisting WLANs, NETMA works properly as well since the objective of NETMA is to maximize the overall throughput of the network in the current wireless environments. The tradeoff between algorithmic performance and convergence speed is controlled by parameter $\tau$ in step 5, where large $\tau$ represents extensive space search with slow convergence. On the contrary, small $\tau$ represents limited space search with fast convergence. Note that the smoothing factor $\tau$ here is analogous to the concept of temperature in simulated annealing [28]. Therefore, it is advisable that at the beginning period of the negotiation, the value of $\tau$ is set with a large number and keeps decreasing as the negotiation iterates. We choose $\tau = 10/k^2$ in our simulations, where $k$ denotes the negotiation step.

In step 1, we require that each AP updates with a probability of $1/m$. For example, we can utilize a random token mechanism where each updating AP randomly selects an AP as the next updating AP, i.e., passing the token. Note that in NETMA, even an erroneous operation happens, for example,
two APs update at the same time in our case, it only prolongs the convergence time for NETMA yet does not affect the final output of NETMA. This is because that such an error, as verified in [22] via extensive simulations, has no influence on the statistically monotonic-increasing tendency of the potential function.

IV. NON-COOPERATIVE ACCESS NETWORKS

In the previous section, we discuss the scenarios where all APs in the wireless mesh access network are cooperative, and the overall throughput is maximized by negotiations among autonomous APs using the NETMA mechanism. However, cooperation is not always attainable. Although the functionality of relaying packets for each other can be achieved by incentive mechanisms such as [29], the adjustable parameters inside each cell cannot be enforced and effectively controlled. The N APs may belong to distinct self-interested users and they care about exclusively their own throughput rather than the overall aggregated throughput. In other words, the utility function of each selfish user is

$$U_i = R_i^s(\gamma_i)$$

(15)

where $R_i^s$ is the throughput of the $i$-th cell, defined in (1). Analogous to CTMG, we can formulate the interaction among $N$ selfish APs as a Non-cooperative Throughput Maximizing Game (NTMG) where each AP is attempting to find the frequency-power pair which maximizes its own SINR value as well as the corresponding throughput. As in the cooperative case, each player’s utility function depends on the frequency and power of itself as well as those of others. However, NTMG is no longer an identical interest game.

**Lemma 2:** In NTMG, all the APs will transmit with the maximal power at the Nash equilibrium, if exists.

**Proof:** The proof of Lemma 2 is straightforward. For a single player, we have

$$\gamma_i = \frac{p_i g_{ii}}{\sum_{k \in F_i(f_i)} p_k g_{ki} + N_i}.$$  

(16)

where $g_{ij}$ is the channel gain from cell $i$’s transmitter to $j$’s receiver and $N_i$ is the Gaussian noise at the $i$’s receiver. $F_i(f_i)$ denotes the set of cells which operate at the same frequency $f_i$ other than cell $i$. Note that given other players’ strategies, $\gamma_i$ is a monotonic increasing function of $p_i$ and so is $U_i$. Assume at a Nash equilibrium of NTMG, the $k$-th AP has a power level of $p_k$ satisfying $0 \leq p_k < p_{max}$, where $p_{max}$ denotes the maximum power defined by MAC layer. The $k$-th AP is inclined to increase its power $p_k$ in order to yield a higher value of $U_i$, which contradicts the definition of Nash equilibrium. Thus, at the Nash equilibrium of NTMG, if exists, all the APs will operate at the same power level, i.e., $p_{max}$.

Based on Lemma 2, the NTMG can be viewed as a simplified game where each player has the same power and only adjusts the frequency to minimize the interference. Moreover, according to (15) (16) and the assumption of uniform environment, the NTMG is equivalent to the following simplified game where each player has the utility function\(^7\) as

$$U_i = -(\sum_{k \in F_i(f_i)} p_{max} g_{ki} + N_i)$$

(17)

and $U_i$ is a function of frequency assignment vector $f$ exclusively.

