A New Mobility Model and its Application in the Channel Holding Time Characterization in PCS Networks

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Abstract—**In order to capture the essence of PCS network behavior, a good mobility model is necessary. This model must be good enough to fit field data and make the resulting queueing model still tractable. In this paper, we propose a new mobility model, called *hyper-Erlang distribution model*, which satisfies the above two requirements. We use this model to characterize the cell residence time and obtain analytical results for the channel holding time, the distribution of which is of primary importance in teletraffic analysis of PCS networks. These results can be used to facilitate the computation together with the use of the partial fractional expansion technique. It is expected that the mobility model and the analytical results for channel holding time presented in this paper will play an significant role for field data processing in PCS network design and performance evaluation.**

Keywords—**PCS, Mobility, Call holding time, Cell residence time, Channel holding time.**

I. Introduction

Channel holding (occupancy) time is an important quantity in teletraffic analysis for PCS networks. This quantity is used to find important design parameters such as the new call blocking probability and the handoff blocking probability ([12]). Bolotin ([1]) studied the CCS (Common-Channel Signaling) systems and found that channel throughput drops significantly more under the exponential call holding time distribution model than under the actual measured call holding time distribution. This observation is expected to be true for PCS networks. Thus it is important to realistically characterize channel holding time in PCS networks and investigate how its distribution affects PCS network traffic. In order to accomplish this, we need to have an appropriate mobility model to reflect the actual traffic situation and the mobility of users.

Most traffic analyses make the assumption that the channel holding time distribution is distributed exponentially ([5], [6], [12], [20], [28], [29]). However, using a simulation model, Guerin ([11]) demonstrated that when the rate of change of direction is "low", the channel holding time is no longer exponentially distributed. Bolotin ([1]) showed that the channel holding time for CCS (Common Channel Signaling) networks is no longer exponentially distributed and showed that a mixture of well-known distributions provides a better model for the channel holding time. Lin et al ([20]) gave a condition under which the channel holding time is exponentially distributed. Namely, they showed that the cell residence time needs to be exponentially distributed and suggested that the cell residence time be modeled directly as a random variable. Fang et al ([8]) presented a necessary and sufficient condition under which the channel holding time is exponentially distributed, showed the limitation of exponential distribution modeling. Recently, Jedrzycki and Leung ([13]) observed through the field data that channel holding time distribution for cellular telephony systems is not exponential, and statistically showed that the lognormal distribution is a better model for the (filtered) data of measurements for channel holding time. Orlik and Rappaport ([24]) observed that the data profile used in [13] can also be approximately modeled by the SOHYP (Sum of Hyper-exponential) distribution. In summary, the above research showed that the exponential assumption for channel holding time is not appropriate.

In this paper we deal with channel holding time (i.e., the time a call spent in a cell) under realistic assumptions. We observe that the channel holding time depends on the users' mobility, which in turn can be characterized by the cell residence time (the time that a mobile user stays in a cell, or dwell time). Thus, in order to appropriately characterize the channel holding time, it is necessary to have an appropriate distribution model for the cell residence time to reflect the mobility of the users consistent with field data. One approach to modeling the cell residence time can be had by assuming specific (hexagon or circle) shapes of a cell, combined with specific distributions of speed and moving direction of a mobile user to determine the probability distribution of cell residence time ([12], [6]). However, in practical systems, cell shapes are irregular, and the speed and direction of mobile users may be hard to characterize, it is more appropriate to directly model the cell residence time as a random variable. Zonoozi and Dassanayake ([30]) used the generalized Gamma distribution to model the cell residence time. Unfortunately, the generalized Gamma distribution leads to the loss of Markovian property in the resulting queueing model of the cellular networks, which makes the traffic analysis difficult. Orlik and Rappaport ([24]) modeled the cell residence time as a SOHYP random variable. The advantage of using the SOHYP distribution is the guarantee of the Markovian property of the queueing network model. It was shown ([25]) that SOHYP models can be tuned to have the coefficient of variation (the ratio of the standard deviation to the mean) of the cell residence time less

This work was supported in part by the U.S. Army Research Office under Contract DAAG55-97-1-0312 and DAAG55-97-1-0382. The work of Yuguang Fang is also suppoted, in part, by the New Jersey Institute of Technology under Grant No. SBR421980.

than, equal to and greater than unity, while the exponential (even Erlang) distribution model for cell residence time can only apply to the cases where the coefficient of variation is less than unity. However, it is not known whether the SOHYP models have the capability of universal approximations. We need a more general distribution model which has the universal approximation capability.

In this work we present a new general distribution model for the cell residence time, what we call *hyper-Erlang distribution model.* The hyper-Erlang distribution preserves the Markovian property of the resulting queueing networks models ([17]) and has universal approximation properties so field data can be readily used to find the model parameters statistically. It is observed that the well-known distributions such as exponential distribution, Erlang distribution and the hyper-Exponential distribution are special cases of hyper-Erlang distribution models. It can be shown that the coefficient of the hyper-Erlang distribution model can be tuned to be less than, equal to and greater than the unity. We also observe that the hyper-Erlang model has the attractive property of having rational Laplace transforms, hence preserves the Markovian property of the resulting queueing networks ([17]), so that the multi-dimensional Markov chain theory can be applied to find all teletraffic parameters such as the call blocking probabilities.

