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Achieving maximum flow in interference-aware wireless sensor networks with smart antennas

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Abstract

Directional antenna offers various benefits for wireless sensor networks, such as increased spatial reuse ratio and reduced energy consumption. In this paper, we formulate the maximum flow problem as an optimization problem in interference-limited wireless sensor networks with switched beam directional antennas. The optimization problem is solvable in the presence of an omniscient controller, but it is NP-hard. Therefore, we seek a distributed algorithm to achieve the maximum flow through jointly routing and scheduling. The maximum flow between given source destination pair is determined forwardly hop by hop and is verified by the proposed feasible condition at downstream nodes. This method works for both single-beam antenna and multi-beam antenna with some variation in the feasibility condition.

Keywords: Routing; Maximum flow; Interference; Wireless sensor networks; Smart antenna

1. Introduction

Due to the hostile wireless channel and interference among flows, how to achieve the maximum throughput in multi-hop wireless networks has been of great interest over the past decades. Especially for resource-constrained wireless sensor networks, how to improve the system capacity is even more important. Sensors are usually powered by batteries and not rechargeable, while they should be functioning for a relatively long period. These requirements pose serious challenges on protocol design. With the switched beam technology, the smart antenna is shown to be an appealing option for wireless sensor networks. By concentrating RF energy in the intended transmission direction, the spatial transmission region shrinks to a sector. Directional antenna is able to reduce energy consumption, and improve spatial reuse ratio, thus can significantly boost the channel capacity. It is feasible to equip sensor nodes with directional antenna, because the switched beam system could be built with fairly inexpensive off-the-shelf components [1]. Particularly at COTS frequencies 2.4 GHz or 5 GHz, switched beam directional antenna can be inexpensive and moderately small. Sensor nodes built with

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fairly powerful capabilities are able to accommodate the switch beam directional antenna.

Various work has been inspired by wireless networks with directional antennas. Although some asymptotic bounds for throughput are derived under certain assumptions about network deployment and node configuration, what the feasible maximum throughput is and how to achieve it are still not answered in the context of general wireless networks. The goal of this paper is to address the "what" and "how" problems in wireless sensor networks with switched beam directional antennas. Both issues are challenging in wireless sensor networks when considering the large scale and limited node capability. For the general purpose, we have no assumption on the traffic pattern. For different applications, various communication pairs may exist, including sensor-to-sensor, sensor-to-sink and sink-to-sensor. We address the problem for any source destination pair so that it applies to any specific traffic pattern.

We attempt to solve the problem in a generalized setting. This problem is inherently a joint multipath routing and optimal scheduling problem. Generally, multipath routing is capable of supporting a larger amount of flow than single path routing. Nevertheless, the interference [18] restricts the efficiency of multipath routing. Taking advantage of mitigated interference, multipath routing is more justifiable in wireless sensor networks with directional antennas. Yet the more involved interference pattern of multipath routing further complicates the problem because of the substantial problem size and searching space. Routing serves as the preliminary of link scheduling and the complexity of the whole problem is actually the multiplication of them.

The contributions of this paper are twofold. First, we formulate the maximum flow problem as centralized MIP problems for both single-beam and multi-beam antennas with slight difference. We show that the maximum flow problem is NP-hard in wireless sensor networks because of the interference constraint. The centralized MIP problem could produce the optimal solution, but it requires an omnipotent coordinator, which is usually unrealistic. In contrast, a distributed algorithm with reasonable computation complexity and better scalability is more practical and computationally efficient. Second, we propose the feasible condition of flow. The maximum flow is attainable only if all constituent links over the paths meet the feasibility condition. With the criterion, the attainable maximum flow from the source node to the destination node is validated at each hop forwardly. Moreover, we discuss the feasibility conditions for both single-beam and multi-beam antenna respectively.

The paper is organized as follows. The next section summarizes the previous work on related topics. Section 3 defines the resource sharing graph based on the antenna model, which leads to general problem formulation for maximum flow. Then we present the problem formulation of maximum flow for switched beam antennas in Section 4. Section 5 illustrates the feasible conditions for feasible arc flows. Numerical result is presented in Section 6. Finally, Section 7 concludes the paper.

2. Related work

There exist many schemes to address problems associated with wireless networks with directional antennas, such as MAC [5], routing and scheduling [6,7]. Some papers have derived the asymptotic throughput bounds under certain assumptions on network topology and node configuration. The seminal paper by Gupta and Kumar [2] studied the network comprising of n randomly placed non-mobile nodes. Subsequent work [3] investigates the capacity gain of wireless ad hoc networks with directional antennas over omni-directional antennas. In [4], Kodialam and Nandagopal consider the problem of joint routing and scheduling to achieve a given rate vector. The only interference constraint they take into consideration is that a node cannot transmit or receive simultaneously. They formulate the scheduling problem as an edge-coloring problem and provide a polynomial time algorithm. The approach achieves at least 67% of the optimal throughput. Jain et al. [9] model the interference between neighboring nodes using a conflict graph and present methods for computing the lower and upper bounds. They focus on the routing component alone. However, they do not propose any approximation algorithm to solve the routing problem. In [10], Peraki and Servetto study the maximum throughput in dense random wireless networks with directional antennas. They derive the asymptotic upper bounds on throughput by solving the minimum cut problem. A coordinated packet injection schedule to minimize inter-flow interference is investigated in multi-hop networks [13]. Joint optimization of two metrics, source throughput and packet head-of-queue delay, are achieved through controlling the packet waiting times of optimal number of flows. An optimal resource allocation scheme is proposed based on the maximal cliques of contention flows in [17]. Some routing algorithms are proposed to achieve the maximum network capacity [15,16]. In [15], Kar et al. propose a routing algorithm aiming to maximize the total amount of data that is successfully carried by the network. The routing algorithm computes the shortest path with link weight, which is characterized by residual nodal energy.

