The Capacity of Wireless Networks

We will discuss Gupta & Kumar's capacity paper [Gupta00] in the class. In the discussion of some technical parts, I will follow the just published book [Xue06] by the same author, not the original paper [Gupta00].

The problem studied in this paper is that: **how the information transfer capability of wireless networks scales as the number of nodes increases**. This leads into the issue of **scaling laws for wireless networks**. Such results can help in understanding the complicated interactions in wireless networks, and to shed light on operating and designing more efficient networks.

Corrections to the Original Paper [Gupta00]:

(1) A correction to the proof of **Lemma 4.8** in pp. 398 is given by the author himself, see: P. R. Kumar, A correction to the proof of a lemma in "The capacity of wireless networks"; *IEEE Transactions on Information Theory*, **49**(11):3117, Nov. 2003.

(2) The formula of **Theorem 2.1 i)** in pp.392 should be:

$$\lambda n\overline{L} \leq \sqrt{\frac{8}{\pi}}\frac{1}{\Delta}W\sqrt{n}.$$

(3) The formula of **Theorem 2.1 ii)** in pp.392 should be:

$$\lambda n\overline{L} \leq \left(\frac{\beta+1}{\beta}\right)^{\frac{1}{\alpha}}\frac{2^{\frac{2\alpha-1}{\alpha-1}}}{\sqrt{\pi}}Wn^{\frac{\alpha-1}{\alpha}}.$$

1 The capacity of Arbitrary Networks

1.1 Definitions in Arbitrary Networks

1.1.1 Arbitrary Network

- *Node location is arbitrary*: n nodes are arbitrarily located in a disk of area A m^2 in the plane[1].
- *Traffic pattern is arbitrary*: each node has an arbitrarily chosen destination to which it wishes to send traffic at an arbitrary rate.
- *Transmission power is arbitrary*: each node can choose an arbitrary range or power level for each transmission.

[1]The author assume $A = 1$ in the original paper [Gupta00]. Even though they say in Theorem 2.1 iv) (pp. 393) that: when the domain is of A square meters rather than 1 m^2, then all the upper bounds are scaled by $\sqrt{A}$, they do not prove it. Here we follow the book [Xue06]. The reason will be clear in the discussion part.

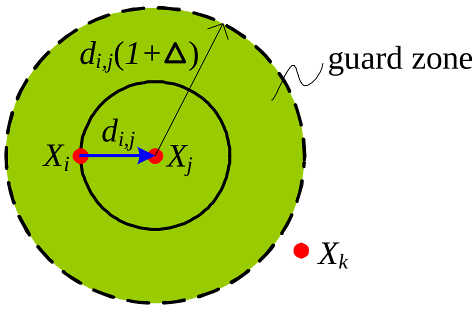

Figure 1: The protocol model. X_i: transmitter, X_j: receiver.

1.1.2 Models for Successful Reception in MAC Layer

We need to describe when a transmission is received successfully by its intended recipient, i.e., for a receiver to correctly decode a packet intended for it.

(1) The protocol model: specifies the condition of successful transmission in terms of geometry

Suppose node X_i transmits to a node X_j. Then this transmission at rate W bits/sec is assumed to be successfully received by node X_j if

$$|X_k - X_j| \geq (1+\Delta)|X_i - X_j|$$

for every other node X_k simultaneously transmitting over the same channel.

X_i: location of a node.

$\triangle > 0$: guard zone parameter[2].

Recall that in 802.11, if transmission range is r, then sensing range is $(1+\Delta)r$.

(2) The physical model: specifies the condition of successful transmission in terms of SINR

The transmission from a node X_i, $i \in \mathcal{T}$, is successfully received by a node X_j if

$$\frac{\frac{P_i}{|X_i - X_j|^\alpha}}{N + \sum_{k \in \mathcal{T}, k \neq i} \frac{P_k}{|X_k - X_j|^\alpha}} \geq \beta.$$

$\mathcal{T}$: let $\{X_k; k \in \mathcal{T}\}$ be the set of nodes simultaneously transmitting at some time instant over a certain channel.

P_k: transmission power level chosen by node X_k.