In this paper we first discuss the hyper-Erlang distribution model and its universal approximation capability, then we derive formulae for the distribution of channel holding time with general cell residence time distributions, and give an easy-to-compute procedure when the cell residence time has rational Laplace transform, in particular for the hyper-Erlang models. Using our results, we show analytically that when the cell residence time is not exponentially distributed, the channel holding time is indeed not exponentially distributed. Surprisingly, a counter-intuitive result is observed: the low variance of the cell residence time leads to the invalidity of the exponential assumption for channel holding time. We observe that while for cellular networks the cell size is large, hence the variance of the cell residence time is high, the exponential assumption for channel holding time may be appropriate for some cellular networks. For the PCS networks, the cell size becomes much smaller, the variance of the cell residence time will be lower, hence the exponential assumption is not valid anymore. Therefore, if the field data for cell residence time shows the low variation, we cannot use the exponential assumption for the channel holding time in the teletraffic analysis for PCS networks. In this instance, our analytical results can be conveniently used to characterize the channel holding time. Another observation made in our work ([8]) shows that when the cell residence time is not exponentially distributed, the handoff traffic is no longer Poisson. This destroys the Poisson property of the cell traffic as well, which invalidates most analytical results such as the Erlang-B formula in telephony and the product-form solution in queueing systems ([17]). With the mobility model for the cell residence time and the analytical results for the channel holding time, we can study the queueing system for the PCS network using the multi-dimensional Markov chain models as illustrated in [17], in this way we can investigate the validity of several classical analytical results in traffic theory for PCS systems. This paper takes the first step towards this goal.

This paper is organized as follows. In the next section, we present the hyper-Erlang distribution model and its approximation capability. We then derive in the third section the analytical formulae for the computation of channel holding time distributions (including the conditional channel holding time distributions) for general cell residence time. In the fourth section, we show how the variance of cell residence time affects the channel holding time distribution. Conclusions are given in the last section.

II. Hyper-Erlang Distribution Model

As we mentioned in the previous section, the cell residence time can be used to characterize the users' mobility. We observe that we can treat the cell residence time as a nonnegative random variable, hence a good distribution model for the random variable will be enough to characterize the users' mobility. In this section, we discuss such a good model, the *hyper-Erlang distribution model.*

The *hyper-Erlang* distribution has the following density function and Laplace transform:

$$\begin{aligned} f_{he}(t) &= \sum_{i=1}^{M} \alpha_i \frac{(m_i\eta_i)^{m_i} t^{m_i-1}}{(m_i-1)!} e^{-m_i\eta_i t} \quad (t \geq 0), \\ f_{he}^*(s) &= \sum_{i=1}^{M} \alpha_i \left(\frac{m_i\eta_i}{s+m_i\eta_i}\right)^{m_i}, \end{aligned} \tag{1}$$

where

$$\alpha_i \geq 0, \quad \sum_{i=1}^{M} \alpha_i = 1,$$

and $M, m_1, m_2, \ldots, m_M$ are nonnegative integers, $\eta_1, \eta_2, \ldots, \eta_M$ are positive numbers.

We next show that these distribution functions provide sufficiently general models, i.e., hyper-Erlang distributions are universal approximators. This can be accomplished by the following result (in what follows we will use star $*$ to denote the Laplace transformation):

Lemma: ([16]) Let $G(t)$ be the cumulative distribution function of a positive random variable. Then it is possible to choose a sequence of distribution functions $G_m(t)$, each of which corresponds to a mixture of Erlang distributions, so that

$$\lim_{m\to\infty} G_m(t) = G(t)$$

for all t at which $G(t)$ is continuous. In particular, $G_m(t)$ can be chosen as

$$G_m(t) = \sum_{k=1}^{\infty} \left[G\left(\frac{k}{m}\right) - G\left(\frac{k-1}{m}\right)\right] G_m^k(t), \; t \geq 0$$

where $G_m^k(t)$ is the distribution function of an Erlang distribution with mean $\frac{k}{m}$ and variance $\frac{k}{m^2}$ (i.e. the distribution of the sum of k exponential random variables each with mean $1/m$). □

Let $g_m(t)$ and $g_m^*(s)$ denote the density function and Laplace transform of $G_m(t)$, and $g_m^k(t)$ denotes the density function of $G_m^k(t)$, then we have

$$g_m(t) = \sum_{k=1}^{\infty} \left[G\left(\frac{k}{m}\right) - G\left(\frac{k-1}{m}\right)\right] g_m^k(t),$$