Several works study the multipath routing in wireless ad hoc networks using directional antennas [11,12]. In [12], Tang et al. define the path interference to find the minimum single path and node-disjoint multiple paths in wireless networks equipped with directional antennas. Since interference affects the network performance, some papers attempt to reduce the interference through topology control. A recent work by Burkahart et al. concisely defines the interference and proposes several interference-aware topology algorithms [19]. In [20], Sundaresan and Sivakumar define a unified MAC layer framework for a variety of smart antennas, and derive the corresponding unified MAC algorithms. In [8], Roy et al. propose a MAC and routing protocol for ad hoc networks with directional antennas to select maximally zone disjoint routes for load balancing. Hou et al. [14] exploit the energy efficiency of directional antenna to maximize the network lifetime.

3. General maximum flow problem subject to interference

3.1. Antenna model

According to beam pattern (beam-radius, beam-width, beam orientation), we have omni-directional antennas, single-beam directional antennas (e.g., single-beam switched beam antennas), multi-beam directional antennas (e.g., multi-beam switched beam antennas or sectorized beam antennas). A directional antenna can transmit and receive directionally. To be clear, for single-beam directional antennas, we assume only one directional transmitting beam or one directional receiving beam can be active at a time; for multi-beam directional antennas, multiple directional transmission beams or multiple directional receiving beams can be active at a time. However, a beam can only be either transmitting or receiving at any instant.

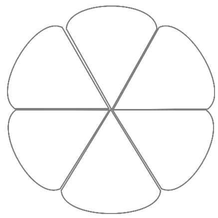

Fig. 1. An illustration of directional antenna model.

An illustration of a switched beam antenna with six beams is shown as Fig. 1. In this paper, we assume that the antenna is directed to discrete directions, with fixed beam-radius and beam-width. Beam-radius is the maximum distance that a transmission can reach. Beam-width is determined by the angle of a sector. For a six-beam directional antenna, the angle of a beam is $\pi/3$. The direction a beam targeting to is defined as the beam orientation. There is a link between two nodes if the distance between them is shorter than the beam-radius. An illustration of a node graph comprising of nodes with directional antennas is shown as Fig. 2, though a realistic node graph is always more complex.

3.2. Link resource sharing graph

Given the toy example of node graph Fig. 2, the link resource sharing graph for the single-beam directional antenna can be represented as Fig. 3. Links in the node graph are numerated on the first

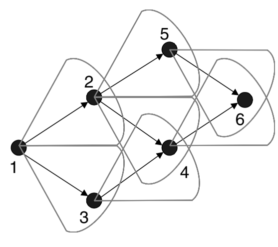

Fig. 2. A simple illustration of node graph $G = (V, E)$.

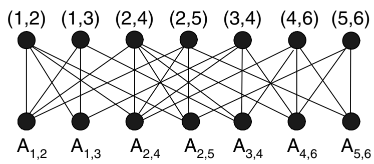

Fig. 3. Link resource sharing graph for single-beam directional antennas.

row in Fig. 3. The bottom row indicates the capacity of the corresponding link. For instance, link (1,2) interferes with link (1,3), (2,3) and (2,4) in Fig. 2, because they cannot be active concurrently given single-beam directional antenna. Therefore, any two of them are not allowed to be active simultaneously. For example, the capacity of (2,4) are shared with those links, as indicated by $A(2,4)$. In other words, (1,2), (2,4), (2,5), (3,4) and (4,6) share the time fraction for using the common wireless channel.

For nodes with multi-beam directional antennas, link (1,2) and (1,3) can be active simultaneously. Only outgoing links (2,4) and (2,5) interfere with link (1,2). As depicted in Fig. 4, similarly for link (2,4), the interfering links are reduced to (1,2) and (4,6). The decrease of interference level is significant. As a result, the link resource sharing graph of the network using multi-beam directional antennas is a subgraph of single-beam directional antennas.