β: required minimum signal-to-noise-interference ratio (SINR) for successful receptions.

N: ambient noise power level.

α: path loss exponent. Signal power decays with distance r as $\frac{1}{r^\alpha}$.

[2]More precisely, a circle of radius $(1+\Delta)|X_i - X_j|$ quantifies a guard zone required around the receiver to ensure that there is no destructive interference from neighboring nodes transmitting on the same channel at the same time.

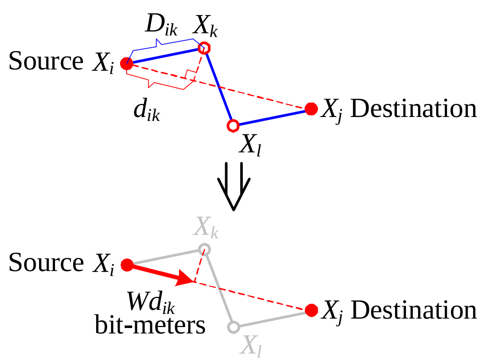

Figure 2: The Transport capacity.

(3) The generalized physical model:[3]

The above model assumes that a transmission can only occur at one of two rates: W bits/sec if the SINR exceeds β, and 0 bits/sec otherwise. This model of data rate can be generalized to be continuous in SINR, based on Shannon's capacity formula for the additive Gaussian noise channel. (This will however require that the modulation or coding scheme for the transmission to be adapted to the existent SINR, a fact that entails further overhead for coordinating between the nodes.) In this case the data rate from transmitter X_i to its receiver $X_{R(i)}$ is assumed to be

$$W = B \log \left(1 + \frac{\frac{P_i}{|X_i - X_j|^\alpha}}{NB + \sum_{k \in \mathcal{T}, k \neq i} \frac{P_k}{|X_k - X_j|^\alpha}} \right).$$

B: bandwidth of channel in hertz.

$N/2$: the noise spectral density in watts/hertz.

1.1.3 The Transport Capacity of Arbitrary Networks

Definition:

transport capacity = (end-to-end data rate)×(source-destination distance)

(1) Given any set of successful transmissions taking place over time and space, let us say that the network transports one *bit-meter* when one bit has been transported a distance of one meter toward its destination.

(2) The *transport capacity* of a specific network is defined as the maximum bit-meters per second the network can achieve in aggregate.

(3) The *transport capacity* of n nodes on a disk of area A (arbitrary network) is the maximum of all achievable transport capacities networks with n nodes. The difference is that in this latter case (3) the locations of the n nodes are also allowed to be optimized, as are the choices of source-destination pairs.

[3]The original paper does not discuss this model. This model is introduced by Xue and Kumar in chapter 4 in [Xue06].

Discussion:

- The transport capacity of arbitrary networks is achieved when the nodes are optimally placed, the traffic pattern is optimally chosen, the range of each transmission is chosen optimally, and the network is optimally operated. By the phrase "optimally operated" we mean optimized over the choice of a route, or a multiple set of routes to be used for each source-destination pair, as well as optimal timing of all transmissions, i.e., spatio-temporal optimization of all transmissions.
- Notice only the distance between the original source and the final destination counts; extra distance travelled due to, say, non-straight line routing is not counted (see Figure 2 for an example).
- We do not consider multicast or broadcast cases, which means we do not give multiple credit for the same bit carried from one source to several different destinations as in the multicast or broadcast cases.

1.2 Main Results for Arbitrary Networks

1.2.1 For protocol model:

(0) **Order of transport capacity:** The transport capacity of an Arbitrary Network of n nodes under the Protocol Model is $\Theta(W\sqrt{An})$ bit-meters/sec.
(1) **Upper-bound of transport capacity:** $\sqrt{\frac{8}{\pi}}\frac{W}{\Delta}\sqrt{An}$ bit-meters/sec.
(2) **Achievable lower-bound of transport capacity:** $\frac{W}{1+2\Delta}\frac{\sqrt{An}}{\sqrt{n}+\sqrt{8\pi}}$ bit-meters/sec (for n a multiple of four) when the networks is with grid node distribution and neighbor-only transmissions.

