Interference-Constrained Maximum Flow in Wireless Sensor Networks with Directional Antennas

Xiaoxia Huang, *Student Member, IEEE,* Jianfeng Wang, *Student Member, IEEE,* and Yuguang Fang, *Senior Member, IEEE*

***Abstract*— Directional antenna offers a variety of benefits for wireless networks, one of which is the increased spatial reuse ratio. This feature gives rise to the improved throughput through multipath routing in resource limited wireless sensor networks. Given a pair of source and destination nodes, we formulate the maximum flow problem in wireless sensor networks with switched beam directional antennas constrained by interference as an optimization problem. The solution of this LP centralized problem determines the maximum flow. The method works for both single beam antenna and multi-beam antenna, with minor variation of the conditions.**

I. Introduction

Due to the hostile wireless channel, contention and interference, how to achieve the maximum throughput in multihop wireless networks has been of great interest over the past decades. Especially for resource-constrained wireless sensor networks, how to improve the capacity is even critical. With the switched beam technology, the directional antenna is shown to be an appealing option for wireless sensor networks. By concentrating RF energy in the intended transmission direction, the spatial transmission region shrinks proportionally to the beam width of a sector. Directional antenna is able to reduce interference and energy consumption, and improve spatial reuse ratio, thus significantly boosting the channel capacity. It is feasible to equip sensor nodes with directional antennas, because the switched beam systems could be built with fairly cheap off-the-shelf components and the size is still moderately small. So this paper focuses on wireless sensor networks with switched beam antennas.

A lot of work is inspired by wireless networks with directional antennas. Some of them focus on the throughput in wireless ad hoc networks. Although some asymptotic bounds for throughput are derived under certain assumptions about network deployment and node configuration, what the feasible maximum throughput is and how to achieve it are still not answered in the context of general wireless ad hoc networks. The goal of this paper is to address the "what" and "how" problems in wireless sensor networks with switched beam directional antennas. Both issues are challenging in wireless sensor networks considering the large scale and limited capability.

Xiaoxia Huang, Jianfeng Wang are with the Department of Electrical and Computer Engineering, University of Florida, Gainesville, FL 32611, USA (phone: 352-392-8576; email: xiaoxiah, jfwang@ufl.edu).

Yuguang Fang is with the Department of Electrical and Computer Engineering, University of Florida, Gainesville, FL 32611, USA (phone: 352-846-3043; fax: 352-392-0044; email: fang@ece.ufl.edu).

This paper also deals with the maximum throughput of a wireless sensor network with switched beam directional antennas. The problem to be addressed in this paper is: Given a network topology and existing traffic load, how can we achieve the maximum flow between a given source-destination pair? For the general purpose, we have no assumption on the traffic pattern, like the source-sink pattern in many papers. Because for some applications, various communication pairs may exist, including sensor-to-sensor, sensor-to-sink and sink-to-sensor. We address the problem for any source destination pair so that it applies to any specific traffic pattern.

Without assumptions on the network topology or homogeneity of link capacity, we attempt to solve the problem in a generalized setting. This problem is inherently a joint multipath routing and optimal scheduling problem. Generally, multipath routing is capable of supporting a larger amount of flow against single path routing. Nevertheless, the interference among multiple paths restricts the efficacy of multipath routing. Taking the advantage of mitigated interference, multipath routing is more justifiable in wireless network with directional antennas. Yet the more involved interference pattern of multipath routing further complicates the problem because of the substantial problem size and searching space. There could be numerous paths between the source and destination pair. Consequently, the searching space for routing is tremendous, which results in formidable computational complexity.

The paper is organized as follows. The next section II summarizes the previous work on related topics. Section III describes the antenna model. Section IV defines the flow contention and resource sharing graph. Then we present the problem formulation of maximum flow for switched beam antennas in Section V. Section VI demonstrates the simulation results for the maximum flow. Finally, Section VII concludes the paper.