3.3. General formulation of maximum flow

The problem here to be addressed is: given network $G(V,E)$ and existing flows, find the maximum flow supported by the network between pair s–d. For clarity, notation used in this paper is listed in Table 1. Based on the resource sharing graph, the maximum flow problem can be formulated as the following optimization problem:

$$\begin{aligned} &\max \quad f \\ &\text{s. t.} \quad \sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i = s, \\ 0 & i = V - \{s,d\}, \\ -f & i = d; \end{cases} \\ &\qquad \sum_{(k,l)\in A_{i,j}} x_{k,l} \leqslant u_{i,j} \quad \forall (i,j) \in E; \\ &\qquad x_{i,j} \geqslant 0 \quad \forall (i,j) \in E. \end{aligned} \tag{1}$$

where $u_{i,j}$ is the normalized remaining capacity ($0 \leqslant u_{i,j} \leqslant 1$) for (i,j). This is a traditional maximum flow problem with added interference (contention) constraint. To straighten the problem formulation, the interference constraint is further explored and characterized in the next section.

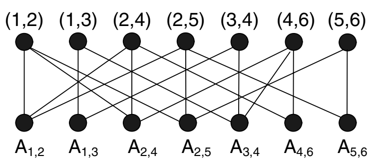

Fig. 4. Link resource sharing graph for multi-beam directional antennas.

Table 1
Notation

B	The total number of beams at each node
E	The set of edges
V	The set of nodes
(i,j)	Link from node i to node j
$x_{i,j}$	Flow over link (i,j)
f	Flow from source node s to destination node d
$f_{i,j}$	Maximum flow over arc (i,j)
d_{ij}	Distance between node i and node j
$\alpha(i,j)$	Transmission direction from node i to node j
θ_i^j	The beam used by node i to transmit to or receive from node j
$b(i,l)$	The lth beam of node i
$b_{i,j}(i,l)$	Whether link (i,j) is in the l beam of node i.
$T(G)$	The flow contention graph of G
$\chi_w^*(G)$	Fractional chromatic number of graph G
λ_j	Fractional color of independent set s_j

4. Formulation of interference-constrained maximum flow

4.1. Interference region

We assume that an antenna both transmits and receives directionally, but it cannot transmit and receive simultaneously. With directional antenna, two links interfere with each other if a receiver is in the transmitting beams of both transmitters, shown in Fig. 5. To guarantee successful reception at node j, any node in the receiving beam of j cannot transmit towards j before current transmission finishes. In wireless sensor networks with directional antennas, the interference region is specified not only by the transmission range or beam-radius, but also the beam orientation.

The protocol model: In the protocol model, the transmission from node i to node j is successful if (1) j is in the transmission range of i, $d_{ij} \leqslant R$, where R is the transmission range; (2) any node u that in

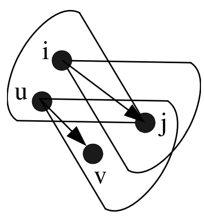

Fig. 5. An illustration of interference caused by (u,v) to (i,j).

the receiving beam of j from i is not transmitting in the beam covering j (when interference range = transmission range). This means that j must be outside of transmission beam of u. Instead of the circular interference area in omni-directional antenna equipped wireless sensor network, the interference region of directional antenna is a beam. Multipath routing, which generally causes more serious interference than single path routing, is more justifiable for network with smart antenna in terms of throughput.

Since the interference region is a beam, the information about the beam to which a link belongs is essential for routing and scheduling. Suppose there are B fixed beams for each antenna. Denote the angle between node i and another node j as $\alpha(i,j)$ as depicted in Fig. 6. With the knowledge of $\alpha(i,j)$, (i,j) can be located in the beam $\theta_i^j = \lceil \alpha(i,j)/2\pi \times B \rceil$ of i, which is the transmission beam for link (i,j).

Now we can recapitulate condition (2) of the protocol model in the following way:

(2′) When (i,j) is active, for any node u in j's receiving beam towards i, the beam θ_u^j should keep silent. Denote $b(i,l)$ as the lth beam of node i, where $l = 1, \ldots, B$.

4.2. MIP formulation for single-beam directional antenna

Because the single-beam directional antenna can only target to one beam at a time. So the channel utilization is shared by all links in all beams. The time sharing constraint is formulated as

$$\sum_{l=1}^{B} \left(\sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l) + \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,l) \right) \leqslant 1 \quad \forall i \in V. \tag{2}$$

For single-beam directional antenna, we can formulate the maximum flow problem as the following mixed integer programming (MIP).

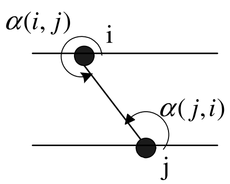

Fig. 6. Illustration of $\alpha(i,j)$ and $\alpha(j,i)$.

Problem formulation 1:

$$\begin{aligned}
\max \quad & f \\
\text{s.t.} \quad & \sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i = s, \\ 0 & i = V - \{s,d\}, \\ -f & i = d; \end{cases} \\
& \underbrace{\sum_{u\in b(i,l)} \sum_{(u,v)\in E} x_{u,v} b_{u,v}(u, \theta_u^i)}_{\text{interfering links in } l\text{th beam}} \\
& + \underbrace{\sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l)}_{\text{incoming flows}} \leqslant 1 \quad \forall l, i, \\
& \sum_{l=1}^{B} \left(\sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l) + \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,l) \right) \leqslant 1 \quad \forall i \in V, \\
& b_{i,j}(i,l) = \begin{cases} 1, & \text{if } (i,j) \in b(i,l), \\ 0, & \text{otherwise}, \end{cases} \\
& x_{i,j} \geqslant 0 \quad \forall (i,j) \in E.
\end{aligned} \tag{3}$$

The first constraint describes the in-flow and the out-flow at each node. The second constraint indicates the flow interference around i as specified by condition (2′) in the protocol model. The first term represents the sum of flows causing interference to i in beam l. When those flows are active, node i must restrain from receiving. The second term stands for the total incoming flows to i in beam l. The second and third constraints aggregately describe the interfering flows at a node.

