Throughput Maximization in Multi-channel Wireless Mesh Access Networks

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***Abstract*—The throughput maximization problem of wireless mesh access networks is addressed. For the case of cooperative access points, we present a negotiation-based throughput maximization algorithm which adjusts the operating frequency and power level among access points autonomously, from a game-theoretical perspective. We show that this algorithm converges to the optimal frequency and power assignment which yields the maximum overall throughput with arbitrarily high probability. Moreover, we analyze the scenario where access points belong to different regulation entities and hence non-cooperative. The long-term behavior and corresponding performance are investigated and the analytical results are verified by simulations.**

I. Introduction

Metropolitan wireless mesh networks gain enormous popularity recently [1]. The deployment of wireless mesh networks not only facilitates the data communication by removing cumbrous wires and cables, but also provides a means of Internet access scheme, which is a further step towards the goal of "communicating anywhere anytime". No matter where the location is or the purpose that the wireless mesh access network is deployed, the same conceptual layered architecture is utilized. Figure 1 illustrates the hierarchical structure of wireless mesh access networks. The peripheral nodes are the access points (AP) which provide wireless access for the end users, or *clients*. Each AP is attached[1] to a mesh router, which is capable of communicating with any other mesh routers. The center node is a gateway mesh router which functions as an information exchange between the wireless mesh access network and other networks such as Internet. Both the routing algorithmic design and channel assignment for backbone mesh routers are interesting issues and attract tremendous attention from the community [5]–[10].

In this paper, we investigate another important issue which needs to be solved in wireless mesh access networks. As in Figure 1, the AP and its associated clients form a regular WLAN *cell*, which operates with the *de facto* IEEE 802.11 standards. The throughput of one cell depends on the signal-to-interference-plus-noise ratio (SINR) experienced at the receiver where the interference mainly comes from the other operating cells. For example, if each of the cell operates with IEEE 802.11b standard, we can utilize a different frequency band such as IEEE 802.11a or WiMAX [11], for the inter-cell communication among mesh routers and hence causes no interference to intra-cell transmissions. However, the co-channel interference from other operating cells is inevitable due to the limitation of available transmission channels, e.g., 3 non-overlapping channels in our example. Most current off-the-shelf APs are capable of adjusting the transmission rate according to the measured channel condition which is indicated by transmission bit error rate (BER). Given a particular modulation scheme, BER is uniquely determined by the SINR experienced by the receiver of the link. Generally speaking, higher SINR value yields lower BER and higher data rate. Therefore, the mutual interference dramatically degrades the transmission rate of each cell and the aggregated throughput of the whole network [12]. Each AP attempts to tune the physical parameters such as operating frequency and transmission power in order to maximize the SINR and hence the throughput. In our work, we investigate the issue of maximizing the overall throughput of the network, defined

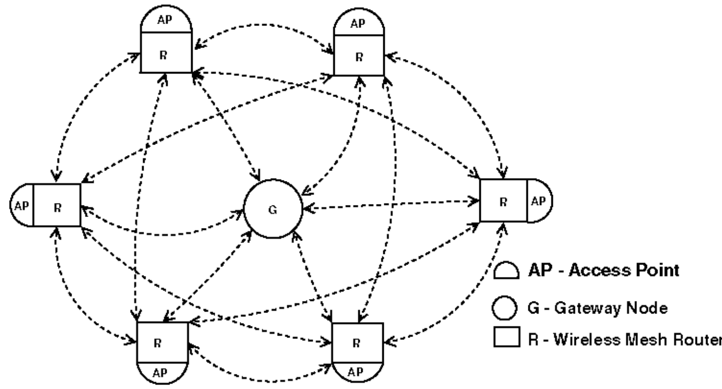

Fig. 1. Hierarchical structure of wireless mesh access networks.

This work was supported in part by the U.S. National Science Foundation under Grant DBI-0529012 and under Grant CNS-0721744.

[1] AP and the associated mesh router can be manufactured in a single device with two separate functional radios [2] [3], or simply connected with Ethernet cables [4].

as the summation of throughput of all cells, by finding the optimal frequency and transmission power allocation strategy. Also, due to the concern of scalability and computational complexity, we prefer a decentralized solution to the throughput maximization problem.

Unfortunately, the throughput maximization problem is challenging. For example, if the APs belong to different regulation entities, the non-cooperative APs may only want to maximize their own cell's throughput rather than the overall one. As shown in the literature, e.g., [13] , the selfish behaviors of independent decision makers usually jeopardise the overall performance from the social-welfare point of view. The performance gap is named the *price of anarchy* and is discussed in different contexts [14]. Another difficulty which makes the throughput maximization problem more challenging is the interdependency among all APs. The frequency and power selected by one AP affects the SINR of other APs, and vice versa. Therefore, the throughput maximization problem becomes coupled and finding the optimum solution is not straightforward.

In this paper, we analyze the throughput maximization problem for both cooperative and non-cooperative scenarios. In the cooperative case, we model the interaction among all APs as an *identical interest game* and present a decentralized negotiation-based throughput maximizing algorithm for the joint frequency and power assignment. We show that this algorithm converges to the optimal frequency and power assignment strategy, which maximizes the overall throughput of the wireless mesh access network, with arbitrarily high probability. In the cases of non-cooperative APs, we prove the existence of Nash equilibria and show that the overall throughput performance is usually inferior to the cooperative cases. To bridge the performance gap, we propose a linear pricing scheme to combat with the selfish behaviors of non-cooperative APs.

The rest of this paper is organized as follows. Section II outlines the system model we considered in the paper. The cooperative wireless mesh access networks and the non-cooperative counterpart are investigated in Section III and Section IV, respectively. The performance evaluation is discussed in Section V and Section VI concludes this paper.

II. System Model

In this paper, we consider a wireless mesh access network illustrated in Figure 1. Each AP and corresponding clients form a cell. Without loss of generality, we assume that all the cells operate with IEEE 802.11b standard and the interference exclusively comes from the cells with same frequency. Furthermore, the distance between cells are sufficiently large in the sense that the accumulated interference experienced at the receiver only affects the SINR value and not block the whole transmission. We consider the worst case where all APs are transmitting under saturated traffic load. In other words, the APs always have packets to transmit and can communicate with each other via the backbone mesh routers with negligible delay. Also, we assume that the APs are transmitters and clients are receivers due to the dominance of downlink traffic, as assumed[2] in [16] [17] [18] [19] and [20]. We only focus on the joint frequency and power allocation where the contention behavior is less relevant and thus omitted. Therefore, we can simplify our model as that all the APs are transmitting data to the associated clients consistently. We assume that each AP is capable of adjusting the operating frequency and power as well as acquiring the SINR values measured at the client by short ACK messages.