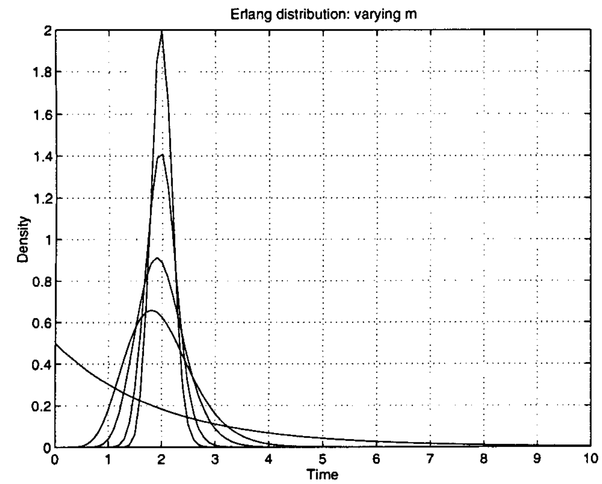

Fig. 1. Erlang distribution

$$g_m^*(s) = \sum_{k=1}^{\infty} \left[G\left(\frac{k}{m}\right) - G\left(\frac{k-1}{m}\right) \right] \left(\frac{k/m}{s+k/m} \right)^k . \tag{2}$$

The resulting distribution is called the mixed Erlang distribution. The advantage using this distribution is that the coefficients can be determined from the experimental data. We can use a finite number of terms to approximate the distribution function, in this case, the resulting distribution approximates the hyper-Erlang distribution.

In fact, we can intuitively illustrate from the Sampling Theorem ([26]) why the distribution $G_m(t)$ provides the universal approximation to general distribution models. Figure 1 shows the density function by varying the shape parameter m (see (25)). We observe that as the shape parameter m becomes sufficiently large, the density function approaches the Dirac δ function. From signal processing theory ([14]) we know that the δ function can be used to sample a function and reconstruct the function from the sampled data (the Sampling Theorem). We can replace the δ function by the Erlang density functions with sufficiently large shape parameters, the resulting approximation is exactly in the form of the hyper-Erlang distribution.

We remark that the hyper-Erlang distribution model is much easier to use than the other models. Let t be the generic form for the cell residence time t_i. If t is modeled by the hyper-Erlang distribution as in equation (1), we can easily find its kth moment given below

$$E[t^k] = (-1)^k f_{he}^{*(k)}(0) = \sum_{i=1}^{M} \alpha_i \frac{(m_i+k-1)!}{(m_i-1)!} (m_i\eta_i)^{-k}.$$

The parameters α_i, m_i and η_i ($i = 1, 2, \ldots, M$) can be found by fitting a number of moments from field data. Moreover, if the number of moments exceeds the number of variables, then the least-square method can be used to find the best fit to minimize the least square error, more importantly, the error function is smooth.

III. Channel Holding Time

Channel holding time distribution depends on the mobility of users, which can be characterized by the cell residence time. As

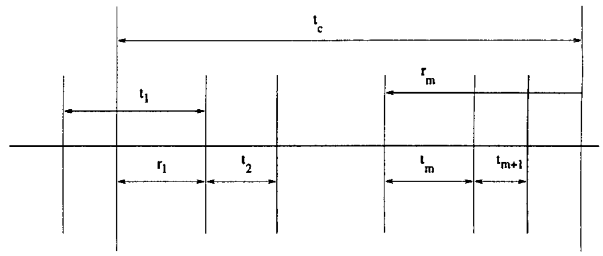

Fig. 2. The time diagram for call holding time and cell residence time

the assumption that the cell residence time is exponentially distributed is too restrictive for real world systems, we wish to relax this assumption. In this section, we will study the channel holding time under the condition that the cell residence time is generally distributed, in particular, hyper-Erlang distributed.

Let the call holding time t_c (i.e., the unencumbered call holding time of requested connection to a PCS network for a new call, as in wireline telephony) be exponentially distributed with parameter μ. Let t_m be the cell residence time, r be the residual life of the cell residence time (i.e., the time between the instant a new call is initiated and the instant the new call moves out of the cell if the new call is not completed), and let r_m ($m > 1$) be the residual life time distribution of call holding time when the call finishes $m-$th handoff successfully (i.e., the time between the instant that the $m-$th handoff is done and the end of the call life). Let t_{nh} and t_{hh} denote the channel holding times for a new call and a handoff call, respectively. Then, from Figure 2, the channel holding time for a new call will be

$$t_{nh} = \min\{t_c, r\}, \tag{3}$$

and the channel holding time for a handoff call is

$$t_{hh} = \min\{r_m, t_m\}. \tag{4}$$

Let t_c, t_m, r, t_{hh} and t_{nh} have density functions $f_c(t)$, $f(t)$, $f_r(t)$, $f_{hh}(t)$ and $f_{nh}(t)$ with their corresponding Laplace transforms $f_c^*(s)$, $f^*(s)$, $f_r^*(s)$, $f_{hh}^*(s)$ and $f_{nh}^*(s)$, respectively, and with cumulative distribution functions $F_c(t)$, $F(t)$, $F_r(t)$, $F_{hh}(t)$ and $F_{nh}(t)$, respectively.