4.3. MIP formulation for multi-beam directional antenna

For multi-beam directional antenna, we have the following time sharing constraint:

$$\max_{l:1\leqslant l\leqslant B} \text{in-flow of beam } l + \max_{l:1\leqslant l\leqslant B} \text{out-flow of beam } l \leqslant 1.$$

We use max function because several beams can transmit or receive simultaneously. This constraint is expanded as

$$\max_{l:1\leqslant l\leqslant B} \sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l) + \max_{l:1\leqslant l\leqslant B} \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,l) \leqslant 1 \quad \forall i \in V. \tag{4}$$

Transforming (4) into the following linear constraints:

$$\sum_{(k,i)\in E} x_{k,i} b_{k,j}(i,l) + \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,m) \leqslant 1 \quad \forall l, m, \ \forall i \in V.$$

Now the maximum flow problem in wireless sensor networks with multi-beam directional antennas can be modeled by the following MIP

Problem formulation 2:

$$
\begin{aligned}
\max \quad & f \\
\text{s.t.} \quad & \sum_{\{j:(i,j)\in E\}} x_{i,j} - \sum_{\{j:(j,i)\in E\}} x_{j,i} = \begin{cases} f & i = s, \\ 0 & i = V - \{s,d\}, \\ -f & i = d; \end{cases} \\
& \sum_{u\in b(i,l)} \sum_{(u,v)\in E} x_{u,v} b_{u,v}(u,\theta_u^i) + \sum_{(k,i)\in E} x_{k,i} b_{k,i}(i,l) \leqslant 1 \quad \forall l, i, \\
& \sum_{(k,i)\in E} x_{k,i} b_{k,j}(i,l) + \sum_{(i,j)\in E} x_{i,j} b_{i,j}(i,m) \leqslant 1, \\
& \forall 1 \leqslant l, m \leqslant B \quad \forall i \in V, \\
& b_{i,j}(i,l) = \begin{cases} 1, & \text{if } l = \theta_i^j, \\ 0, & \text{otherwise}, \end{cases} \\
& x_{i,j} \geqslant 0, \quad \forall (i,j) \in E.
\end{aligned} \tag{5}
$$

The first two constraints are the same as those in (3). The last constraint guarantees that the flow is feasible because the in-flow and out-flow share the capacity at the node. Observe that the constraint becomes linear at the cost of adding more constraints. The number of constraints is increased by a factor of $B^2 - 1$.

Until now, the maximum flow has been formulated for single-beam and multi-beam directional antennas, respectively. Except one constraint, the formulations of them are the same. It is easy to solve the MIPs in (3) and (5). So we give a brief description of the algorithm. First, we obtain $b_{i,j}(i,l)$s after establishing the neighbor list of every beam at each node. The constants are determined by the relative positions between nodes. Then we need to calculate the remaining capacity in each beam for every node. The remaining capacity is the total capacity minus the capacity used by interfering flows. With remaining capacity in each beam, the MIP can be transformed to a standard MIP. Applying an existing optimization algorithm like branch and bound method [22,23], we can obtain the optimal solution.

4.4. Problem complexity

Many papers have analyzed the flow contention problem by resorting to graph theory. The independence number of the network can be reduced to a maximum feasible network throughput problem even if we only consider the routing with perfect scheduling [9]. Based on flow contention graph for given paths, some papers attempt to maximize flow by finding out the minimum clique cover of the network. However, the minimum clique cover problem is also NP-complete [21]. So the maximum flow problem in multi-hop wireless networks is generally at least NP-hard. Although smart antenna reduces the interference region, the contention graph still makes the maximum flow problem hard.

Theorem 1. *Maximum flow problem in wireless sensor networks with directional antennas is NP-hard.*

5. Distributed routing and scheduling

The optimization problem in the previous section has to be solved through a centralized algorithm, which is computationally expensive and not feasible in large-scale networks. In this section, we search for a distributed method to achieve the maximum flow more efficiently in resource limited wireless sensor networks. The feasibility of the end-to-end flow is validated by decomposing it into arc flows [23]. By an arc flow we mean a vector $x = x_{i,j}$ that satisfies the constraints in the maximum flow problem. Each element in the arc flow corresponds to the flow over an arc. In the rest of our paper, we refer to a link as an arc. If all constituent arc flows are feasible, then the end-to-end flow is feasible.