Let us first consider the simplest case where there is only one cell in the wireless mesh access network, i.e., a single WLAN. Upon receiving the SINR value[3] measured by the client, denoted by γ, the AP tunes the physical parameters in order to maximize the *throughput*, which is defined as

$$R^*(\gamma) = \max_{R_i} R_i \times (1 - P_e(\gamma, R_i)) \qquad (1)$$

where R_i is the raw data rate specified by the IEEE 802.11 standard and R^*, i.e., the throughput of this cell, is a non-decreasing function of received SINR γ. P_e is the error probability of the transmission channel, which is a function of SINR value providing the transmission rate [21]. Apparently, if there is only one cell in the mesh access network, the AP will boost the power as much as possible to increase the value of γ and thus the throughput is maximized.

We now consider the cases where N cells coexist in the wireless mesh access network. Let p_i and f_i denote the power and frequency for the i-th AP, respectively. We use $\mathbf{p} = [p_1, p_2, \cdots, p_N]$ and $\mathbf{f} = [f_1, f_2, \cdots, f_N]$ to represent the power and frequency assignment vector for all N APs. Therefore, for each cell i, the value of SINR, i.e., γ_i, is a function of $(\mathbf{p}, \mathbf{f})$. The throughput of one cell depends not only on the power level and frequency of itself, but also those of other APs in the network. Therefore, the throughput maximization problem is coupled and by no means straightforward.

In the following sections, we will discuss the scenarios where the APs are cooperative and non-cooperative, respectively. The performance evaluation of the two scenarios are provided by simulations in Section V.

III. Cooperative Access Networks

In this section, we consider the scenarios where all APs in the wireless mesh access network are cooperative. The transmission power of APs are quantized into discrete power levels for simplicity. From the system point of view, we want to find a joint frequency and power level assignment such that the overall throughput in the whole network is maximized. Our objective function can be written as

$$U_{network}(\mathbf{p}, \mathbf{f}) = \sum_{i=1}^{N} R_i^*(\gamma_i) = \sum_{i=1}^{N} R_i^*(\mathbf{p}, \mathbf{f}) \qquad (2)$$

where R_i^* is defined in (1).

[2]The dominance of the downlink traffic is verified by the experimental measurements in [15] as well.

[3]Although there is no interference in this case, we adopt SINR instead of SNR for notation consistency.

However, finding the optimal frequency and power assignment which maximizes (2) is non-trivial. The interdependency makes the problem coupled and difficult to solve by traditional optimization methods [22]. A combination of $(\mathbf{p}, \mathbf{f})$ is named a *profile* and a naive approach to solve the problem is to investigate all profiles exhaustively. However, this is impossible in practice. For example, in a medium-size wireless mesh access network with 20 APs where each has 3 frequency channels and 10 power levels, the search space is $(3 \times 10)^{20}$ profiles! Obviously, the centralized algorithms are not favorable in the wireless mesh access network due to the scalability concern. Moreover, the traditional site-planning methods are not feasible either. For example, the network administrator may want to add more APs when more users are joining the network or disable some APs where the associated users fail to pay the bill. The network topology is not static, although the change takes place slowly. Therefore, the demand for adaptability and light computation burden requires a decentralized solution for the throughput maximization problem. Next, we will introduce a decentralized negotiation-based throughput maximization algorithm, from a game-theoretical perspective.

A. Cooperative Throughput Maximization Game

The APs in the wireless mesh access networks are considered as *players*, i.e., *decision makers* of the game. We model the interaction among APs as a *Cooperative Throughput Maximization Game (CTMG)*, where each player has an identical objective function U_i, as

$$U_i(\mathbf{p}, \mathbf{f}) = U_{network}(\mathbf{p}, \mathbf{f}) = \sum_{i=1}^{N} R_i^*(\mathbf{p}, \mathbf{f}) \ \forall i. \qquad (3)$$

For each player i, all possible frequency and power level pairs form a *strategy space* Φ_i which has a size of $c \times l$, where c is the number of frequency channels available and l is the number of feasible power levels. Define

$$\Omega = \Phi_1 \times \Phi_2 \times \cdots \times \Phi_N. \qquad (4)$$

Then, the N players autonomously negotiate about the joint frequency-power profile in Ω in order to find the optimal profile which maximizes (3). However, due to the interdependency among N players caused by mutual interference, one question of interest is that whether this negotiation will eventually meet an agreement, a.k.a., a Nash equilibrium. The importance of Nash equilibria lies in that a possible steady state of the system is guaranteed. If the game has no Nash equilibrium, the negotiation process never stops and oscillates in an everlasting fashion. In addition, we are concerning about what the performance of the steady states would be, if exist, in terms of overall throughput of the whole network. We provide answers to these questions in the following.

Lemma 1: The CTMG is a *potential game*.

A potential game is defined as a game where there exists a *potential function* P such that

$$P(a', a_{-i}) - P(a'', a_{-i}) = U_i(a', a_{-i}) - U_i(a'', a_{-i}) \ \forall i, a', a'' \qquad (5)$$

where U_i is the utility function for player i and a', a'' are two arbitrary strategies in Φ_i. In our case, we have $a' = [p_i', f_i']$ and $a'' = [p_i'', f_i'']$. The notation of a_{-i} denotes the vector of choices made by all players *other than* i. Potential games have been broadly applied in modeling the interactions in communication networks [23]. The popularity is on account of the nice properties of potential games, such as

- Potential games have at least one Nash equilibrium.
- All Nash equilibria are the maximizers of the potential function, either locally or globally.
- There are several learning schemes available which are guaranteed to converge to a Nash equilibrium, such as *better response* and *best response* [24] [25].

For detailed description about potential games, readers are referred to [24] and [26], which investigates the potential game theory in engineering context.

We observe that in the cooperative case, each player has the same utility function as in (3), which is the overall throughput of the network. Apparently, one potential function of the game is the common utility function itself, i.e.,

$$P = U_1 = U_2 = \cdots = U_N. \qquad (6)$$

In fact, the games where all players share the same utility function are called *identical interest games* [27], which is a special case of potential games and hence all the properties of potential games can be applied directly.

In the literature, both best response and better response are popular learning mechanisms that have been utilized in potential games [28]–[30]. At each step of the best response approach, one of the players investigates its strategy space and chooses the one with maximum utility value. This updating procedure is carried out sequentially. The primary drawback of the best response is the computational complexity, which grows linearly with the cardinality of the strategy space. An improvement of the best response is the so-called better response, where at each step, the player updates as long as the randomly selected strategy yields a better performance. The dramatically reduced computation is the tradeoff with the convergence speed. Both the best response and the better response dynamics are guaranteed to converge to a Nash equilibrium in potential games [23]. However, there may be multiple Nash equilibria in a potential game and the performance of different equilibria may vary dramatically. Therefore, although the best response and the better response could guarantee the convergence, they may reach an undesirable Nash equilibrium with inferior performance.