II. Related Work

Many papers have proposed schemes to address problems associated with wireless sensor networks with directional antennas, such as MAC, routing, scheduling and etc. Two important issues are pointed out in designing contention-based MAC for wireless sensor networks using directional antennas [1]. Implementing the proposed simple scheduling strategy in a tree-like sensor network [2], the performance of a sensor network equipped with directional antennas outperforms one with omnidirectional antennas. To provide point-to-point,

1-4244-0065-1/06/$20.00 ©2006 IEEE

point-to-area and one-to-all communications in randomly deployed wireless sensor networks with directional antennas, the authors [3] propose a method to establish a reliable high level communication system which is independent of physical capacities of the network. In [4], the two tier architecture consists of higher-tier nodes with directional antennas used for data aggregation and forwarding, and lower tier monitoring nodes with omnidirectional antennas. A single beam flow routing which maximizes the network lifetime is proposed for higher tier nodes.

Many papers have derived the asymptotic throughput bounds under certain assumptions on network topology and node configuration. The seminal paper by Gupta and Kumar [5] studied the network comprising of n randomly placed non-mobile nodes. The throughput per node for a randomly chosen destination is $\Theta(\frac{1}{\sqrt{n \log n}})$, where $\Theta(x)$ is a quantity on the same order of x. Even under the optimal node placement and communication pattern, the per-node throughput is $\Theta(\frac{1}{\sqrt{n}})$. In this case, the total end-to-end capacity is roughly $\Theta(\frac{n}{\sqrt{n}})$, which is $\Theta(\sqrt{n})$. Subsequent work [6] investigates the capacity gain of wireless ad hoc networks with directional antennas over omni-directional antennas. In [7], Kodialam and Nandagopal consider the problem of joint routing and scheduling to achieve a given rate vector. The only interference constraint they take into consideration is that a node cannot transmit or receive simultaneously. They formulate the scheduling problem as an edge-coloring problem and provide a polynomial time algorithm. The approach achieves at least 67% of the optimal throughput. Jain et. al [8] model the interference between neighboring nodes using a conflict graph and present methods for computing the lower and upper bounds. They focus on the routing component alone. But they do not propose any approximation algorithm to solve the routing problem. In [9], Peraki and Servetto study the maximum throughput in dense random wireless networks with directional antennas with bounded queue. They derive the asymptotic upper bounds on throughput by solving the minimum cut problem. An optimal resource allocation scheme is proposed based on the maximal cliques resulted from contention flows in [10]. A distributed pricing algorithm is provided to approximate global optimum and fairness among end-to-end flows.

Several works study the multipath routing in wireless ad hoc networks using directional antennas [11] [12]. In [12], the authors define the path interference to find the minimum single path and node-disjoint multiple paths in wireless networks equipped with directional antennas. Since interference affects the network performance, some paper attempts to reduce the interference through topology control. A recent work concisely defines the interference and proposes several interference-aware topology algorithms [13].

III. System Model

A. Antenna Model

According to beam pattern (beam-radius, beam-width, beam orientation), we have omni-directional antennas, single-beam directional antennas (e.g., single-beam switched beam antennas), multi-beam directional antennas (e.g., multi-beam switched beam antennas or sectorized beam antennas). For directional antennas, both directional transmission and directional reception are enabled. To be clear, for single-beam directional antennas, we assume only one directional transmitting beam or one directional receiving beam can be active at a time; for multi-beam directional antennas, multiple directional transmission beams or multiple directional receiving beams can be active at a time. Assume that the antenna is directed to discrete directions, with fixed beam-radius and beam-width.

An illustration of a node graph is shown as Figure 1, though an realistic node graph is always more complex. Node 1 and node 6 are considered source node and destination node, respectively.

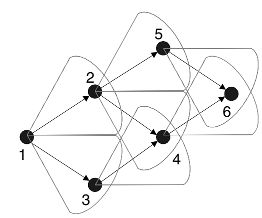

Fig. 1. A simple illustration of node graph $G = (V, E)$.