Obviously, each node must have the knowledge of its neighbors' available bandwidth to make routing decision. The total available bandwidth is shared by all interfering flows. Flows within two hops are potential interfering flows, because they may have en edge in the flow contention graph. A node overhears neighbors' transmissions, so it is able to calculate the available bandwidth in each beam for flow scheduling.

5.1. Control information for distributed routing and scheduling

Based on the information of available resource in each beam and the received information from neighbors, a node can perform jointly routing and scheduling. The indispensable control information to be exchanged between neighbors in case of single-beam and multi-beam directional antennas is summarized in Tables 2 and 3 respectively.

For each beam of node i, denote $a_{i,l}$ the allocated resource

Table 2
The exchanging control information for single-beam directional antenna

$R(i)$	Available bandwidth in terms of time fraction at node i

Table 3
The exchanging control information for multi-beam directional antenna

$R(i)_l$	Available time fraction for reception in beam l at node i
$a(i)_{r,\theta_i^j}$	The bandwidth used by incoming flows and interfering flows in the beam towards node j
$a(i)_r$	The bandwidth used by incoming flows and interfering flows in all beams at node i
$T(i)_{\bar{l}}$	Utilized resource for outgoing flows in all beams except the lth beam at node i

$$a_{i,l} = \sum_{(i,j)\in b(i,l)} x_{i,j} + \sum_{(m,i)\in b(i,l)} x_{m,i} + \sum_{(u,i)\in b(i,l)} \sum_{(u,v)\in E, v\neq i} x_{u,v} b_{u,v}(u, \theta_u^i). \quad (6)$$

The first and second terms are the outgoing and incoming flows in beam l of node i, respectively. The third term indicates the interfering flows. So the sum of them gives the occupied bandwidth in beam l of node i. For single-beam antenna, $R(i)$ is

$$R(i) = 1 - \sum_{l=1}^{B} a_{i,l}.$$

While for multi-beam antenna, the available bandwidth for reception in beam l is

$$R(i)_l = 1 - \max_{1\leqslant l\leqslant B}\left(\sum_{(i,j)\in b(i,l)} x_{i,j} + \sum_{(u,i)\in b(i,l)} \sum_{(u,v)\in E, v\neq i} x_{u,v} b_{u,v}(u, \theta_u^i)\right) - \sum_{(m,i)\in b(i,l)} x_{m,i}.$$

The max function calculates the bandwidth occupied by outgoing flows and interfering flows. The last term is the total amount of existing incoming flows in beam l at node i. Denote $T(i)_l$ the bandwidth used for transmissions in each beam

$$T(i)_l = \sum_{(i,j)\in b(i,l)} x_{i,j}.$$

Then the total bandwidth occupied by transmissions in all beams except l is

$$T(i)_{\bar{l}} = \sum_{l=1, l\neq\theta_i^k}^{B} T(i)_l.$$

Variables $a(i)_r$ and $a(i)_{r,\theta_i^j}$ will be explained in the next subsection. After calculating the information specified in Table 2 or 3, a node informs its neighbors. This information is identical for neighboring nodes located in the same beam of i, but may vary with beams. Once a new flow establishes or an existing flow terminates, involved nodes update the information about the available resource to neighbors. After exchanging the control information, every node has the knowledge of available resource in its own beams, and the utilized resource of its neighbors. That information collectively builds up a view of two hop resource utilization at each node, which is fundamental for feasible flow distribution.

Definition 1. A flow over an arc is feasible if it can be routed over succeeding link(s).

The flow $x_{i,j}$ over the arc (i,j) emanates from the *tail*, which is node i, and enters the head, which is node j. We can observe that the feasibility of arc flows implies the agreement of both head and tail nodes of the flow. Arc flows cannot exceed the beam capacity at both nodes.

5.2. Feasible condition of a flow over an arc

A feasible flow has to satisfy the following *feasibility condition* at each node on the path(s) except the last hop.

Feasibility condition

1. the flow over the arc must be no greater than the receiving capacity of the corresponding beam at the receiver,
2. the flow should be no greater than the transmitting capacity of the corresponding beam at the sender,
3. the flow can be supported at the receiver, which means that the flow can be routed by the receiver further towards the destination without congestion. For the last hop, this condition is skipped because the destination node just receives and does not need to forward packets.

To formulate this feasibility condition set, we consider single-beam and multi-beam directional antenna separately.

5.2.1. Feasibility condition for single-beam directional antenna

Suppose node k is transmitting to node i. Denote c_{k,θ_k^i} the transmission capacity of the beam towards i

$$c_{k,\theta_k^i} = 1 - a(k)_r - \sum_{l=1}^{B} T(k)_l.$$

According to the aforementioned feasibility condition, flow $x_{k,i}$ should satisfy

$$x_{k,i} \leqslant R(i), \quad x_{k,i} \leqslant c_{k,\theta_k^i}, \quad x_{k,i} \leqslant R(i)/2.$$