Let us consider an illustrative example in Figure 2. There are four labeled APs in the network. A and B are close to each other, and so are C and D. Without loss of generality, we assume that the APs have the same power and only adjust the operating frequencies in an order of $A \to B \to C \to D$ to avoid the interference. The adaptation continues with the best response mechanism until a Nash equilibrium is reached. Suppose there are two frequency channels available, say 1 and 2. First, A randomly selects one channel, say 1. B will pick 2.

Fig. 2. An illustrative example of multiple Nash equilibria.

Next, C has the chance to update. Since C is closer to B than A, channel 1 will be selected. Finally, D will choose channel 2. By inspection, we claim that profile $1-2-2-1$ is a Nash equilibrium since no player is willing to update its strategy unilaterally. Meanwhile, we observe that another profile $1-2-1-2$ is also a Nash equilibrium. Obviously, the second Nash equilibrium generates much less interference than the first Nash equilibrium and hence yields superior performance in terms of overall throughput. However, the best response only leads to the less desirable Nash equilibrium.

In fact, the existence of multiple Nash equilibria is observed in [28] by simulations. However, the authors fail to specify which one would be the steady state of their game due to the limitation of the best response, even in a statistical fashion. Recall that the Nash equilibria are the maximizers of the potential function in potential games, converging to an inferior Nash equilibrium analogously indicates being trapped at a local optimum of the potential function. However, it is the global optimum, i.e., the optimal Nash equilibrium, that is the desirable steady state which we are yearning for.

Next, we introduce a negotiation-based throughput maximization algorithm (NETMA) which can converge to the optimal Nash equilibrium with arbitrarily high probability.

B. NETMA- NEgotiation-based Throughput Maximization Algorithm

We assume that the APs are homogeneous and each has a unique ID for routing purpose. Each AP maintains two valuables D_{pre} and D_{cur}. The AP has the knowledge of its current throughput and records it in D_{cur}. Whenever there is a change of throughput caused by exterior interference[4], the AP sets $D_{pre} = D_{cur}$ and resets D_{cur} with the newly measured throughput. When the wireless mesh access network enters the *negotiation phase*[5], NETMA is executed. The detailed procedure of NETMA is provided as follows.

NETMA:

– **Initialization:** For each AP, a pair of frequency and power level is randomly selected. Set $D_{pre} = D_{cur}$ equals the current throughput.

– **Repeat:**
  1) Randomly choose one of the AP, say k, as the updating one, i.e., each AP updates with a probability of $1/N$.
  2) For the updating AP k:
     a) Randomly chooses a pair of frequency and power level, say f' and p', from the strategy space Φ_k. Then the AP computes the current throughput with f' and p' and records it into D_{cur}.
     b) Broadcasts a short notifying message which contains its unique ID_k to all the other APs in the mesh access network.
  3) For each AP other than k, say j:
     a) If the γ_j value changes, records the previous throughput into D_{pre} and the current throughput into D_{cur}. Remains unchanged otherwise.
     b) Upon receiving the notifying message, a three-value vector of $[D_{pre}, D_{cur}, ID_j]$ is sent back to the k-th AP.
  4) After receiving all the three-value vectors by counting the identifiers ID_j, the k-th AP computes the sum throughput before and after f' and p' are selected, which are denoted by P_{pre} and P_{cur}.
  5) For a *smoothing factor* $\tau > 0$, the k-th AP keeps f' and p' with a probability of
  $$\frac{e^{P_{cur}/\tau}}{e^{P_{cur}/\tau} + e^{P_{pre}/\tau}} \tag{7}$$
  6) The k-th AP broadcasts another short notifying message, which indicates the end of updating process and a specific number δ, to all the other APs.

– **Until:** The stopping criteria Γ is met.

Note that in step 6, the specific format of δ depends on the predefined stopping criterion Γ. For example,

- If the stopping criterion is the maximum number of negotiation steps, δ is a counter which adds one after each updating process.
- If the stopping criterion is that no AP has updated for a certain number of steps, δ is a binary number where 1 means updating.
- If the stopping criterion is that the difference between sum throughput obtained in consecutive steps are less than a predefined threshold ϵ, δ is the calculated sum throughput after each updating process.

We can have other stopping criteria Γs and corresponding formats of δ as well.

The NETMA algorithm is inspired by the work in [31], where a similar algorithm was first introduced in the context of stream control in MIMO interference networks. The distinguishing feature of this type of negotiation algorithms, from the better response and the best response, is the randomness deliberately introduced on the decision making in step 5. The rationale can be illustrated in Figure 2 intuitively. If there is no

[4]We assume the channel is slow-varying and the change of throughput for a single cell is due to the mutual interference only.

[5]The negotiation phase can be initiated by the network administrator after a new contracted user joins or a current user terminates the service, or on a daily basis.

randomness in decision making , i.e., $\tau = 0$, the four APs may get trapped at a low efficiency Nash equilibrium $1-2-2-1$. However, with the randomness caused by nonzero τ, they may reach an intermediate state $1-2-2-2$ and arrive at the optimum Nash equilibrium $1-2-1-2$ eventually.

The steady state behavior of NETMA is characterized in the following theorem.

Theorem 1: NETMA converges to the optimal Nash equilibrium in CTMG with arbitrarily high probability.

Proof: The proof of Theorem 1 follows similar lines of the proof in [31] and [32].

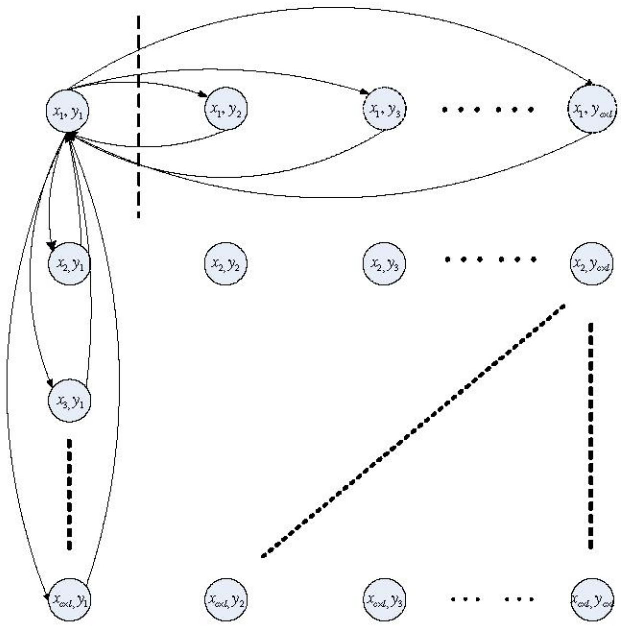

Fig. 3. Markovian chain of NETMA with two players.