IV. Resource Sharing Graph

A. Flow Contention Graph

Given the toy example of node graph Figure 1, the flow contention graphs for the single beam directional antenna case and multi-beam directional antenna case are shown as Figure 2 and 3, respectively. The vertices in the flow contention graph are the links in G. There is an edge between two vertices in the flow contention graph if the corresponding two links in G interfere with each other. Since the multi-beam directional antenna is able to receive or transmit towards several directions concurrently, the contention is only a portion of the single beam counterpart. As a result, the flow contention graph for the network using multi-beam directional antennas is a subgraph of single beam directional antennas.

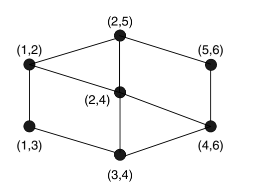

Fig. 2. Flow contention graph for single beam directional antennas.

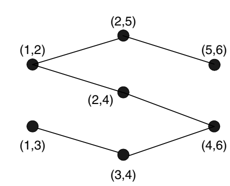

Fig. 3. Flow contention graph for multi-beam directional antennas.

B. Link Resource Sharing Graph

Given flow contention graph Figure 2, the link resource sharing graph for the single beam directional antenna can be represented as Figure 4. For link $(2,4)$, the contention links are $(1,2)$, $(2,5)$, $(3,4)$ and $(4,6)$. Hence, no two contending links are allowed to be active simultaneously. Thus the capacity of $(2,4)$ are shared with those links, as indicated by $A(2,4)$.

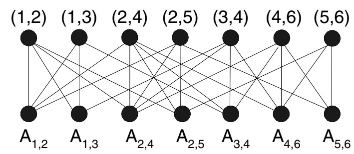

Fig. 4. An illustration of link resource sharing graph for single beam directional antennas.

When using multi-beam directional antennas, the resource sharing graph is disparate according to Figure 3. Typically, the resource contention in networks with multi-beam directional antennas is moderate compared to single-beam directional antennas. As depicted in Figure 5, for link $(2,4)$, the contention links are reduced to $(1,2)$ and $(4,6)$. The decrease of contention level is significant.

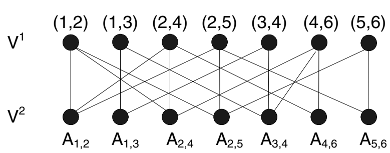

Fig. 5. An illustration of link resource sharing graph for multi-beam directional antennas.

C. Formulation of Maximum Flow Problem for a Single Source-destination Pair

The problem here to be addressed is: given network $G(V,E)$ and existing flows, find the maximum flow can be supported by the network between pair $s-d$. Based on the resource sharing graph, the maximum flow problem can be formulated as the following optimization problem.

$$\begin{aligned}
&\max \quad f \\
&\text{s. t.} \\
&\sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i = s, \\ 0 & i = V - \{s,d\}, \\ -f & i = d; \end{cases} \\
&\sum_{(k,l)\in A_{i,j}} x_{k,l} \le u_{i,j}, \forall (i,j) \in E; \\
&x_{i,j} \ge 0, \forall (i,j) \in E.
\end{aligned} \tag{1}$$

where $u_{i,j}$ is the normalized remaining capacity ($0 \le u_{i,j} \le 1$) for (i,j). The second constraint specifies the contention for resource of each link according to the resource sharing graph. This is a traditional maximum flow problem with added interference (contention) constraint. To straighten the problem formulation, the interference constraint is further explored and characterized in the next section.

V. Maximum Flow with Switched Beam Directional Antennas

A. Contention Region

In wireless networks, a transmission collision occurs when both receivers are in the communication range of a transmitter. Here we do not consider the contention at the transmitter. Suppose the antenna both transmits and receives directionally, but it cannot transmit and receive simultaneously. With directional antenna, two links interfere with each other if the two receivers are in the same beam of at least one transmitter. We call it R-R interfere, shown in Figure 6. The protocol model in wireless ad hoc networks with directional antennas differs from those with omni-directional antennas. Because the interference region is specified not only by the interference range, but also the beam orientation.