As in the cooperative case, the frequency selection among $N$ players is mutually dependent. The question arises that whether this frequency adjusting dynamic converges, or equivalently, whether NTMG has a Nash equilibrium. We provide the answer of this question in the following theorem.

**Theorem 2:** There exists at least one Nash equilibrium in NTMG.

The proof of Theorem 2 can be found in [30]. As shown in Lemma 2, at the equilibrium, the non-cooperative APs will always transmit at the maximum power level. This seems to be the best choice for each one of the APs. However, it is usually not a favorable strategy from a social-welfare point of view. To bridge the performance gap, we propose a linear pricing scheme to combat with the selfish behaviors, i.e., the players are forced to pay a tax proportional to the utilized resources. For example, we could impose a price to all selfish APs for the power they utilize. Hence for each AP, the utility function becomes

$$U_i = R_i^s(\gamma_i) - \lambda_i^p p_i$$

(18)

where $\lambda_i^p$ represents the power utilization price specific for the $i$-th AP and $p_i$ is its transmission power. Therefore, the more power AP uses, the more tax it has to pay. By imposing power prices properly, a more desirable equilibrium may be induced, from a social-welfare point of view. We define the corresponding game as a Non-cooperative Throughput Maximation Game with Pricing (NTMGP).

Let us first investigate the impact of prices on the behaviors of players. If $\lambda_i^p = 0$, where no price is imposed, the $i$-th AP will transmit at the the maximum power and causes extra interference to other APs. However, if we impose an unbearably high price, say $\lambda_i^p = \infty$, the AP would rather not to transmit at all. Based on these observations, we propose a heuristic linear pricing scheme to improve the overall throughput in non-cooperative wireless mesh access networks.

To enforce the scheme, we introduce a pricing dictator (PDU) into the network which determines the prices for all APs and informs them timely. In addition, we assume that the PDU has the monitoring capability and is aware of the operating frequencies of each cell. There are two prices charged by the PDU for each non-cooperative AP. Besides the power utilizing price $\lambda_i^p$, a frequency switching price $\lambda_f^i$ is imposed on the $i$-th AP whenever it changes the operating frequency. The price setting process is described as follows.

**Price setting process:**

**Phase I:**

- The PDU sets $\lambda_f^1 = \cdots \lambda_f^N = 0$ and $\lambda_i^p = \cdots \lambda_N^p = 0$ and all APs play NTMG until converges, i.e., a

\(^7\)The negative sign comes from the convention that utility functions are the ones to be maximized.
Nash equilibrium is reached.

The PDU collects the current throughput information from each cell, denoted by $M_i$, where $i$ is the index of the cell.

**Phase II:**

- The PDU sets $\lambda_i^1 = \cdots \lambda_i^N = \infty$.
- For each AP indexed by $i = 1, \cdots, N$:
  1) The PDU sets $\lambda_p^i = \infty$ for the $i$-th AP and let the APs play the NTMGP. Upon convergence, the PDU collects the overall throughput, say $V_i$, in the current price setting.
  2) Calculate the power utilizing price for the $i$-th AP as
    \[
    \lambda_p^i = \frac{V_i - \sum_{j=1, j \neq i}^N M_j}{p_{max}}
    \] (19)
  3) Reset $\lambda_p^i = 0$.

**Output:**

- Power utilizing price vector $\lambda_p = [\lambda_p^1, \cdots, \lambda_p^N]$
- Frequency switching price vector $\lambda_F = [\infty, \cdots, \infty]$.