First, we observe that the joint frequency-power negotiation generates an N-dimensional Markovian chain. Figure 3 illustrates the Markovian chain introduced by NETMA with two players, say A and B. Let x and y be the choices for each player , where $x \in \Phi_A$ and $y \in \Phi_B$. In other words, player A can choose a frequency-power pair from $[x_1, \cdots, x_{c\times l}]$ and player B can choose from $[y_1, \cdots, y_{c\times l}]$. Note that at an arbitrary time instant, only one of the players can update. In Figure 3, for example, state (x_1, y_1) can only transit to a state either in the same row or the same column, not anywhere else. This is true for every state in the Markovian chain. Let $S_{i,j}$ denote the state of (x_i, y_i). We have

$$Pr(S_{m,n}|S_{i,j}) = \begin{cases} \frac{e^{P(S_{m,n})/\tau}}{2\times c\times l\times(e^{P(S_{m,n})/\tau}+e^{P(S_{i,j})/\tau})}, \\ \text{ if } m = i \text{ or } n = j \\ 0, \\ \text{ otherwise.} \end{cases} \quad (8)$$

where τ is the smoothing factor in step 5 of NETMA.

Let us derive the stationary distribution Pr^* for each state. We examine the balanced equations for the directions. Writing the balance equations [33] at the place marked with dashed line, we obtain

$$\sum_{k=2}^{c\times l} Pr^*(S_{1,1})\times Pr(S_{1,k}|S_{1,1}) = \sum_{k=2}^{c\times l} Pr^*(S_{1,k})\times Pr(S_{1,1}|S_{1,k}). \quad (9)$$

By substituting (9) with (8), we have

$$\begin{aligned} &\textstyle\sum_{k=2}^{c\times l} Pr^*(S_{1,1}) \times \frac{e^{P(S_{1,k})/\tau}}{e^{P(S_{1,1})/\tau}+e^{P(S_{1,k})/\tau}} \\ = &\textstyle\sum_{k=2}^{c\times l} Pr^*(S_{1,k}) \times \frac{e^{P(S_{1,1})/\tau}}{e^{P(S_{1,1})/\tau}+e^{P(S_{1,k})/\tau}}. \end{aligned} \quad (10)$$

Observing the symmetry of equation (10) as well as the Markovian chain, we note that the set of equations as (10) are all balanced if for arbitrary state $\tilde{S}$ in the strategy space Ω, the stationary distribution is

$$Pr^*(\tilde{S}) = \mathcal{K}e^{P(\tilde{S})/\tau} \quad (11)$$

where $\mathcal{K}$ is a constant. By applying the probability conservation law [34] [33], we obtain the stationary distribution for the Markovian chain as

$$Pr^*(\tilde{S}) = \frac{e^{P(\tilde{S})/\tau}}{\sum_{S_i\in\Omega} e^{P(S_i)/\tau}} \quad (12)$$

for arbitrary state $\tilde{S} \in \Omega$.

In addition, we observe that the Markovian chain is irreducible and aperiodic. Therefore, the stationary distribution given in (12) is valid and unique.

Let S^* be the optimal state which yields the maximum value of potential function P, i.e.,

$$S^* = \text{argmax}_{S_i\in\Omega} P(S_i). \quad (13)$$

From (12), we have

$$\lim_{\tau\to 0} Pr^*(S^*) = 1 \quad (14)$$

which substantiates that NETMA converges to the optimal state in probability.

Finally, the analogous analysis can be straightforwardly extended to an N-dimensional Markovian chain and thus completes the proof. ■

In NETMA, there is no central computational unit required. The joint frequency-power assignment is achieved by negotiations among cooperative APs and the maximum overall throughput is achieved with arbitrarily high probability. The autonomous behavior and decentralized implementation make NETMA suitable for large scale wireless mesh access networks. Moreover, NETMA has fast adaptability for the topology change of the wireless mesh access networks. NETMA does not depend on any rate adaption algorithms, nor on any underlying MAC protocols. In our simulation in Section V, we use IEEE 802.11b as the MAC layer protocol. However, it can be easily extended to arbitrary MAC protocol with multi-rate multi-channel capability, such as IEEE 802.11a. NETMA mechanism can also be applied in the cases where non-overlapping channels are utilized [35]. In addition, even with the existence of exterior interference source, such as coexisting WLANs, NETMA works properly as well since the objective of NETMA is to maximize the overall throughput of the network in the current wireless environments. The tradeoff between algorithmic performance and convergence speed is controlled by parameter τ in step 5, where large τ represents extensive space search with slow convergence. On the contrary, small τ represents limited space search with fast convergence. Note that the smoothing factor τ here is analogous to the concept of *temperature* in simulated annealing [36]. Therefore, it is advisable that at the beginning period of the negotiation, the value of τ is set with a large number and keeps deceasing as the negotiation iterates. We choose $\tau = 10/k^2$ in our simulations, where k denotes the negotiation step.

In step 1, we require that each AP updates with probability $1/N$. For example, each AP may randomly set a backoff counter as in IEEE 802.11 DCF protocol. In the case of collision, which means two APs update at the same time in our case, it only prolongs the convergence time for NETMA and does not affect the final output of NETMA. This is because that the conflict, as verified in [29] via extensive simulations, has no influence on the statistically monotonic-increasing tendency of the potential function. We believe that by applying carefully designed backoff mechanisms as in IEEE 802.11 standards, the successive collisions are very rare and the convergence speed of NETMA is subtly lessened.

IV. Non-cooperative Access Networks

In the previous section, we discuss the scenarios where all APs in the wireless mesh access network are cooperative, and the overall throughput is maximized by negotiations among autonomous APs using the NETMA mechanism. However, cooperation is not always attainable. Although the functionality of relaying packets for each other can be achieved by incentive mechanisms such as [37], the adjustable parameters inside each cell cannot be enforced and effectively controlled. The N APs may belong to distinct self-interested users and they care about exclusively their own throughput rather than the overall aggregated throughput. In other words, the utility function of each selfish user is

$$U_i = R_i^*(\gamma_i) \tag{15}$$

where R_i^* is the throughput of the i-th cell, defined in (1). Analogous to CTMG, we can formulate the interaction among N selfish APs as a *Non-cooperative Throughput Maximizing Game (NTMG)* where each AP is attempting to find the frequency-power pair which maximizes its own SINR value as well as the corresponding throughput. As in the cooperative case, each player's utility function depends on the frequency and power of itself as well as those of others. However, NTMG is no longer an identical interest game.

Lemma 2: In NTMG, all the APs will transmit with the maximal power at the Nash equilibrium, if exists.