The last constraint guarantees that the flow is acceptable and can be routed through other beams of i. Apparently, the feasibility condition can be written concisely as

$$x_{k,i} \leqslant \min\{c_{k,\theta_k^i}, R(i)/2\}. \tag{7}$$

5.2.2. Feasibility condition for multi-beam directional antenna

For multi-beam directional antenna, the first two constraints are obvious,

$$x_{k,i} \leqslant R(i)_l, \quad x_{k,i} \leqslant c_{k,\theta_k^i},$$

where

$$c_{k,\theta_k^i} = 1 - a(k)_r - T(k)_{\theta_k^i}.$$

Remind that the incoming and outgoing flows share the bandwidth in multi-beam directional antenna, the transmission capability of all beams at node i after accepting $x_{k,i}$ is

$$\sum_{l=1, l\neq\theta_i^k}^{B} (1 - \max\{a(i)_r, a(i)_{r,\theta_i^k} + x_{k,i}\} - T(i)_l).$$

Only $B - 1$ beams, except the receiving beam, can be used for forwarding

$$a(i)_{r,l} = \sum_{(m,i)\in b(i,l)} x_{m,i} + \sum_{(u,i)\in b(i,l)} \sum_{(u,v)\in E, v\neq i} x_{u,v} b_{u,v}(u, \theta_u^i),$$

$$a(i)_r = \max_{1\leqslant l\leqslant B} \left(\sum_{(m,i)\in b(i,l)} x_{m,i} + \sum_{(u,i)\in b(i,l)} \sum_{(u,v)\in E, v\neq i} x_{u,v} b_{u,v}(u, \theta_u^i) \right).$$

The max function computes the total bandwidth used for reception after accepting $x_{k,i}$. To ensure that $x_{k,i}$ can be supported as specified in the feasibility condition (3), we have

$$x_{k,i} \leqslant (B - 1)(1 - \max\{a(i)_r, a(i)_{r,\theta_i^k} + x_{k,i}\}) + T(i)_{\bar{l}}.$$

To sum up, the feasibility condition for multi-beam directional antenna is

$$x_{k,i} \leqslant \min\{R(i)_l, c_{k,\theta_k^i}\},$$
$$x_{k,i} \leqslant (B - 1)(1 - \max\{a(i)_r, a(i)_{r,\theta_i^k} + x_{k,i}\}) + T(i)_{\bar{l}}. \tag{8}$$

The feasibility condition at all nodes is aggregately equivalent to the constraints in the centralized MIP formulation. The tremendous computation cost of the centralized MIP is significantly reduced and becomes affordable for sensor nodes.

5.3. Flow decrementing algorithm

At the very beginning, the source manages to push the amount of flow $f = \sum_{(s,i)\in E} x_{s,i}$ subject to the feasible condition into the network. However, the feasibility of the flow is to be explored in the network through probing.

In the probing procedure, the feasibility of the end-to-end flow is examined at each hop. Also, the maximum feasible flow can be discovered adaptively. When node i attempts to transfer packets to node j, $x_{i,j}$ is computed through the feasibility condition at node i. Node i includes the calculated value of $x_{i,j}$ in the probe packet and sends over the corresponding link. So multiple probing packets may be created at a node if multiple paths are used. Upon receiving the probe packet, node j checks the feasibility condition for the requested flow. If it is capable of supporting the requested flow, then the flow is admitted. In other words, a flow over an arc is acceptable if there exists a feasible solution satisfying the feasibility condition at node j. After node j decides to accept the flow, it will immediately update its available resource to all neighbors. Otherwise, it stops propagating the probe packet and indicates the maximum amount of flow viable for the node in the feedback packet. The upstream node i will adjust the outgoing flows based on the feedback information until all involved nodes agree on the arc flows. If the node is unable to reschedule the flow according to the feedback information, the amount of feasible flow is piggybacked over the reverse path until a node is able to reschedule the requested flow. If all intermediate nodes fail to reschedule, the feedback information will trace back to the source node. When the source node receives the feedback, it decrements the amount of outgoing flow accordingly. The probing process involves the verification of the feasibility condition at all intermediate nodes. Probing terminates at the destination node, indicating that the source and destination nodes agree on the routing and scheduling.

Theorem 2. *In a network of n nodes connected by m links, the flow decrementing algorithm runs in* O(*nm*) *time.*

Suppose node i performs a push on arc (i,j). If node j cannot support it, node i has to reschedule the flows over all arcs that the flow has traversed. Therefore, a failed push causes a reschedule of m links in the worst case. On the other hand, the flow just progresses through only one push if it is feasible. So the number of reschedules dominates the complexity of the algorithm. As the flow progresses a hop further, the total number of pushes, including rescheduling, increases m at most. For each denied flow schedule, the bandwidth of one node is used up and the node will not be visited. Therefore, the flow schedule can be denied at most n times as the flow propagates. Consequently, the algorithm performs $O(nm)$ pushes. When there are multiple flows, the algorithm's running time scales linearly.

6. Simulation results

In this section, the simulation results of the distributed algorithm implemented in C++ are compared with the numerical results of the two MIP problems. Nodes are randomly deployed in a 10×10 square, with transmission range of 2.5 units. The link capacity is normalized to 1. The network size varies from 20 to 40 nodes. A source–destination pair is randomly chosen from all nodes. There exist other random flows which may interfere with the flow between the source–destination pair. Constant traffic flows are generated in the stationary network. Each instance is repeated for 30 runs.