From (4), the independency of r_m and t_m, and the strong memoryless property of the exponential distribution of t_c (See Appendix), we can obtain

$$\begin{aligned} f_{hh}(t) &= f_c(t) + f(t) - f_c(t)\Pr(t_m \le t) - \Pr(t_c \le t) f(t) \\ &= f_c(t) \int_t^{\infty} f(\tau) d\tau + f(t) \int_t^{\infty} f_c(\tau) d\tau. \end{aligned} \tag{5}$$

From (5), applying Laplace transform, we obtain

$$\begin{aligned} f_{hh}^*(s) &= f^*(s) + f_c^*(s) - \int_0^{\infty} e^{-st} f(t) \int_0^t f_c(\tau) d\tau dt \\ &\quad - \int_0^{\infty} e^{-st} f_c(t) \int_0^t f(\tau) d\tau dt \\ &= f^*(s) + f_c^*(s) - \mu \int_0^{\infty} e^{-(s+\mu)t} \int_0^t f(\tau) d\tau dt \end{aligned}$$

$$
\begin{aligned}
& - \int_0^{\infty} e^{-st}(1 - e^{-\mu t}) f(t) dt \\
= \ & \frac{\mu}{s+\mu} + \frac{s}{s+\mu} f^*(s+\mu).
\end{aligned}
$$

From this, it is easy to obtain that the expected handoff call channel holding time is given by (we will use $h^{(i)}(x)$ to denote the ith derivative of any function $h(x)$ at point x in the subsequent development)

$$E[t_{hh}] = -f_{hh}^{*(1)}(0) = \frac{1}{\mu}(1 - f^*(\mu)).$$

From (3) and a similar argument, we obtain

$$f_{nh}(t) = f_c(t) \int_t^{\infty} f_r(\tau) d\tau + f_r(t) \int_t^{\infty} f_c(\tau) d\tau.$$

Applying Laplace transform, we have

$$f_{nh}^*(s) = \frac{\mu}{s+\mu} + \frac{s}{s+\mu} f_r^*(s+\mu), \tag{6}$$

and the expected new call channel holding time is

$$E[t_{nh}] = -f_{nh}^{*(1)}(0) = \frac{1}{\mu}[1 - f_r^*(\mu)].$$

In the preceding discussion we differentiate new calls from handoff calls when considering the channel holding times. If such distinction is not made, we need to consider the channel holding time distribution for any call (either new call or handoff call), i.e., the channel holding time for the merged traffic of new calls and handoff calls, as used in current literature. We will simply call this the channel holding time without any modifiers such as new call or handoff call. Let t_h denote the channel holding time, λ_h the handoff call arrival rate and λ the new call arrival rate. Then, it is easy to show that $t_h = t_{nh}$ with probability $\lambda/(\lambda + \lambda_h)$ and $t_h = t_{hh}$ with probability $\lambda_h/(\lambda + \lambda_h)$. Let $f_h(t)$ and $f_h^*(s)$ be its density function and the corresponding Laplace transform. It is easy to obtain

$$
\begin{aligned}
f_h(t) &= \frac{\lambda}{\lambda+\lambda_h} f_{nh}(t) + \frac{\lambda_h}{\lambda+\lambda_h} f_{hh}(t), \\
f_h^*(s) &= \frac{\lambda}{\lambda+\lambda_h} f_{nh}^*(s) + \frac{\lambda_h}{\lambda+\lambda_h} f_{hh}^*(s),
\end{aligned} \tag{7}
$$

Summarizing the above we arrive at:

Theorem 1: For a PCS network with exponential call holding time and Poisson new call arrivals with arrival rate λ, we have:

(1) the Laplace transform of the density function of the new call channel holding time is given by

$$f_{nh}^*(s) = \frac{\mu}{s+\mu} + \frac{s}{s+\mu} f_r^*(s+\mu), \tag{8}$$

and the expected new call channel holding time is

$$E[t_{nh}] = -f_{nh}^{*(1)}(0) = \frac{1}{\mu}[1 - f_r^*(\mu)]. \tag{9}$$

(2) the Laplace transform of the density function of the handoff call channel holding time is given by

$$f_{hh}^*(s) = \frac{\mu}{s+\mu} + \frac{s}{s+\mu} f^*(s+\mu), \tag{10}$$

and the expected handoff call channel holding time is

$$E[t_{hh}] = \frac{1}{\mu}(1 - f^*(\mu)). \tag{11}$$

(3) let λ and λ_h denote the new call arrival rate and the handoff call arrival rate, respectively, then the Laplace transform of the density function of channel holding time is given by

$$f_h^*(s) = \frac{\lambda}{\lambda+\lambda_h} f_{nh}^*(s) + \frac{\lambda_h}{\lambda+\lambda_h} f_{hh}^*(s). \tag{12}$$

and the expected channel holding time is given by

$$E[t_h] = \frac{\lambda}{\mu(\lambda+\lambda_h)}(1 - f_r^*(\mu)) + \frac{\lambda_h}{\mu(\lambda+\lambda_h)}(1 - f^*(\mu)). \tag{13}$$

□

When the residual life r is exponentially distributed with parameter μ_r, then its Laplace transform $f_r^*(s)$ is $\mu_r/(s + \mu_r)$. Taking this into (8), we obtain

$$f_{nh}^*(s) = \frac{\mu}{s+\mu} + \frac{\mu_r s}{(s+\mu)(s+\mu+\mu_r)} = \frac{\mu+\mu_r}{s+\mu+\mu_r},$$

which implies that the new call channel holding time is exponentially distributed with parameter $\mu + \mu_r$. Similarly, if the cell residence time t_i is exponentially distributed with parameter η, then the handoff call channel holding time is also exponentially distributed with parameter $\mu + \eta$. In this case, the channel holding time is hyper-exponentially distributed. If $\mu_r = \eta$, then the channel holding time (see (12)) is exponentially distributed with parameter $\mu + \eta$. In fact, since r is the residual life of t_1, from the Residual Life Theorem ([17]), we have

$$f_r^*(s) = \frac{\eta[1 - f^*(s)]}{s} = \frac{\eta}{s+\eta} = f^*(s),$$

hence the channel holding time is exponentially distributed with parameter $\mu + \eta$ when the cell residence time is exponentially distributed.