Proof: The proof of Lemma 2 is straightforward. For a single player, we have

$$\gamma_i = \frac{p_i g_{ii}}{\sum_{k\in\mathcal{F}_i(f_i)} p_k g_{ki} + N_i}. \tag{16}$$

where g_{ij} is the channel gain from cell i's transmitter to j's receiver and N_i is the Gaussian noise at the i's receiver. $\mathcal{F}_i(f_i)$ denotes the set of cells which operate at the same frequency f_i other than cell i. Note that given other players' strategies, γ_i is a monotonic increasing function of p_i and so is U_i. Assume at a Nash equilibrium of NTMG, the k-th AP has a power level of p_k satisfying $0 \le p_k < p_{max}$, where p_{max} denotes the maximum power defined by MAC layer. The k-th AP is inclined to increase its power p_k in order to yield a higher value of U_i, which contradicts the definition of Nash equilibrium. Thus, at the Nash equilibrium of NTMG, if exists, all the APs will operate at the same power level, i.e., p_{max}. ■

Based on Lemma 2, the NTMG can be viewed as a simplified game where each player has the same power and only adjusts the frequency to minimize the interference. Moreover, according to (15) (16) and the assumption of uniform environment, the NTMG is equivalent to the following simplified game where each player has the utility function[6] as

$$U_i = -(\sum_{k\in\mathcal{F}_i(f_i)} p_{max} g_{ki} + N_i) \tag{17}$$

and U_i is a function of frequency assignment vector $\mathbf{f}$ exclusively.

As in the cooperative case, the frequency selection among N players is mutually dependent. For example, we have two frequency channels available, 1 and 2. At a time instance t_0, channel 2 has fewer APs. Therefore, the APs in channel 1 are inclined to switch. However, this may make 2 much more crowded and the APs want to switch back. The question arises that whether this frequency adjusting dynamic converges, or equivalently, whether NTMG has a Nash equilibrium. The existence of Nash equilibrium is crucial for the analysis of interactive dynamics since the lack of Nash equilibrium indicates that the interaction will never converge. The whole network will be overwhelmed by oscillating adjustments and will never reach a steady state. We provide the answer of the question in the following theorem.

Theorem 2: There exists at least one Nash equilibrium in NTMG.

Proof: Let us first consider the simplified game. For each player, the utility function is given as

$$\begin{aligned} U_i &= -(\sum_{k\in\mathcal{F}_i(f_i)} p_k g_{ki} + N_i) && (18)\\ &= -(\sum_{k\neq i, k\in N} p_k g_{ki} \times \delta(f_i - f_k) + N_i) && (19) \end{aligned}$$

where

$$\delta(k) = \begin{cases} 1, & \text{if } k = 0 \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

[6] The negative sign comes from the convention that utility functions are the ones to be maximized.

We conjecture that one of the feasible potential functions is

$$P = -\frac{1}{2} \times \sum_{i \in N} \sum_{k \in \mathcal{F}_i(f_i)} p_k g_{ki}. \tag{21}$$

The verification is as follows.

$$\begin{aligned}
2P &= -\sum_{i \in N} \sum_{k \in \mathcal{F}_i(f_i)} p_k g_{ki} \\
&= -\left\{ \sum_{k \in \mathcal{F}_i(f_i)} p_k g_{ki} + \sum_{j \neq i, j \in N} \sum_{k \in \mathcal{F}_j(f_j)} p_k g_{kj} \right\} \\
&= -\left\{ \sum_{k \in \mathcal{F}_i(f_i)} p_k g_{ki} + \sum_{j \neq i, j \in N} \{ p_i g_{ij} \delta(f_i - f_j) \right. \\
&\quad \left. + \sum_{k \in \mathcal{F}_j(f_j), k \neq i} p_k g_{kj} \} \right\} \\
&= -\left\{ \sum_{k \neq i, k \in N} p_k g_{ki} \delta(f_k - f_i) \right. \\
&\quad + \sum_{j \neq i, j \in N} p_i g_{ij} \delta(f_i - f_j) \\
&\quad \left. + \sum_{j \neq i, j \in N} \sum_{k \in \mathcal{F}_j(f_j), k \neq i} p_k g_{kj} \right\}.
\end{aligned} \tag{22}$$

Note that

$$p_k g_{ki} \delta(f_k - f_i) = p_i g_{ik} \delta(f_i - f_k) \tag{23}$$

for any pair of i, k. We have

$$P = -\left\{ \sum_{k \neq i, k \in N} p_k g_{ki} \delta(f_k - f_i) + \frac{1}{2} \times Q(-i) \right\} \tag{24}$$

where

$$Q(-i) = \sum_{j \neq i, j \in N} \sum_{k \in \mathcal{F}_i(f_i), k \neq i} p_k g_{kj} \tag{25}$$

and $Q(-i)$ is independent of f_i. Therefore, for arbitrary two frequencies a' and a'' of player i, we have

$$Q_{a'}(-i) = Q_{a''}(-i) \tag{26}$$

and

$$\begin{aligned}
& P(a', a_{-i}) - P(a'', a_{-i}) \\
= & \left\{ \textstyle\sum_{k \in \mathcal{F}_i(a')} p_k g_{ki} + \frac{1}{2} \times Q_{a'}(-i) \right\} \\
& - \left\{ \textstyle\sum_{k \in \mathcal{F}_i(a'')} p_k g_{ki} + \frac{1}{2} \times Q_{a''}(-i) \right\} \\
= & \; U_i(a', a_{-i}) - U_i(a'', a_{-i}).
\end{aligned} \tag{27}$$

Therefore, according to the definition in (5), the simplified game is a potential game and has at least one Nash equilibrium. Thus, the existence of Nash equilibrium in NTMG is obtained from the equivalence derived from Lemma 2. ■

Lemma 2 shows that at the equilibrium, the non-cooperative APs will always transmit at the maximum power level. This seem to be the best choice for each one of the APs. However, it is usually not a favorable strategy from a social-welfare point of view. The price of anarchy is owing to the non-cooperative nature. To bridge the performance gap, we propose a *linear pricing* scheme to combat with the selfish behaviors, i.e., the players are forced to pay a *tax* proportional to the utilized resources. For example, we could impose a price to all selfish APs for the power they utilize. Hence for each AP, the utility function becomes

$$U_i = R_i^*(\gamma_i) - \lambda_p^i p_i \tag{28}$$

where λ_p^i represents the *power utilization price* specific for the i-th AP and p_i is its transmission power. Therefore, the more power AP uses, the more tax it has to pay. By imposing power prices properly, a more desirable equilibrium may be induced, from a social-welfare point of view. We define the corresponding game as a Non-cooperative Throughput Maximization Game with Pricing (NTMGP).