- The Protocol Model: In the protocol model, the transmission from node i to node j is successful if (1) j is in the transmission range of i, $d_{ij} \le R$, where R is the transmission range; (2) any node u that in the receiving beam of j from i is not transmitting in the beam covering j (when interference range = transmission range). This means that j must be outside of transmission beam of u, to avoid the R-R interference.

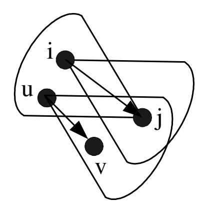

Fig. 6. An illustration of interference caused by (u,v) to (i,j)

Instead of the circular interference area in omni-directional antenna network, the interference region in directional antenna equipped wireless network is a beam. The smaller interference area significantly reduces the interference, thus improves the capacity comparing to the network with omni-directional antennas. Multipath routing, which generally causes more serious interference than single path routing, is more justifiable for network with smart antennas in terms of throughput. In addition, multipath routing achieves better load-balancing and end-to-end delay.

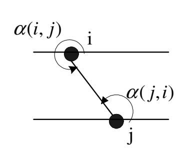

Fig. 7. Illustration of $\alpha(i,j)$ and $\alpha(j,i)$

Our work is based on this protocol model. Since the contention region is a beam, the information about the beam to which a link belongs is essential for routing and scheduling. Suppose there are fixed B beams for each antenna, labeled from 1 to B counterclockwise. Then a beam is specified by the transmission range and the direction pointed to. Denote the angle between node i and another node j as $\alpha(i,j)$ as depicted in Figure 7. The transmission beam and reception beam of (i,j) are different by $B/2$ beams. With the knowledge of $\alpha(i,j)$, (i,j) can be located in the beam $\theta_i^j = \lceil \alpha(i,j)/2\pi \times B \rceil$ of i, which is the transmission beam for link (i,j).

Now we can recapitulate condition (2) of the protocol model in the following way:
(2') When (i,j) is active, for any node u in the j's receiving beam towards i, beam θ_u^j should keep silent. Denote $b(i,l)$ as the lth beam of node i, where $l = 1, \ldots, B$.

The problem formulation is mostly the same for the single beam and the multi-beam cases.Due to the different number of transceivers, only one constraint is different, which is the time sharing constraint as described in the following subsections. With the protocol model, we are now ready to expand the second constraint in (1).

B. Single beam directional antenna

Because the single beam directional antenna can only target to one beam at a time. So the time sharing constraint is formulated as,

$$\sum_{l=1}^{B} \left(\sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l) + \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,l) \right) \leq 1, \forall i \in V \tag{2}$$

Thus, for single beam directional antennas, we can formulate the maximum flow problem as the following linear programming.
Problem formulation 1:

$$\begin{aligned}
&\max \quad f \\
&\text{s. t.} \\
&\sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i = s, \\ 0 & i = V - \{s,d\}, \\ -f & i = d; \end{cases} \\
&\underbrace{\sum_{u\in b(i,l)} \sum_{(u,v)\in E} x_{u,v} b_{u,v}(u,\theta_u^i)}_{\text{R-R contention links in l-th beam}} + \underbrace{\sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l)}_{\text{incoming links}} \leq 1, \forall l, i, \\
&\sum_{l=1}^{B} \left(\sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l) + \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,l) \right) \leq 1, \forall i \in V, \\
&b_{i,j}(i,l) = \begin{cases} 1, & \text{if } (i,j) \in b(i,l), \\ 0, & \text{otherwise} \end{cases} \\
&x_{i,j} \geq 0, \forall (i,j) \in E.
\end{aligned} \tag{3}$$

The first constraint describes the in-flow and the out-flow at each node. The second constraint indicates the flow contention around i as described as condition (2) in the protocol model. The first term represents the links causing $R-R$ interference to i in beam l. The second term stands for the total incoming flows to i in beam l. The total sum of these two parts should be less than the normalized beam capacity 1. The third constraint describes the time sharing constraint. From the resource constraint graph, we observe that the second and third constraints aggregately describe the contention flows at a node. The contention region includes all link flows in the 1-hop area of a node.