Based on the field data collected for channel holding time, Jedrzycki and Leung ([13]) demonstrated that the channel holding time is not exponentially distributed, that the log-normal distribution provides a satisfactory approximation after the spikes of data are removed (the spikes correspond to the handoff calls). Orlik and Rappaport ([24]) interpreted the distribution reported in ([13]) as the *conditional distribution given that the call completes in its current cell*, and derived the results for conditional distributions for the channel holding time when the cell residence time is SOHYP distributed. We adopt a different approach and use more general distribution model for cell residence time. Simple results for the conditional distributions for channel holding time when the cell residence time is generally distributed are presented next.

Let $f_{cnh}(t)$, $f_{chh}(t)$ and $f_{ch}(t)$ denote the conditional density functions for the new call channel holding time, for the handoff call channel holding time and for the (absolute) channel holding time (i.e., without distinguishing the new calls and handoff calls), respectively, with Laplace transforms $f_{cnh}^*(s)$, $f_{chh}^*(s)$ and $f_{ch}^*(s)$, and with cumulative distribution functions $F_{cnh}(t)$, $F_{chh}(t)$ and $F_{ch}(t)$.

We first study the conditional distribution for the handoff call channel holding time. We have

$$\begin{aligned} F_{chh}(h) &= \Pr(t_{hh} \le h | r_m \le t_m) = \frac{\Pr(r_m \le h,\, r_m \le t_m)}{\Pr(r_m \le t_m)} \\ &= \frac{\int_0^h f_c(t) \int_t^\infty f(\tau) d\tau dt}{\Pr(r_m \le t_m)} = \frac{\int_0^h f_c(t)[1 - F(t)]dt}{\Pr(r_m \le t_m)}, \end{aligned}$$

Differentiating both sides, we obtain the conditional density function

$$f_{chh}(h) = \frac{f_c(h)[1 - F(h)]}{\Pr(r_m \le t_m)}. \tag{14}$$

We observe that

$$\begin{aligned} \Pr(r_m \le t_m) &= \int_0^\infty \int_0^t f(t) f_c(\tau) d\tau dt \\ = &= \int_0^\infty f(t)[1 - e^{\mu t}]dt1 - \int_0^\infty f(t) e^{-\mu t} dt = 1 - f^*(\mu). \end{aligned}$$

Taking this into (14), we obtain

$$f_{chh}(h) = \frac{[1 - F(h)]\mu e^{-\mu h}}{1 - f^*(\mu)},$$

hence

$$f^*_{chh}(s) = \frac{\mu}{s + \mu} \cdot \frac{1 - f^*(s + \mu)}{1 - f^*(\mu)}. \tag{15}$$

In a similar fashion, we obtain the following result for the new call channel holding time:

$$\begin{aligned} f_{cnh}(h) &= \frac{[1 - F_r(h)]\mu e^{-\mu h}}{1 - f_r^*(\mu)}, \\ f^*_{cnh}(s) &= \frac{\mu}{s + \mu} \cdot \frac{1 - f_r^*(s + \mu)}{1 - f_r^*(\mu)}. \end{aligned}$$

The conditional channel holding time distribution $f_{ch}(t)$ ($f^*_{ch}(s)$) is the average of the conditional new call channel holding time distribution and handoff call channel holding time distribution. In summary, we therefore have

Theorem 2: For a PCS network with exponential call holding time and Poisson call arrivals, the conditional distributions for the new call channel holding time, the handoff call channel holding time and the channel holding time can be characterized by their Laplace transforms as follows:

$$f^*_{cnh}(s) = \frac{\mu}{s + \mu} \cdot \frac{1 - f_r^*(s + \mu)}{1 - f_r^*(\mu)}, \tag{16}$$

$$f^*_{chh}(s) = \frac{\mu}{s + \mu} \cdot \frac{1 - f^*(s + \mu)}{1 - f^*(\mu)}, \tag{17}$$

$$\begin{aligned} f^*_{ch}(s) &= \frac{\mu}{s + \mu} \left(\frac{\lambda}{\lambda + \lambda_h} \cdot \frac{1 - f_r^*(s + \mu)}{1 - f_r^*(\mu)} \right. \\ &\quad \left. + \frac{\lambda_h}{\lambda + \lambda_h} \cdot \frac{1 - f^*(s + \mu)}{1 - f^*(\mu)} \right). \end{aligned} \tag{18}$$