Let us first investigate the impact of prices on the behaviors of players. If $\lambda_p^i = 0$, where no price is imposed, the i-th AP will transmit at the the maximum power and causes extra interference to other APs. However, if we impose an unbearably high price, say $\lambda_i^p = \infty$, the AP would rather not to transmit at all. Based on these observations, we propose a heuristic linear pricing scheme to improve the overall throughput in non-cooperative wireless mesh access networks.

To enforce the scheme, we introduce a *pricing dictator unit* (PDU) into the network which determines the prices for all APs and informs them timely. In addition, we assume that the PDU has the monitoring capability and is aware of the operating frequencies of each cell. There are two prices charged by the PDU for each non-cooperative AP. Besides the power utilizing price λ_p^i, a *frequency switching price* λ_f^i is imposed on the i-th AP whenever it changes the operating frequency. The price setting process is described as follows.

Price setting process:

Phase I:

- The PDU sets $\lambda_f^1 = \cdots \lambda_f^N = 0$ and $\lambda_p^1 = \cdots \lambda_p^N = 0$ and all APs play NTMG until converges, i.e., a Nash equilibrium is reached.
- The PDU collects the current throughput information from each cell, denoted by M_i, where i is the index of the cell.

Phase II:

- The PDU sets $\lambda_f^1 = \cdots \lambda_f^N = \infty$.
- For each AP indexed by $i = 1, \cdots, N$:
  1) The PDU sets $\lambda_p^i = \infty$ for the i-th AP and let the APs play the NTMGP. Upon convergence, the PDU collects the overall throughput, say V_i, in the current price setting.
  2) Calculate the power utilizing price for the i-th AP as

$$\tilde{\lambda}_p^i = \frac{V_i - \sum_{j=1, j \neq i}^{N} M_j}{p_{max}} \tag{29}$$

3) Reset $\lambda_p^i = 0$.

Output:

- Power utilizing price vector $\tilde{\lambda}_{\rm p} = [\tilde{\lambda}_p^1, \cdots, \tilde{\lambda}_p^N]$
- Frequency switching price vector $\tilde{\lambda}_{\rm f} = [\infty, \cdots, \infty]$

In the price setting process above, the PDU imposes zero prices for all APs initially. As a consequence, all APs will transmit with p_{max} at the equilibrium, as shown in Lemma 2. Upon convergence, the PDU fixes the frequency switching price to infinity which discourages the non-cooperative APs from switching channels thereafter. In (29), $\sum_{j=1,j\neq i}^{N} M_j$ is the sum throughput for all cells other than i, when the i-th AP transmits with the maximal power due to the zero power price. Similarly, V_i is the sum throughput of other cells when the i-th AP is silent due to the unaffordable power price. Therefore, in (29), the power utilization price charged for the i-th AP, a.k.a., $\tilde{\lambda}_p^i$, can be viewed as a compensation to the impact it causes on the overall throughput of other cells. The more power it utilizes, the more severe it affects the other players and thus the more it pays, as illustrated in (28). Hence, by imposing taxes deliberately, the selfish behaviors of non-cooperative APs are effectively discouraged and a more desirable equilibrium, in term of overall throughput of the whole network, can be induced. We will present the detailed performance evaluation of CTMG, NTMG and NTMGP in the next section.

V. Performance Evaluation

We consider a wireless mesh access network with N homogeneous APs. The simulation parameters are summarized as follows.

- Each AP has a maximum power $p_{max} = 100mW$ and a minimum power $p_{min} = 10mW$ and 10 different power levels as $[10mW, 20mW, \cdots, 100mW]$.
- The noise experienced at each receiver is assumed identical and has a power of $2mW$.
- All APs use IEEE 802.11b standard as the MAC protocol. In other words, each AP has four feasible data rate, $1, 2, 5.5, 11$ Mbps and 3 non-overlapping channels, i.e., $c = 3$.
- Without loss of generality, we assume that the received power is inversely proportional to the square of the Euclidian distance.
- The smoothing factor τ decreases as $\tau = 10/k^2$, where k is the negotiation step.
- The stopping criteria for NETMA and NTMG are the maximum number of iterations, denoted by ω.

For the sake of simplicity, we utilize a table-driven rate adaption algorithm, where a data rate is selected if and only if a certain SINR threshold is met. The mapping relationship is shown in Table I, provided by [38]. Note that our results can also be applied to arbitrary propagation models, rate adaptation algorithms and underlying multi-channel multi-rate MAC protocols.

TABLE I

Data rates v.s. SINR thresholds with maximum BER = 10^{-5}

Rate(Mbps)	Minimum SINR (dB)
1	-2.92
2	1.59
5.5	5.98
11	6.99

A. Example of Small Networks

We first consider a small wireless mesh access network with 5 APs, i.e., $N = 5$. All APs are randomly located in a square of 10-by-10 area. The global optimum solution is obtained by enumerating all feasible strategies, i.e., $(3 \times 10)^5$ profiles, as the performance benchmark. We first investigate the cooperative scenario where NETMA mechanism is applied. Next, the non-cooperative scenario is considered and each AP operates at the maximum power and adjusts the frequency only. The stopping criteria for both NETMA and NTMG are the maximum number of iterations where $\omega = 200$. The performance comparison is shown in Figure 4.

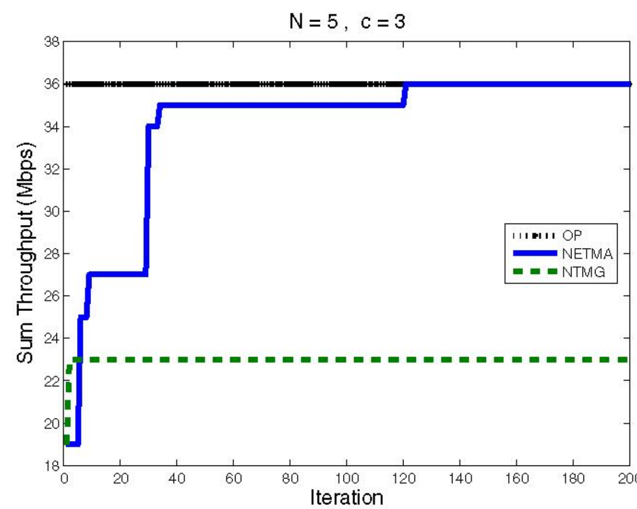

Fig. 4. Performance evaluation of the wireless mesh access network with N = 5 and c = 3.