C. Multi-beam directional antenna

For multi-beam directional antenna, multiple incoming flows and outgoing flows could share the time resource, which is normalized to 1. So we obtain

$$\max_{l:1\leq l\leq B} \text{in-flow of beam } l + \max_{l:1\leq l\leq B} \text{out-flow of beam } l \leq 1$$

We need *max* functions because several beams can transmit or receive simultaneously.

Now for a single pair of source and destination nodes in wireless networks with multi-beam directional antennas, the problem formulation in (1) can be expanded more specifically as follows,

$$\begin{aligned}
&\max \quad f \\
&\text{s. t.} \\
&\sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i = s, \\ 0 & i = V - \{s,d\}, \\ -f & i = d; \end{cases} \\
&\underbrace{\sum_{u\in b(i,l)} \sum_{(u,v)\in E} x_{u,v} b_{u,v}(u,\theta_u^i)}_{\text{R-R contention links in l-th beam}} + \underbrace{\sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l)}_{\text{incoming links}} \leq 1, \forall l, i \\
&\max_{l:1\leq l\leq B} \sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l) + \max_{l:1\leq l\leq B} \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,l) \leq 1, \\
&\forall i \in V, \\
&b_{i,j}(i,l) = \begin{cases} 1, & \text{if } (i,j) \in b(i,l), \\ 0, & \text{otherwise} \end{cases} \\
&x_{i,j} \geq 0, \forall (i,j) \in E.
\end{aligned} \tag{4}$$

The first two constraints are the same as those in (3). In the last constraint, the first maximum value is the load of the beam with the most incoming flow to node i, while the second maximum value is the load of the beam with the most outgoing flow. The last constraint guarantees that the flow is feasible because the in-flow and out-flow share the capacity at the node. This constraint also implies that the in-flow from any beam should not be greater than 1. However, the constraint is nonlinear.

Notice that the relationship between the link and active beam is determined by the positions of both transmitter and receiver. Therefore

$$(i,j) \in b(i,\theta_i^j), \tag{5}$$

$b_{i,j}(i,l)$ in (4) can be calculated by

$$b_{i,j}(i,l) = \begin{cases} 1, \text{ if } l = \theta_i^j, \\ 0, \text{otherwise} \end{cases}$$

The formulation in (4), the third constraint is non-linear because of the max function. To transform it into a linear constraint, we use the following constraint:

$$\sum_{(k,i)\in E} x_{k,i} b_{k,j}(i,l) + \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,m) \leq 1, \forall l, m, \forall i \in V$$

Observe that the constraint becomes linear at the cost of adding more constraints. The number of constraints is increased by a factor of $B^2 - 1$. Thus, the maximum flow problem can be modeled by the linear programming problem *Problem formulation 2*:

$$\begin{aligned} &\max \quad f \\ &\text{s. t.} \\ &\sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i = s, \\ 0 & i = V - \{s, d\}, \\ -f & i = d; \end{cases} \\ &\sum_{u\in b(i,l)} \sum_{(u,v)\in E} x_{u,v} b_{u,v}(u, \theta_u^i) + \sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l) \leq 1, \forall l, i \\ &\sum_{(k,i)\in E} x_{k,i} b_{k,j}(i,l) + \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,m) \leq 1, \\ &\forall 1 \leq l, m \leq B, \ \forall i \in V, \\ &b_{i,j}(i,l) = \begin{cases} 1, \text{ if } l = \theta_i^j, \\ 0, \text{otherwise} \end{cases} \\ &x_{i,j} \geq 0, \ \forall (i,j) \in E. \end{aligned} \tag{6}$$

VI. Numerical Results

In this section, we give some preliminary results of the two LP problems, which are solved using MATLAB [14]. Simulations are done on a machine with 3GHz processor and 2GB of RAM. Nodes are randomly deployed in a 10×10 square, with transmission range of 2.5 units. The link capacity is normalized to 1. The network size varies from 20 to 40 nodes. A source-destination pair is randomly chosen from all nodes. There exist other random flows which may interfere with the flow between the source-destination pair. Each scenario is repeated for 30 runs. The maximum amount of flow in 20, 30 and 40-node network is shown in Table I to Table III.