In the price setting process above, the PDU imposes zero prices for all APs initially. As a consequence, all APs will transmit with $p_{max}$ at the equilibrium, as shown in Lemma 2. Upon convergence, the PDU fixes the frequency switching price to infinity which discourages the non-cooperative APs from switching channels thereafter. In (19), $\sum_{j=1, j \neq i}^N M_j$ is the sum throughput for all cells other than $i$, when the $i$-th AP transmits with the maximal power due to the zero power price. Similarly, $V_i$ is the sum throughput of other cells when the $i$-th AP is silent due to the unaffordable power price. Therefore, in (19), the power utilization price charged for the $i$-th AP, a.k.a., $\lambda_p^i$, can be viewed as a compensation to the impact it causes on the overall throughput of other cells. The more power it utilizes, the more severe it affects the other players and thus the more it pays, as illustrated in (18). Hence, by imposing taxes deliberately, the selfish behaviors of non-cooperative APs are effectively discouraged and a more desirable equilibrium can be induced, in term of overall throughput of the whole network. The proof of existence of Nash equilibrium in NTMGP is straightforward. Note that the utility function of (18) is quasi-concave with respect to power. The existence of pure strategy Nash equilibrium follows directly from the results of [31]. We will present the detailed performance evaluation of CTMG, NTMG and NTMGP in Section VI.

**V. AN EXTENSION TO ADAPTIVE CODING AND MODULATION CAPABLE DEVICES**

So far, we have assumed that the throughput of a cell is given in the form of (1), which is not a continuous function. To be precise, in both Section III and Section IV, we are confined to the traditional IEEE 802.11 family devices where the modulation and coding schemes are pre-determined and fixed. For example, Table I provides a mapping between SINR values and corresponding rates [32], the feasible data transmission rate is a discrete set and is usually much less than the theoretical channel capacity. We name such devices as legacy IEEE 802.11 devices.

However, thanks to the advance of coding techniques, the maximum data transmission rate can be largely closed to the theoretical Shannon capacity in AWGN channels [33]. Note that achieving this requires variable-rate transmissions by matching to the instantaneous SINR, which can be implemented in practice through adaptive coding and modulation (ACM) techniques [34]. This motivates us to extend our results to ACM-capable devices. More specifically, we are considering more advanced and powerful APs which attempt to tune the frequency and power in order to maximize

\[
C_i(\gamma_i) = W_i \log_2(1 + \gamma_i)
\] (20)

where $C_i$ is the Shannon capacity of the $i$-th cell, and $W_i$ is the bandwidth. Note that in this scenario, the maximum achievable transmission rate, as denoted by the Shannon capacity, is a continuous variable with respect to $\gamma_i$, rather than discrete, as exemplified in Table I. It is worth noting that the only difference in this scenario is the alternative objective function of each AP. Therefore, all the results we have obtained so far are extendable to this special scenario with merely a change of the objective function.

Furthermore, due to the continuity of the objective function in (20), the aforementioned heuristic pricing scheme in Section IV can be improved. Without loss of generality, we assume that all the cells have unity bandwidth. Restated, we assume that the PDU is a centralized device which knows the channel environment sufficiently by greedy acquiring. In addition, the PDU is assumed to have monitoring capability and is aware of the operating frequency of each cell. The tailored pricing scheme for this ACM-capable scenario is described as follows.

- First, the PDU sets $\lambda_i^1 = \cdots \lambda_i^N = 0$ and $\lambda_p^i = \cdots \lambda_p^N = 0$ and all APs play selfishly, until a Nash equilibrium is achieved.
- The PDU sets $\lambda_i^1 = \cdots \lambda_i^N = \infty$.
- For each AP indexed by $i = 1, \cdots, N$, the PDU sets

\[
\lambda_p^i = \left| \sum_{k \in F_i} \frac{\partial C_k}{\partial p_i} \right| = \left| \sum_{k \in F_i} - \frac{p_k \times g_{kk} \times g_{jk}}{\ln 2 \times (1 + \gamma_k) \times (\sum_{j \in F_k} p_j \times g_{jk} + N_k)^2} \right| = \left| \sum_{k \in F_i} \frac{p_k \times g_{kk} \times g_{jk}}{\ln 2 \times (1 + \gamma_i) \times (\frac{p_k \times g_{kk} \times g_{jk}}{\gamma_k})^2} \right|
\]