As indicated by the *OP* curve, the global optimum obtained by enumeration approach functions as the upper bound of the overall throughput. In Figure 4, we observe that NETMA gradually catches up with the global optimum as negotiations go. As expected, the non-cooperative APs yield remarkably inferior performance in terms of overall throughput, depicted by the *NTMG* curve. The inefficiency is due to the selfish behavior that APs transmit at the maximum power and are regardless of the interference. The existence of Nash equilibrium in both CTMG and NTMG are substantiated by the convergence of curves in Figure 4. Figure 5 and Figure 6 depict the trajectories of frequency negotiations and power level negotiations in NETMA, respectively. At the initialization, each AP randomly picks a frequency and a power level and negotiates with each

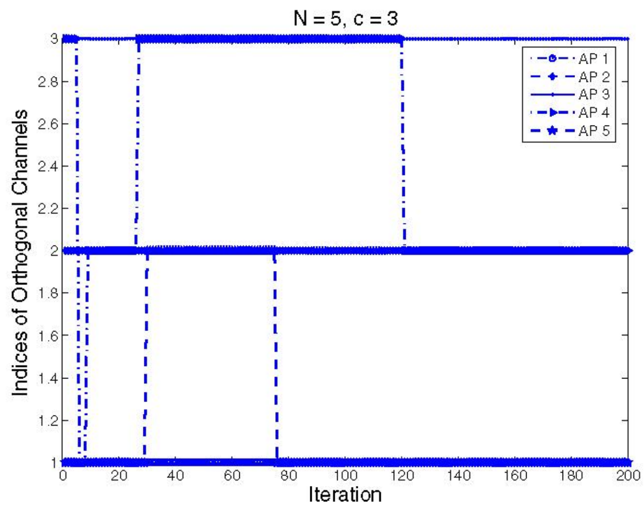

Fig. 5. The trajectory of frequency negotiations in NETMA when N = 5 and c = 3.

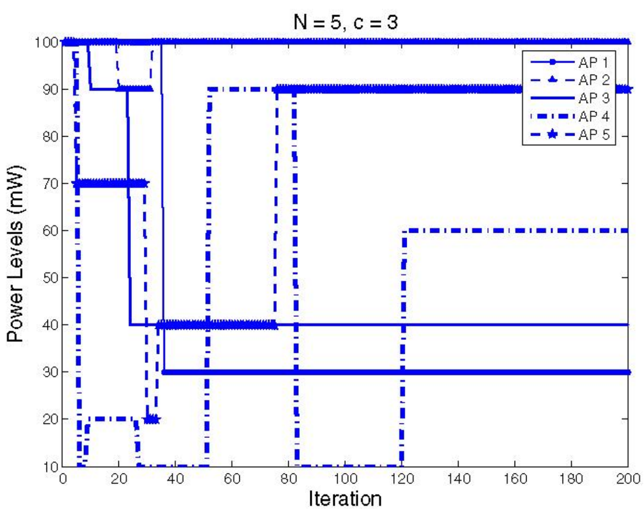

Fig. 6. The trajectory of power negotiations in NETMA when N = 5 and c = 3.

other following NETMA mechanism, until the optimum Nash equilibrium is achieved. Note that when the frequency vector and power vector converge in Figure 5 and Figure 6, the corresponding overall throughput obtained by NETMA catches the global optimum in Figure 4 simultaneously.

B. Example of Large Networks

We now consider a large wireless mesh access network with 20 APs. The enumeration approach is no longer feasible in this scenario due to the enormous strategy space. The 20 APs are randomly scattered in a d-by-d square, where the *side length* d is a tunable parameter in simulations. We investigate both cooperative and non-cooperative cases represented by *NETMA* and *NTMG* curves, where the maximum number of iterations is set to $\omega = 1000$. Figure 7 pictorially depicts the performance inefficiency of NTMG caused by the non-cooperative APs which transmit at the maximum power. The average throughput per AP is calculated by averaging the results of 50 simulations, for each value of the side length d. In Figure 7, it is worth noting that as the side length d gets bigger, the performance gap between NETMA and NTMG reduces. The reason is that when the area is large, the impact of mutual interference is less severe and so is the performance deterioration. However, when the network is crowded, i.e., d is small, the selfish behaviors are remarkably devastating.

To alleviate the throughput degradation by the non-cooperative APs, we implement the linear pricing scheme introduced in Section IV. The throughput improvement is illustrated as *NTMGP* in Figure 7. It is noticeable that by utilizing the proposed pricing scheme, the efficiency of Nash equilibrium is dramatically enhanced, especially for crowded networks. Therefore, the selfish incentives of the non-cooperative APs have been effectively suppressed.

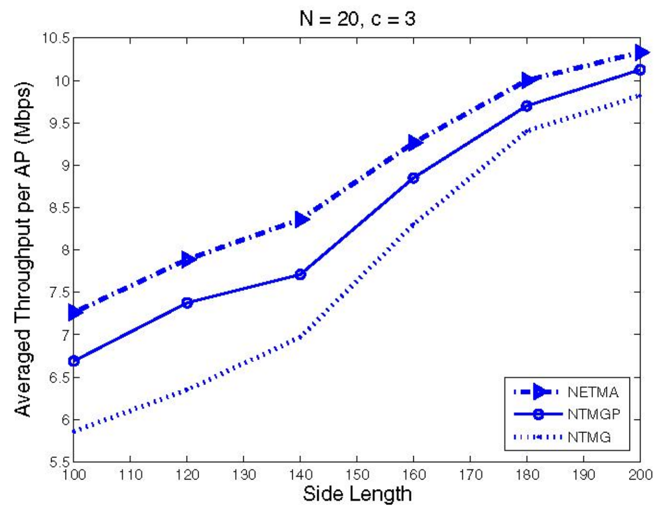

Fig. 7. Performance evaluation of the wireless mesh access network with N = 20 and c = 3.

VI. Conclusions and Future Work

In this paper, we investigate the throughput maximization problem in wireless mesh access networks. The problem is coupled due to the mutual interference and hence challenging. We first consider a cooperative case where all APs collaborate with each other in order to maximize the overall throughput of the network. A negotiation-based throughput maximization algorithm, a.k.a., NETMA, is introduced. We prove that NETMA converges to the optimum solution with arbitrarily high probability. For the non-cooperative scenarios, we show the existence and the inefficiency of Nash equilibria due to the selfish behaviors. To bridge the performance gap, we propose a linear pricing scheme which tremendously improves the performance in terms of overall throughput. The analytical results are verified by simulations.

In our work, we consider a saturated wireless mesh access network with orthogonal channels. Our future work will extend to non-saturated networks with partially overlapping channels.

References

[1] I. F. Akyildiz and X. Wang, "A survey on wireless mesh networks," *Communications Magazine, IEEE*, vol. 43, no. 9, pp. S23–S30, 2005.

[2] H.-J. Ju and I. Rubin, "Backbone topology synthesis for multiradio mesh networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, Nov.2006.

[3] J. Zhu and S. Roy, "802.11 mesh networks with two-radio access points," *IEEE International Conference on Communications. ICC'05*, vol. 5, pp. 3609– 3615, 2005.