TABLE I
Performance of 20-node network

	20 nodes	
	average	time (seconds)
single beam	0.2674	12
multi-beam	0.4387	25

As expected, the maximum flow in case of multi-beam directional antenna is greater than using single beam directional antenna. An interesting observation is that the maximum flow decreases inversely to the network size for networks with single beam directional antenna, while the maximum flow increases for networks with multi-beam directional antenna. The reason for the different performance is that the single beam directional antenna is more sensitive to the contention. As the network aggrandizes, more contention among flows is introduced, so the maximum flow declines. But multi-beam directional antenna is capable to harness the advantage of space reuse more efficiently, so the contention for time fraction is still low even the network size increases. The maximum flow does not deteriorate with the densities in the simulation. On the contrary, the flow increases because more space-separated paths are available. We would expect the maximum flow of the network with multi-beam directional antennas to degrade when the node density reaches a certain degree.

TABLE II
Performance of 30-node network

	30 nodes	
	average	time (minutes)
single beam	0.2664	5
multi-beam	0.5066	9

TABLE III
Performance of 40-node network

	40 nodes	
	average	time (minutes)
single beam	0.2048	49
multi-beam	0.6095	112

The computational cost is also listed in the table. The computational cost is measured in time. The computational time of multi-beam case is longer than its single beam counterpart, because there are more constraints using multi-beam directional antennas.

The distribution of the maximum flow is also plotted in Figure 8 to Fig. 10. Apparently, the maximum flow obtained when we use multi-beam directional antennas shifts to the higher end of the flow compared to when using single beam antennas. The results comply to the fact that multi-beam directional antenna is less resource contending in terms of time.

A. Discussion

To sum up, we have formulated the maximum flow using multipath routing subject to interference as an LP for multi-hop wireless sensor networks using directional antennas. The problem is disparate from the traditional maximum flow problem because of the interference constraints. It can be solved by a centralized algorithm at an omniscient base station. This is feasible because a base station is usually available for commanding and data collection. Typically, the base station has greater computation capacity and higher energy level; thus, it is able to carry out complex computing.

Although the centralized LP solution gives the optimal

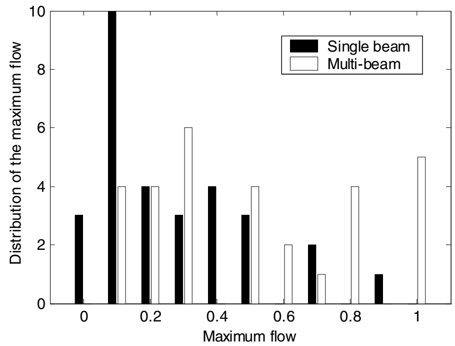

Fig. 8. The distribution of maximum flow in network with 20 nodes

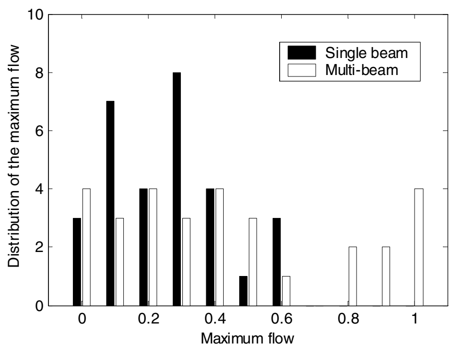

Fig. 9. The distribution of maximum flow in network with 30 nodes

multipath flow, it has the inherent and common disadvantages of all centralized algorithms – it is not scalable to the network size and cannot quickly adapt to changes in link condition and topology. The simulation time shows that the computation load skyrockets quickly as the network increases. So developing a distributed algorithm, which is jointly routing and scheduling is our future work.