The maximum flow rates in 20, 30 and 40-node networks are shown in Fig. 7. As expected, the maximum flow in case of multi-beam directional antenna is greater than that using single-beam directional antenna. The performance of the distributed algorithm is comparable to the optimal result obtained in the centralized algorithm. The distributed algorithm is able to achieve approximately 95% of the flow that can be supported by the centralized algorithm. An interesting observation is that the maximum flow rate decreases inversely to the network size for networks with single-beam directional antenna, while the maximum flow increases for networks with multi-beam directional antenna. This trend is shown in both centralized and distributed algorithms. The reason for the different trend is that the single-beam directional antenna is more sensitive to interference caused by increased flows. As the number of nodes increases, the network density grows as the network field is fixed. Consequently, more interference among flows is introduced, so the maximum flow supportable for the given source destination pair decreases. However multi-beam directional antenna is capable of harnessing the advantage of space reuse more efficiently, so the interference level is still low even the network density increases. The maximum flow does not deteriorate with the increased density in the simulation. On the contrary, the maximum flow increases because more space-separated paths are available. We expect the maximum flow of the network with multi-beam directional antennas to degrade when the node density reaches a certain degree.

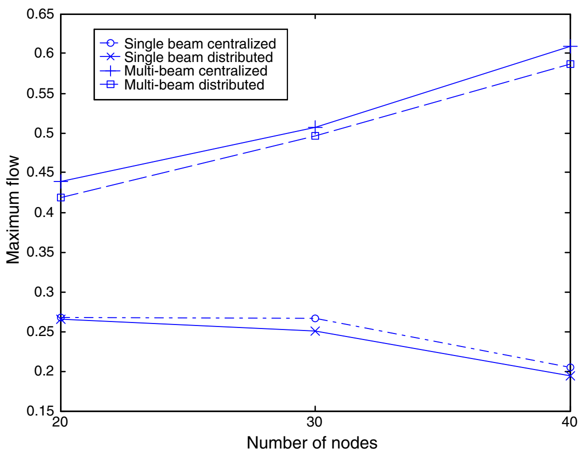

Fig. 7. Maximum flow in networks with different size.

We do not know how frequently the performance of using single-beam directional antenna and multi-beam directional antenna differs. The histograms of the maximum flow demonstrate the difference in Figs. 8–13. The histogram shows the frequency of maximum flow rate that falls into the corresponding interval. Apparently, the frequency of maximum flow obtained when using multi-beam directional antennas shifts to the higher value comparing to the case when using single-beam directional antennas. Using single-beam directional antenna tends to achieve maximum flow at relatively lower value. The maximum flow rate in case of single-beam directional antenna rarely reaches 0.5. On the other hand, the maximum flow rate can achieve the maximum capacity of 1 in some scenarios when using multi-beam directional antennas. The result is consistent with the fact that multi-beam directional antenna is less resource demanding in terms of channel reuse.

From the histograms, we observe that the distributed algorithm attains high flow rate less frequently than the centralized algorithm. This is due to the

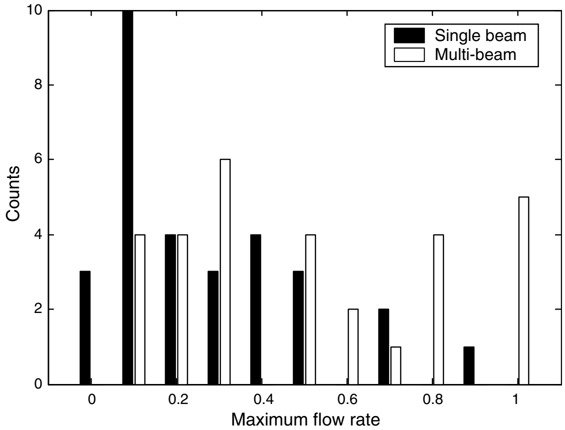

Fig. 8. Histogram of maximum flow in network with 20 nodes (centralized algorithm).

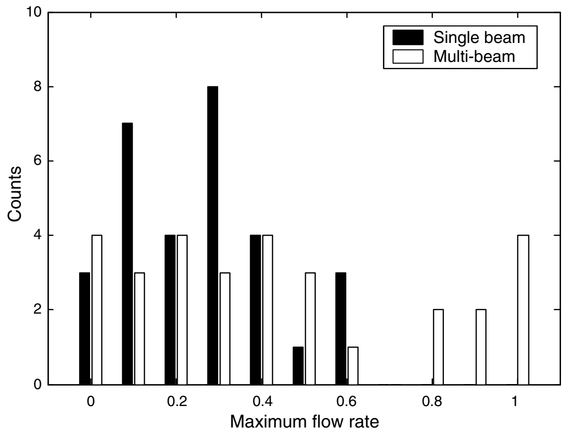

Fig. 9. Histogram of maximum flow in network with 30 nodes (centralized algorithm).