One **implication** of this result is that each node, on average, obtains $\Theta(W\sqrt{\frac{A}{n}})$ bit-meters/sec. Since this quantity diminishes to zero as n goes large, we see that there is a law of diminishing returns in our model where the area of the domain is fixed, while the number of nodes is allowed to grow.

1.2.2 For physical model:

(0) **Order of transport capacity:** $\Theta(W\sqrt{An})$ bit-meters/sec.
(1) **Upper-bound of transport capacity:**

i) For the general cases, the upper-bound is $\left(\frac{\beta+1}{\beta}\right)^{\frac{1}{\alpha}}\frac{2^{\frac{2\alpha-1}{\alpha-1}}}{\sqrt{\pi}}W\sqrt{A}n^{\frac{\alpha-1}{\alpha}}$ (or $O\big(W\sqrt{A}n^{\frac{\alpha-1}{\alpha}}\big)$) bit-meters/sec. Note that $\alpha \geq 2$, therefore $O\big(W\sqrt{A}n^{\frac{\alpha-1}{\alpha}}\big) \geq O(W\sqrt{An})$.

ii) If the ratio $\frac{P_{max}}{P_{min}}$ between the maximum and minimum powers that transmitters can employ is strictly bounded above by β, then the upper-bound is $\sqrt{\frac{8}{\pi}}\frac{1}{\left(\frac{\beta P_{min}}{P_{max}}\right)^{\frac{1}{\alpha}}-1}W\sqrt{An}$ bit-meters/sec.
(2) **Achievable lower-bound of transport capacity:** $\frac{W\sqrt{A}}{1+2\Delta}\frac{n}{\sqrt{n}+\sqrt{8\pi}}$ bit-meters/sec (for n a multiple of four) when the networks is with grid node distribution and neighbor-only transmissions.

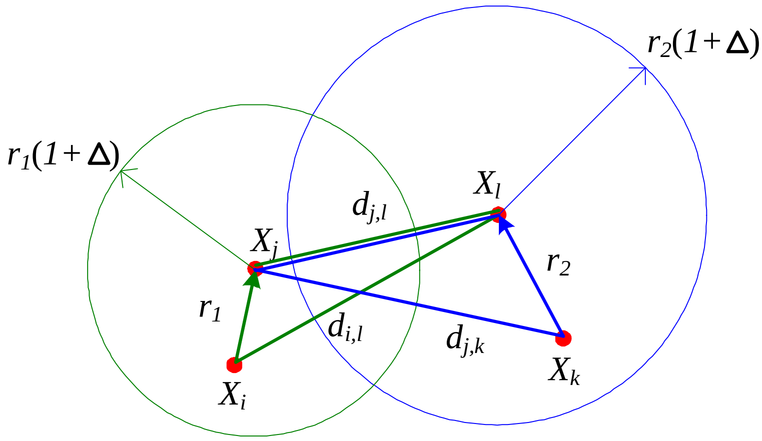

Figure 3: Area constraint for two transmitter-receiver pairs.

Discussion: At the first sight, the upper-bound becomes a little larger compared to the protocol model. The reason is that the techniques used in the original paper [Gupta00] are not enough. An improved upper-bound of $O(W\sqrt{An})$ is given in [Agarwal04]. Therefore, we can include that the results are the same as for the protocol model.

1.3 An Upper Bound on Transport Capacity for Arbitrary Networks under the Protocol Model

Main ideas behind the proof:

The essential idea to upper-bound the transport capacity in the proof below is to observe that successful transmissions "consume" area as they happen. Moreover the radius of such a consumed area is proportional to the transmission range. Since the sum of such areas is upper-bounded by the limited total area A, it follows from invoking the convexity of quadratic function, that one can transform this upper-bound into an upper-bound for the bit-meters per second – the transport capacity.