\footnotetext{8}{More specifically, in the cooperative case, we replace (2) with $U_{network}(p, f) = \sum_{i=1}^N C_i(\gamma_i)$ whereas in the non-cooperative case, (15) and (18) are replaced by $U_1 = C_i(\gamma_i)$ and $U_i = C_i(\gamma_i) - \lambda_p^i p_i$, respectively.}
Therefore, illustrated in the next section. The performance evaluation of this tailored pricing scheme is scenarios we considered in Section III and Section IV. We NETMA, NTMG and NTMGP with legacy devices, i.e., the power vector only local information and can be implemented in a distributed fashion. Pricing Scheme:

1) The PDU initially sets zero prices. As a consequence, all APs will converge at a Nash equilibrium with maximum power.
2) The PDU sets infinity for the frequency switching price which prevents the whole network from unstable frequency oscillations.
3) The PDU sets and announces the power prices for each AP according to (21).
4) Informed by the PDU, each AP optimizes its transmission power following (24).
5) Go back to step 3 until the iteration converges.

The performance evaluation of this tailored pricing scheme is illustrated in the next section.

VI. PERFORMANCE EVALUATION
A. Legacy IEEE 802.11 devices

In this subsection, we first investigate the performance of NETMA, NTMG and NTMGP with legacy devices, i.e., the scenarios we considered in Section III and Section IV. We assume the wireless mesh access network has N homogeneous APs. The other simulation parameters are summarized as follows.

- Each AP has a maximum power \( p_{\text{max}} = 100 \text{mW} \) and a minimum power \( p_{\text{min}} = 10 \text{mW} \) and 10 different power levels as \([10 \text{mW}, 20 \text{mW}, \cdots, 100 \text{mW}]\).
- The noise experienced at each receiver is assumed identical and has a power of \( 2nW \).
- All APs use IEEE 802.11b standard as the MAC protocol. In other words, each AP has four feasible data rate, \([1, 2, 5, 5, 11 \text{ Mbps}] \) and 3 non-overlapping channels, i.e., \( c = 3 \).
- Without loss of generality, we assume that the received power is inversely proportional to the square of the Euclidian distance.
- The smoothing factor \( \tau \) decreases as \( \tau = 10/k^2 \), where \( k \) is the negotiation step.
- The stopping criteria for NETMA and NTMG are the maximum number of iterations, denoted by \( \omega \).

For the sake of simplicity, we utilize a table-driven rate adaption algorithm provided in Table I. Note that our results can also be applied to arbitrary propagation models, rate adaptation algorithms and underlying multi-channel multi-rate MAC protocols.

1) Example of Small Networks: We first consider a small wireless mesh access network with 5 APs, i.e., \( N = 5 \). All APs are randomly located in a square of 10-by-10 area. The global optimum solution is obtained by enumerating all feasible strategies, i.e., \((3 \times 10)^5 \) profiles, as the performance benchmark. We first investigate the cooperative scenario where NETMA mechanism is applied. Next, the non-cooperative scenario is considered and each AP operates at the maximum power and adjusts the frequency only. The stopping criteria for both NETMA and NTMG are the maximum number of iterations where \( \omega = 200 \). The performance comparison is shown in Figure 4.

As indicated by the \( OP \) curve, the global optimum obtained by enumeration approach functions as the upper bound of the overall throughput. In Figure 4, we observe that NETMA gradually catches up with the global optimum as negotiations...
where the maximum number of iterations is set to $\omega = 1000$. As expected, the non-cooperative APs yield remarkably inferior performance in terms of overall throughput, depicted by the NTMG curve. The inefficiency is due to the selfish behavior that APs transmit at the maximum power and are regardless of the interference. The existence of Nash equilibrium in both CTMG and NTMG are substantiated by the convergence of curves in Figure 4. Figure 5 and Figure 6 depict the trajectories of frequency negotiations and power level negotiations in NETMA, respectively. At the initialization, each AP randomly picks a frequency and a power level and negotiates with each other following NETMA mechanism, until the optimum Nash equilibrium is achieved. Note that when the frequency vector and power vector converge in Figure 5 and Figure 6, the corresponding overall throughput obtained by NETMA catches the global optimum in Figure 4 simultaneously.