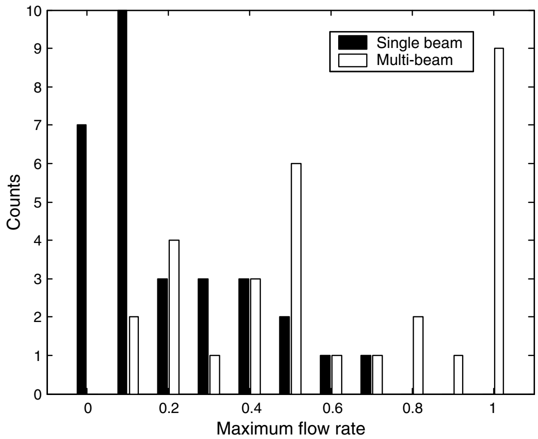

Fig. 10. Histogram of maximum flow in network with 40 nodes (centralized algorithm).

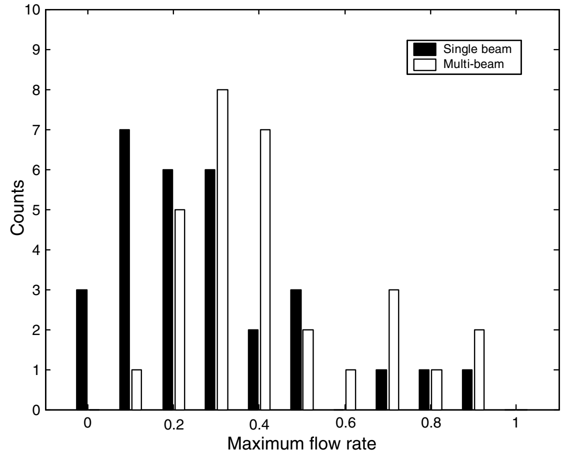

Fig. 11. Histogram of maximum flow in network with 20 nodes (distributed algorithm).

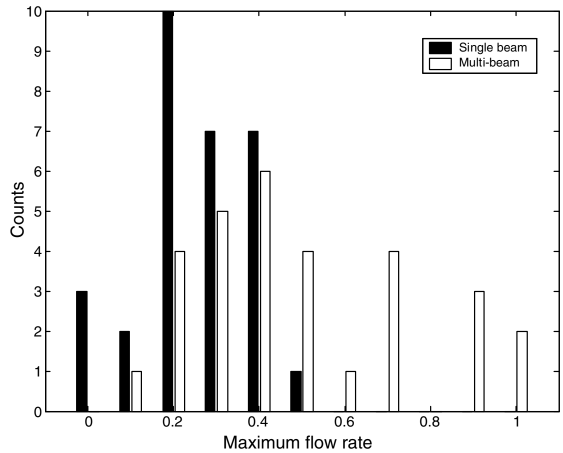

Fig. 12. Histogram of maximum flow in network with 30 nodes (distributed algorithm).

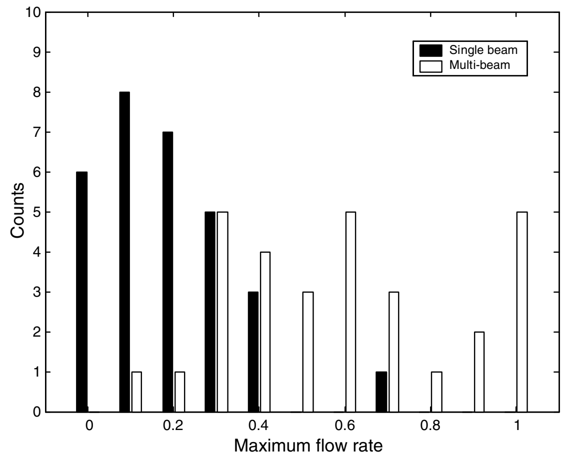

Fig. 13. Histogram of maximum flow in network with 40 nodes (distributed algorithm).

lack of global knowledge of available link bandwidth. So the distribution of the flow at succeeding links may not be optimal. When the outgoing flow is not assigned in the optimal way, some link capacity may be unused. Especially due to suboptimal distribution, a node has higher capacity than all upstream nodes. Then its capacity can never be fully utilized. This leads to the suboptimal performance of the distributed algorithm.

7. Conclusion

We study the joint routing and scheduling in multi-hop wireless networks with directional antennas in this work. The goal is to maximize the throughput between given s–d pair. A key distinction of our work compared to previous work is that our approach answers the questions of what the optimal flow is and how to realize it, with a practical interference model. Based on the protocol model, the maximum throughput problem constrained by interference is formulated as an optimization problem. The large searching space due to myriad paths between a s–d pair, combined with the inherent hardness of link scheduling, leads to an NP-hard centralized MIP problem. To search for a feasible approach, we decompose the end-to-end flow into arc flows. Integrating routing with scheduling, the answer for "what" and "how" is unveiled concurrently. The method is applicable to both single-beam and multi-beam directional antennas, with some variation in information exchange and feasibility condition.

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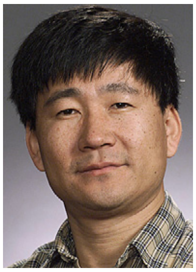

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