Let T_{cnh}, T_{chh} and T_{ch} denote the expected conditional new call channel holding time, the expected conditional handoff call channel holding time and the expected conditional channel holding time, respectively, then we have

$$T_{cnh} = \frac{1}{\mu} + \frac{f_r^{*(1)}(\mu)}{1 - f_r^*(\mu)} \tag{19}$$

$$T_{chh} = \frac{1}{\mu} + \frac{f^{*(1)}(\mu)}{1 - f^*(\mu)} \tag{20}$$

$$\begin{aligned} T_{ch} &= \frac{1}{\mu} + \frac{\lambda}{\lambda + \lambda_h} \cdot \frac{f_r^{*(1)}(\mu)}{1 - f_r^*(\mu)} \\ &\quad + \frac{\lambda_h}{\lambda + \lambda_h} \cdot \frac{f^{*(1)}(\mu)}{1 - f^*(\mu)}. \end{aligned} \tag{21}$$

□

When the residual life r is exponentially distributed with parameter μ_r, from (16) we obtain $f^*_{cnh}(s) = (\mu + \mu_r)/(s + \mu + \mu_r)$, hence the conditional new call channel holding time is also exponentially distributed. Moreover, this holding time has the same distribution as the unconditional new call channel holding time, this contributes to the memoryless property of the exponential distribution. Similarly, the handoff call channel holding time is also exponentially distributed if the cell residence time is exponentially distributed.

To apply the results above, i.e. Theorem 1 and 2, it remains to determine the handoff call arrival rate λ_h. This parameter depends on the new call arrival rate, the new call blocking probability and handoff call blocking probability. Let p_o and p_f denote the new call and handoff call blocking probabilities, respectively, let H be the number of handoffs for a call. The expectation $E[H]$ is also called the handoff rate. Using a procedure similar to the one in [8] or [9], we obtain

$$E[H] = \frac{(1 - p_o) f_r^*(\mu)}{1 - (1 - p_f) f^*(\mu)}. \tag{22}$$

Since each unblocked call initiates $E[H]$ handoff calls on the average, hence the handoff call arrival rate can be obtained

$$\lambda_h = \lambda E[H] = \frac{(1 - p_o)\lambda f_r^*(\mu)}{1 - (1 - p_f) f^*(\mu)}. \tag{23}$$

We observe from the above discussion that as long as $f_r^*(s)$ and $f^*(s)$ are proper rational functions, then the Laplace transforms of distribution functions of all channel holding times (either conditional or unconditional) are all rational functions (see Theorem 1 and Theorem 2). To find the corresponding density functions, we only need to find the inverse Laplace transforms. This can be accomplished by using the partial fraction expansion ([14]). To illustrate the idea, we present the following procedure. Suppose that $g(s)$ is a proper rational function with poles $p_1, p_2, \ldots, p_k$ with multiplicities $n_1, n_2, \ldots, n_k$. Then $g(s)$ can be expanded as

$$g(s) = \sum_{i=1}^{k} \sum_{j=0}^{n_i} A_{ij} \frac{s^j}{(s + p_i)^{n_i}}, \tag{24}$$

where the constants A_{ij} can be found easily by the formula

$$A_{ij} = \frac{d^j}{ds^j}[(s + p_i)^{n_i} g(s)] \bigg|_{s=0}, \; j = 0, 1, \ldots, i, \; i = 1, 2, \ldots, k.$$

Notice the relationship ($\mathcal{L}^{-1}$ denotes the inverse Laplace transform operator)

$$\mathcal{L}^{-1}[s^j f(s)] = \frac{d^j}{dt^j}\{\mathcal{L}^{-1}[f(s)]\},\ \mathcal{L}^{-1}[1/(s+\beta)^h] = \frac{t^h}{h!}e^{-\beta t}.$$

Taking this into (24), we obtain the inverse Laplace transform

$$\mathcal{L}^{-1}[g(s)] = \sum_{i=1}^{k}\sum_{j=0}^{m_i} A_{ij}\frac{d^j}{dt^j}\left(\frac{t^j}{j!}e^{-p_i t}\right).$$

We also notice that the inverse Laplace transform of a rational function is in fact the impulse response of linear system in which the rational function is the system transfer function of the resulting linear system ([14]), and the cumulative distribution function is the step response of the linear system. By studying the impulse response and step response of the linear system, we can characterize the properties of the channel holding time. Several ready-to-use software packages for the study of the impulse response and step response in signals and systems ([14]) are available. In the well-known software package Matlab, the commands *impulse* and *step* can be used to find the density function and the distribution function.

When we apply the hyper-Erlang distribution model for cell residence time, we can in fact reduce the computation further. As an example, we use Theorem 1 (2) to illustrate this point. If we substitute $f^*(s)$ with $f^*_{he}(s)$, we obtain

$$\begin{aligned} f^*_{hh}(s) &= \sum_{i=1}^{M}\alpha_i\left[\frac{\mu}{s+\mu}+\frac{s}{s+\mu}\left(\frac{m_i\eta_i}{s+m_i\eta_i}\right)^{m_i}\right] \\ &= \sum_{i=1}^{M}\alpha_i f^*_e(s;m_i,\eta_i), \end{aligned}$$

where $f^*_e(s;m_i,\eta_i)$ corresponds to the handoff call channel holding time when the cell residence time is Erlang distributed with parameters (m_i,η_i). Thus, we can concentrate on finding the algorithm for computing the channel holding time for the case when the cell residence time is Erlang distributed.