[4] [Online]. Available: http://moment.cs.ucsb.edu/meshnet/

[5] A. K.Das, R. Vijayakumar, and S. Roy, "Static channel assignment in multi-radio multi-channel 802.11 wireless mesh networks: Issues, metrics, and algorithms," *Globecom'06*, 2006.

[6] R. Vedantham, S. Kakumanu, S. Lakshmanan, and R. Sivakumar, "Component based channel assignment in single radio, multi-channel ad hoc networks," *Proceedings of the 12th annual international conference on Mobile computing and networking,MOBICOM'06*, pp. 378–389, 2006.

[7] A. Raniwala, K. Gopalan, and T. cker Chiueh, "Centralized channel assignment and routing algorithms for multi-channel wireless mesh networks," *ACM SIGMOBILE Mobile Computing and Communications Review*, pp. 50–65, 2004.

[8] M. Alcherry, R. Bhatia, and L. E. Li, "Joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks," *Proceedings of the 11th annual international conference on Mobile computing and networking, Mobicom'05*, pp. 58–72, 2005.

[9] K. N. Ramachandran, E. M.Belding, K. C.Almeroth, and M. M. Buddhikot, "Interference-aware channel assignment in multi-radio wireless mesh networks," *INFOCOM'06*, 2006.

[10] A. H. M. Rad and V. W. Wong, "Joint channel allocation, interface assignment and mac design for multi-channel wireless mesh networks," *INFOCOM'07*, 2007.

[11] C. Eklund, R. Marks, K. Stanwood, and S. Wang, "Ieee standard 802.16: a technical overview of the wirelessman-tm air interface for broadband wireless access," *IEEE Communications Magazine*, vol. 40, pp. 98–107, 2002.

[12] C. cheng Chen, E. Seo, H. Luo, N. H. Vaidya, and X. Wang, "Rate-adaptive framing for interfered wireless networks," *INFOCOM'07*, 2007.

[13] P. Dubey, "Inefficiency of nash equilibria," *Mathematics of Operations Research*, vol. 11, pp. 1–8, 1986.

[14] T. Roughgarden, *Selfish Routing and the Price of Anarchy*. The MIT Press, 2005.

[15] C. Na, J. K.Chen, and T. S.Rappaport, "Measured traffic statistics and throughput of ieee 802.11b public wlan hotspots with three different applications," *IEEE Transactions on Wireless Communications*, vol. 5, pp. 3296–3305, Nov.2006.

[16] X. Yang, G. Feng, and D. S. C. Kheong, "Call admission control for multiservice wireless networks with bandwidth asymmetry between uplink and downlink," *IEEE Transactions on Vehicular Technology*, vol. 55, Jan.2006.

[17] Q. Pang, S. Liew, and V. Leung, "Performance improvement of 802.11 wireless network with tcp ack agent and auto-zoom backoff algorithm," *IEEE 61st Vehicular Technology Conference*, vol. 3, pp. 2046–2050, 2005.

[18] F. Anjum, M. Elaoud, D. Famolari, A. Ghosh, R. Vaidyanathan, A. Dutta, and P. Agrawal, "Voice performance in wlan networks - an experimental study," *Globecom'03*, 2003.

[19] S. Pilosof, R. Ramjee, D. Raz, Y. Shavitt, and P. Sinha, "Understanding tcp fairness over wireless lan," *In Proceedings of IEEE INFOCOM '03.*, vol. 13, 2003.

[20] S. W. Kim, B.-S. Kim, and Y. Fang, "Downlink and uplink resource allocation in ieee 802.11wireless lans," *IEEE Transactions on Vehicular Technology*, vol. 54, Jan.2005.

[21] J. Proakis, *Digital Communication*. McGraw-Hill Science/Engineering/Math (4 edition), 2000.

[22] C. Peng, F. Yang, Q. Zhang, D. Wu, M. Zhao, and Y. Yao, "Impact of power and rate selection on the throughput of ad hoc networks," *IEEE International Conference on Communications. ICC '06.*, vol. 9, pp. 3898–3902, 2006.

[23] J. Neel, J. Reed, and R. Gilles, "Game models for cognitive radio algorithm analysis," *SDR Forum Technical Conference*, 2004.

[24] D.Monderer and L.Shapley, "Potential games," *Journal of Games and Economic Behavior*, vol. 14, pp. 124–143, 1996.

[25] V. Srivastava, J. Neel, A. Mackenzie, R. Menon, L. Dasilva, J. Hicks, J. Reed, and R. Gilles, "Using game theory to analyze wireless ad hoc networks," *IEEE Communications Surveys and Tutorials*, vol. 7, pp. 46–56, 2005.

[26] J. Neel, "Analysis and design of cognitive radio networks and distributed radio resource management algorithms," Ph.D. dissertation, Virginia Polytechnic Institute and State University, 2006.

[27] J. Marden and G. Arslan, "Joint strategy fictitious play with inertia for potential games," *In Proceeding of IEEE Conference on Decision and Control*, 2005.

[28] N. Nie, C. Comaniciu, and P. Agrawal, "A game theoretic approach to interference management in cognitive networks," *In Proceeding of IMA*, 2006.

[29] N. Nie and C. Comaniciu, "Adaptive channel allocation spectrum etiquette for cognitive radio networks," *In Proceedings of IEEE DySPAN'05*, 2005.

[30] J. Neel and J. Reed, "Performance of distributed dynamic frequency selection schemes for interference reducing networks," *In Proceeding of Milcom'06*, 2006.

[31] G. Arslan, M. F. Demirkol, and Y. Song, "Equilibrium efficiency improvement in mimo interference systems: A decentralized stream control approach," *IEEE Transaction on Wireless Communications*, 2007, to appear.

[32] H. P. Young, *Individual Strategy and Social Structure*. Princeton, NJ: Princeton University Press, 1998.

[33] D. Bertsekas and R. Gallager, *Data Networks*. Prentice Hall; 2 edition, 1991.

[34] L. Kleinrock, *Queueing Systems, Volume 1, Theory*. Wiley-Interscience; 1 edition, 1975.

[35] A. Mishra, V. Shrivastava, S. Banerjee, and W. Arbaugh, "Partially overlapped channels not considered harmful," *ACM SIGMETRICS International Conference on Measurement and Modeling of Computer Systems (ACM SIGMETRICS Performance)*, 2006.

[36] P. J. M. van Laarhoven and E. H. L. Aarts, *Simulated Annealing: Theory and Applications*. Holland: Reidel, 1987.

[37] R. K.Lam, D.-M. Chiu, and J. C.S.Lui, "On the access pricing issues of wireless mesh networks," *IEEE International Conference on Distributed Computing Systems, ICDCS'06*, 2006.

[38] J. Yee and H. Pezeshki-Esfahani, "Understanding wireless lan performance trade-offs," *Communication Systems Design*, Nov.2002.