Step 1: Get the necessary condition for successful concurrent transmitter-receiver pairs: *exclusion disks are disjoint.*

We consider two pairs: $X_i \to X_j$ and $X_k \to X_l$. Based on the protocol model, we have the conditions for this two pairs successful are:

$$|X_k - X_j| \geq (1 + \Delta)|X_i - X_j|, \tag{1}$$

$$|X_i - X_l| \geq (1 + \Delta)|X_k - X_l|. \tag{2}$$

See Figure 3 for an illustration. For pair $X_k \to X_l$, we have:

$$\begin{aligned} |X_j - X_l| + |X_i - X_j| &\geq |X_i - X_l| \text{ (Note: using triangle inequality)} \\ &\geq (1 + \Delta)|X_k - X_l| \text{ (Note: using equation (2))} \end{aligned} \tag{3}$$

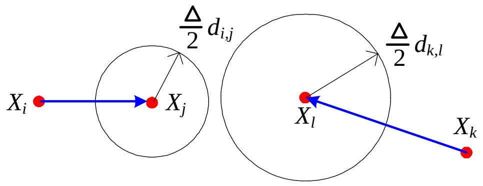

Figure 4: The exclusion disk around each receiver node.

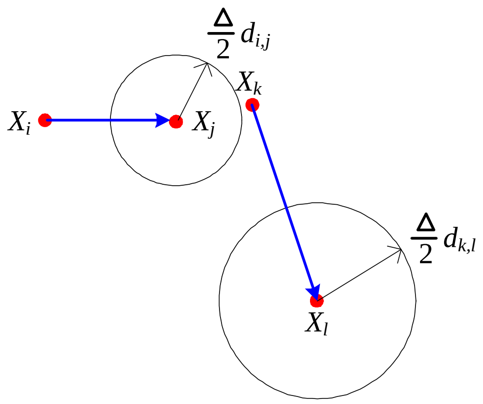

Figure 5: The necessary condition for successful concurrent transmissions.

Similarly for pair $X_i \rightarrow X_j$, we have:

$$\begin{aligned} |X_j - X_l| + |X_k - X_l| &\geq |X_k - X_j| \text{ (Note: using triangle inequality)} \\ &\geq (1+\Delta)|X_i - X_j| \text{ (Note: using equation (1))} \end{aligned} \tag{4}$$

Adding the two inequalities (3) (4), we obtain:

$$\begin{aligned} \underbrace{|X_l - X_j|}_{\text{distance between two receivers}} &\geq \frac{\Delta}{2}(|X_k - X_l| + |X_i - X_j|) \\ &= \frac{\Delta}{2} \underbrace{|X_k - X_l|}_{\text{distance between (k,l) pair}} + \frac{\Delta}{2} \underbrace{|X_i - X_j|}_{\text{distance between (i,j) pair}} \end{aligned} \tag{5}$$

Geometric interpretation of inequality (5): two disks, one of radius $\frac{\Delta}{2}|X_i - X_j|$ centered at X_j , and the other of radius $\frac{\Delta}{2}|X_k - X_l|$ centered at X_l, are disjoint; as shown in Figure 4. We call these disks "exclusion disks."

Condition that exclusion disks are disjoint is only the necessary condition for successful concurrent transmissions, not the sufficient condition. See Figure 5 for an example.

Step 2: Sum of areas of exclusion disks is upper-bounded by A.

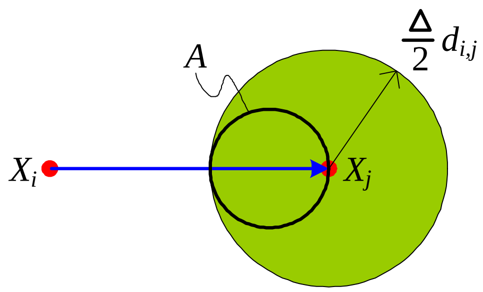

Figure 6: At least a quarter of each exclusion disk is within the disk region.

At least a quarter of each exclusion disk is within the disk region (see Figure 6). Hence we get

$$\sum_{(i,j)\in\mathcal{T}(t)} \frac{1}{4}\pi\left(\frac{\Delta}{2}d_{i,j}\right)^2 \leq A \Rightarrow \sum_{(i,j)\in\mathcal{T}(t)} (d_{i,j})^2 \leq \frac{16A}{\pi\Delta^2}. \tag{6}$$

$d_{i,j}$: the distance between node i and j.
$\mathcal{T}(t)$: the set of all transmitter-receiver pairs ongoing at time t.