VII. Conclusion

We studied the multipath routing in wireless sensor networks with directional antennas in this work. The goal is to maximize the throughput between given s-d pair over multiple paths. A key distinction of our work compared to previous work is that our approach answers the questions of what the optimal flow is and how to realize it, with a practical interference model. Based on the protocol model, the maximum throughput problem constrained by interference is formulated as an optimization problem. By solving the LP at the powerful base station, the optimal flow can be determined. The method applies to both single-beam and multi-beam directional antennas, with minor modifications.

VIII. Acknowledgment

This work was supported in part by the U.S. Office of Naval Research under Young Investigator Award N000140210464 and under grant N000140210554.

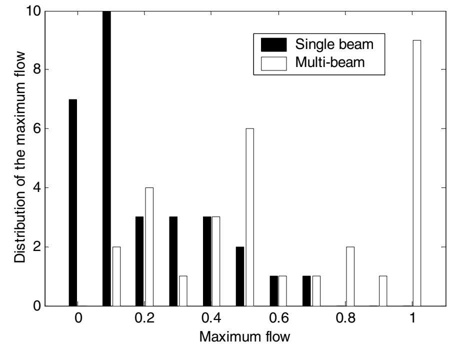

Fig. 10. The distribution of maximum flow in network with 30 nodes

References

[1] C. Santivanez and J. Redi, "On the use of directional antennas for sensor networks," *MILCOM 2003*, pp. 670 - 675, Boston, MA, 2003.

[2] C. Florens and R.McEliece, "Scheduling algorithms for wireless adhoc sensor networks," *GLOBECOM 2002*, pp. 6- 10, Taiwan, 2002.

[3] C. Àvarez, J. Díaz, J. Petit, J. Rolim and M. Serna, "Efficient and reliable high level communication in randomly deployed wireless sensor networks," *Proc. of the second international workshop on Mobility management and wireless access protocols*, pp. 106 - 110, Philadelphia, PA, 2004.

[4] Y. T. Hou, Y. Shi, J. Pan, S. F. Midkiff, and K. Sohraby," Single-Beam Flow Routing for Wireless Sensor Networks," to appear in *Proc. IEEE Globecom 2005*, St Louis, MO, 2005.

[5] R. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388-404, March 2000.

[6] S. Yi, Y. Pei and S. Kalyanaraman, "On the capacity improvement of ad hoc wireless networks using directional antennas," *Proc. of the 4th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2003)*, pp. 108-116, Annapolis, MD, 2003.

[7] M. Kodialam and T. Nandagopal, "Characterizing achievable rates in multi-hop wireless networks: the joint routing and scheduling problem," *Proc. of ACM MOBICOM*, pp. 42-54, San Diego, CA, 2003.

[8] K. Jain, J. Padhye, V. N. Padmanabhan, and L. Qiu, "Impact of interference on multi-hop wireless network performance," *Proc. of ACM MOBICOM*, pp. 66-80, San Diego, CA, 2003.

[9] C. Peraki, S. D. Servetto, "On the maximum stable throughput problem in random networks with directional antennas," Proc. of the 4th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2003), pp. 76-87, Annapolis, MD, 2003.

[10] Y. Xue, B. Li and K. Nahrstedt, "Optimal resource allocation in wireless ad hoc networks: a price-based approach," *IEEE Transactions on Mobile Computing*, in press.

[11] Y. Li and H. Man, "Analysis of multipath routing for ah hoc networks using directional antennas," *IEEE Vehicular Technology Conference 2004*. vol. 4, pp. 2759-2763, 2004.

[12] J. Tang, G. Xue, C. Chandler and W. Zhang, "Interference-aware routing in multihop wireless networks using directional antennas," *Proc. of IEEE INFOCOM 2005*.

[13] M. Burkhart, P. V. Rickenbach, R. Watternhofer and A. Zollinger, "Does topology control reduce interference?", Proc. of the 5th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2004), pp. 9-19, Roppongi, Japan, 2004.

[14] Matlab version 6.1. http://www.matlab.com/.