2) Example of Large Networks: We now consider a large wireless mesh access network with 20 APs. The enumeration approach is no longer feasible in this scenario due to the enormous strategy space. The 20 APs are randomly scattered in a $d$-by-$d$ square, where the side length $d$ is a tunable parameter in simulations. We investigate both cooperative and non-cooperative cases represented by NETMA and NTMG curves, where the maximum number of iterations is set to $\omega = 1000$.

Figure 7 pictorially depicts the performance inefficiency of NTMG caused by the non-cooperative APs which transmit at the maximum power. The average throughput per AP is calculated by averaging the results of 50 simulations, for each value of the side length $d$. In Figure 7, it is worth noting that as the side length $d$ gets bigger, the performance gap between NETMA and NTMG reduces. The reason is that when the area is large, the impact of mutual interference is less severe and so is the performance deterioration. However, when the network is crowded, i.e., $d$ is small, the selfish behaviors are remarkably devastating.

To alleviate the throughput degradation by the non-cooperative APs, we implement the linear pricing scheme introduced in Section IV. The throughput improvement is illustrated as NTMGP in Figure 7. It is noticeable that by utilizing the proposed pricing scheme, the efficiency of Nash equilibrium is dramatically enhanced, especially for crowded networks. Therefore, the selfish incentives of the non-cooperative APs have been effectively suppressed.

B. ACM-capable devices

In this subsection, we investigate the performance of our model with ACM-capable devices. Since the major difference lies in the improved pricing scheme tailored for this specific scenario, we will only provide the performance evaluation of the tailored pricing scheme, i.e., NTMGP, in order to avoid duplicate results. We consider a populated network where 30 APs are randomly scattered in an $100m$-by-$100m$ square, i.e., $N = 30$. All other simulation parameters are the same as in the previous subsection except that the power is a continuous variable in this scenario. We first investigate the performance in terms of overall achievable rate of the network when no pricing scheme is applied, as a performance benchmark. Afterwards, the tailored pricing scheme is implemented to improve the equilibrium efficiency, a.k.a., the overall performance at the equilibrium. As shown in Figure 8, the overall achievable rate of the network is dramatically improved by the pricing scheme. The PDU adapts the announced price for each AP according to the optimum power, which is calculated by the previous announced price, in an iterative fashion.
The aggregated achievable rate converges as the price setting iteration goes, as depicted by Figure 8.

Figure 9 shows the trajectories of the power utilization prices for each AP. The counterpart of actual utilized power is illustrated in Figure 10, where the curves start at \( p_{\text{max}} \) and evolve with the announced price in Figure 9.

As observed in Figure 8, to achieve a better performance, the price setting process needs to be executed iteratively, comparing with the “one shot” heuristic pricing scheme proposed for legacy devices. Therefore, the further improvement of the equilibrium efficiency is achieved as a tradeoff of communication overheads. However, it is worth noting that even at the first iteration, where the prices are determined by \( p_{\text{max}} \), the induced equilibrium yields remarkable superior performance than the case where no pricing scheme is applied.

**VII. CONCLUSIONS**

In this paper, we investigate the throughput maximization problem in wireless mesh networks. The problem is coupled due to the mutual interference and hence challenging. We first consider a cooperative case where all APs collaborate with each other in order to maximize the overall throughput of the network. A negotiation-based throughput maximization algorithm, a.k.a., NETMA, is introduced. We prove that NETMA converges to the optimum solution with arbitrarily high probability. For the non-cooperative scenarios, we show the existence and the inefficiency of Nash equilibria due to the selfish behaviors. To bridge the performance gap, we propose a pricing scheme which tremendously improves the performance in terms of overall throughput. The analytical results are verified by simulations. In addition, we extend our model and analytical results to the scenarios where more advanced APs are utilized, i.e., the devices with the adaptive coding and modulation capability. In this scenario, we propose a tailored pricing scheme which remarkably improves the overall performance in an iterative fashion.

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