As a final remark in this section, we illustrate the relationship between $f^*_r(s)$ and $f^*(s)$. If we are interested in all calls for a long run (i.e., we have large samples for cell residence time), the residual life time r can be regarded as the residual life of the cell residence time because it is the time that a mobile user spends in the initiating cell (where the call is made). From the Residual Life Theorem ([17]), we obtain

$$f^*_r(s) = \frac{\eta[1-f^*(s)]}{s},$$

where $1/\eta$ is the average cell residence time. If we only have small number of samples for cell residence time, then the Residual Life Theorem may not be appropriate ([17]), we can only use the best distribution fit for r from the available samples. In this research, we can regard the cell residence time sequence $r, t_2, t_3, \cdots$ as a renewal process ([4]).

IV. Effect of Distribution of the Cell Residence Time on the Channel Holding Time Distribution

It is well-known that the exponential distribution is completely determined by a single parameter, i.e., the average value. Thus, if we use exponential distribution to model the cell residence time for the field trials, the fitted distribution is completely determined by the average value of the field data. Therefore, this model hardly captures the variation of the cell residence time for a mobile user. In this case, the resulting channel holding time, which is also exponentially distributed, is also completely determined by the average channel holding time (or the average cell residence time). In real situation, however, a mobile user's cell residence time significantly deviates from the average value from time to time and from cell to cell. It is important to understand how distribution of the cell residence time affects the channel holding time distribution. One statistical quantity to characterize the deviation of the field data from the average value is the variance. In fact, the variance of the cell residence time is one of the reasons why the channel holding time is not exponentially distributed when the cell residence time is not exponentially distributed.

In this section, we use our analytical results to study a few examples and analytically show how variance of cell residence time affects the channel holding time distribution. We show that when the cell residence time is not exponentially distributed, the channel holding time is not exponentially distributed either. In fact, for some cases (where the variance is small), the approximation using the exponential distribution is severely inappropriate. This suggests that a careful study is needed for the channel holding time in teletraffic analysis.

We first study the channel holding time for the case when the cell residence time is Erlang distributed. The Erlang distribution is characterized by its density function and Laplace transform as follows:

$$f(t) = \frac{\beta^m t^{m-1}}{(m-1)!}e^{-\beta t},\ f^*(s) = \left(\frac{\beta}{s+\beta}\right)^m, \qquad (25)$$

where $\beta = m\eta$ is called the scale parameter and m is called the shape parameter. The mean of this Erlang distribution is η and its variance is $1/(m\eta^2)$. When the mean η is fixed, varying the value m is equivalent to varying the variance, larger m means smaller variance, lesser spread of the cell residence time.

Due to the similarity of the formulae for new call and handoff call, we only study the handoff call channel holding time. Figure 3 shows the handoff call channel holding time distribution functions with different variance of cell residence time distributed according to Erlang distribution with the same mean. It can be observed that when the cell residence time become less spread, the handoff call channel holding time shows severe mismatch to the exponential distribution. This implies that we can not simply apply the exponential distributions for handoff call channel holding time during the network study of PCS networks where mobility is a major issue.

Now we study the case when the cell residence time is hyper-Erlang distributed with two terms as follows

$$f^*(s) = \alpha_1\left(\frac{m_1\eta}{s+m_1\eta}\right)^{m_1} + \alpha_2\left(\frac{m_2\eta}{s+m_2\eta}\right)^{m_2}.$$

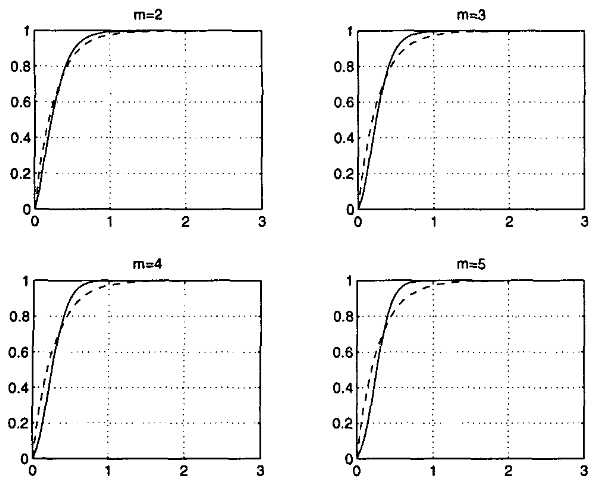

Fig. 3. Distribution of handoff call channel holding time (solid line) and its exponential fitting (dashed line) when cell residence time is Erlang distributed with parameter (m, η)

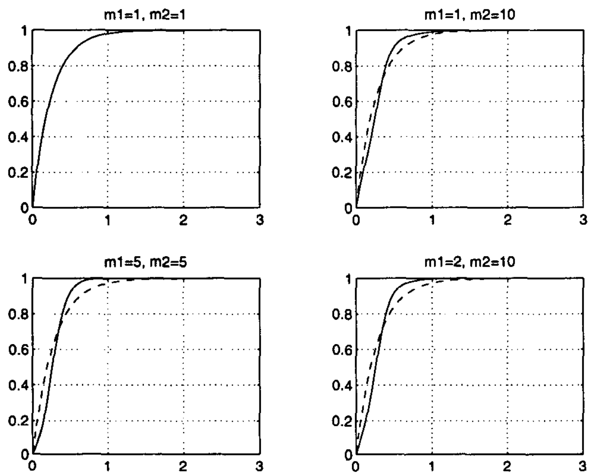

Fig. 4. Distribution of handoff call channel holding time (solid line) and its exponential fitting (dashed line) when cell residence time is hyper-Erlang distributed with parameter (m_1, m_2, η)

The handoff call channel holding time can be written as

$$f^*_{hh}(s) = \alpha_1 \left\{ \frac{\mu}{s+\mu} + \frac{s}{s+\mu} \cdot \left(\frac{m_1 \eta}{s + m_1 \eta} \right)^{m_1} \right\} + \alpha_2 \left\{ \frac{\mu}{s+\mu} + \frac{s}{s+\mu} \cdot \left(\frac{m_2 \eta}{s + m_2 \eta} \right)^{m_2} \right\}.$$

This representation illustrates that the handoff call channel holding time is in fact an average of two channel holding times each of which is obtained from Erlang distributed cell residence time cases. The mean value of the cell residence time for this case is still η, which is the same as in the Erlang case. Different values of m_1 and m_2 signify the different variances. Figure 4 shows the distribution plotting. It can be theoretically proved that when $m_1 = m_2 = 1$ and $\eta_1 = \eta_2$, the cell residence time is in fact exponentially distributed. In this case, we have an exponentially distributed handoff call channel holding time as evidenced in the figure ($m_1 = m_2 = 1$). However, when m_1 and m_2 are different, i.e., the variances of cell residence time are different, the handoff call channel holding time is no longer exponentially distributed.

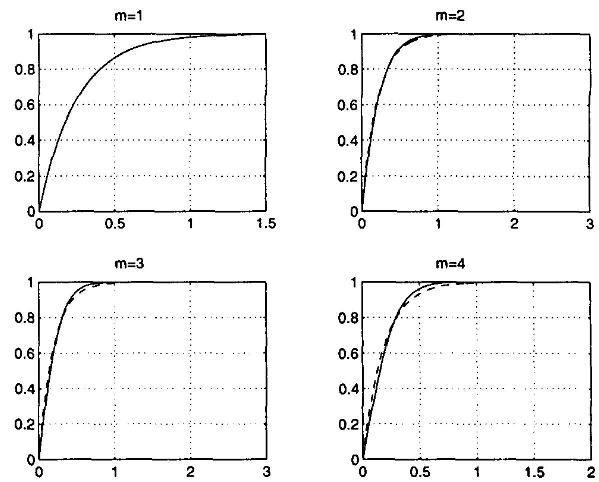

Fig. 5. Conditional distribution of handoff call channel holding time (solid line) and its exponential fitting (dashed line) when cell residence time is Erlang distributed with parameter (m, η)

Figure 5 shows the conditional distribution for the handoff call channel holding time when the cell residence time is Erlang distributed. From this example, we observe the conditional distribution for handoff call channel holding time is a better match to the exponential distribution when the variance of cell residence time is large. However, when the variance becomes small, i.e., the cell residence time is less spread, this match disappears.

V. Conclusions

In this paper we propose a new mobility model and analytically characterize the distribution of the channel holding time under the realistic assumption that the cell residence time is generally distributed. Our modeling effort focuses on the characterization of the channel holding time under the assumption that distribution of the cell residence time has rational Laplace transform, hence our analytical results can be readily applicable to the hyper-Erlang distribution models for the cell residence time. We demonstrate that hyper-Erlang distribution model is general enough to fit field data for system analysis and design. The analytical results presented in this work provide a general framework for further study of teletraffic aspects in PCS networks in which classical assumptions may not be satisfied.

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APPENDIX

Strong Memoryless Property of Exponential Distribution: If ξ is exponentially distributed, then for any nonnegative random variable ζ, we have

$$\Pr(\xi \geq \zeta + x | \xi \geq \zeta) = \Pr(\xi \geq x), \quad x \geq 0. \tag{26}$$

Proof: Let $1/\mu = E[\xi]$ and let $f_\zeta(y)$ denote the probability density function of ζ, then we have

$$\begin{aligned}
\Pr(\xi \geq \zeta + x | \xi \geq \zeta) &= \frac{\Pr(\xi \geq \zeta + x)}{\Pr(\xi \geq \zeta)} \\
&= \frac{\int_0^\infty \Pr(\xi \geq y + x) f_\zeta(y) dy}{\Pr(\xi \geq \zeta)} = \frac{\int_0^\infty e^{-\mu(y+x)} f_\zeta(y) dy}{\Pr(\xi \geq \zeta)} \\
&= \frac{e^{-\mu x} \int_0^\infty e^{-\mu y} f_\zeta(y) dy}{\Pr(\xi \geq \zeta)} = \frac{e^{-\mu x} \Pr(\xi \geq \zeta)}{\Pr(\xi \geq \zeta)} \\
&= e^{-\mu x}.
\end{aligned}$$

This completes the proof. □

Remark: Traditionally, the memoryless property is equivalent to saying that (26) holds for any constants $\zeta \geq 0$ and $x \geq 0$. The strong memoryless property seems to be more appropriate for many applications.