Step 3: Number of concurrent transmissions is upper-bounded by $n/2$.

Since at most half the nodes can be transmitting at any time while the other half nodes are receiving, there are at most $n/2$ concurrent transmissions at time t, i.e., $|\mathcal{T}(t)| \leq n/2$.

Step 4: Get the upper-bound of transport capacity.

Inequality (6) only give us the upper-bound of $\sum_{(i,j)\in\mathcal{T}(t)}(d_{i,j})^2$, but what we need is the upper-bound of $\sum_{(i,j)\in\mathcal{T}(t)} d_{i,j}$, the trick here is to use the Cauchy-Schwartz inequality:

$$\left(\sum_{i=1}^{n} x_i \cdot y_i\right)^2 \leq \left(\sum_{i=1}^{n} x_i^2\right) \cdot \left(\sum_{i=1}^{n} y_i^2\right). \tag{7}$$

We have:

$$\begin{aligned} \sum_{(i,j)\in\mathcal{T}(t)} d_{i,j} &\leq \sqrt{\sum_{(i,j)\in\mathcal{T}(t)} (d_{i,j})^2 \cdot \sum_{(i,j)\in\mathcal{T}(t)} 1^2} \text{ (Note: using inequality (7))} \\ &\leq \sqrt{\sum_{(i,j)\in\mathcal{T}(t)} (d_{i,j})^2 \cdot \frac{n}{2}} \leq \sqrt{\frac{8An}{\pi\Delta^2}} \text{ (Note: using inequality (6))} \end{aligned} \tag{8}$$

So the instantaneous rate in bit-meters/sec at time t, is upper-bounded by

$$W \sum_{(i,j)\in\mathcal{T}(t)} d_{i,j} \leq \sqrt{\frac{8A}{\pi}} \frac{W}{\Delta}\sqrt{n}.$$

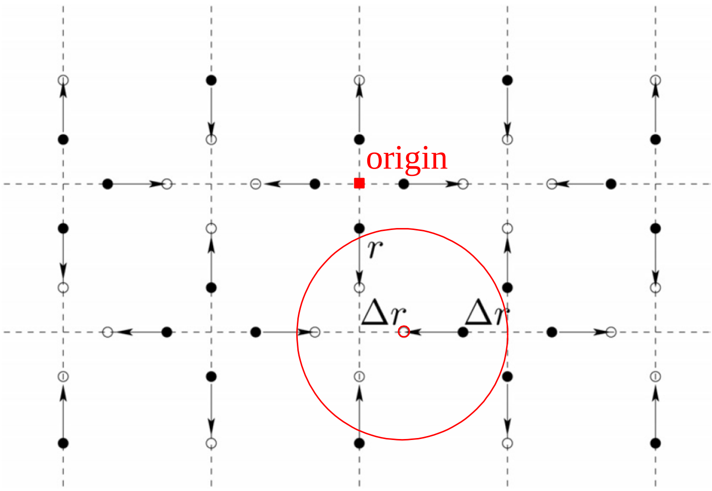

Figure 7: The node arrangement for achieving the lower bound.

1.4 A Constructive Lower Bound on Transport Capacity for Arbitrary Networks under the Protocol Model

We need to show the upper-bound just obtained is a tight one, which means we need to give an example to show that this upper-bound is achievable.

Main ideas behind the proof:

To show the achievability of $\Theta\sqrt{An}$ bit-meters/sec, one can first arrange the n nodes in grid-like positions, then choose $n/2$ nodes as senders with each of them transmitting only to one of its nearest neighbors.

Node arrangement:

Let $r = \frac{1}{1+2\Delta}\frac{\sqrt{A}}{\sqrt{\frac{n}{4}+\sqrt{2\pi}}}$. Center the domain of radius $\sqrt{\frac{A}{\pi}}$ at the origin. Then place transmitters (solid nodes) and receivers (open nodes) as shown in Figure 7.

A simple calculation shows that:

(1) All the transmitters can transmit concurrently, there is no interference from any other transmitter-receiver pair.

(2) There are $n/2$ concurrent transmissions feasible with the same range r and rate W; thus $\frac{W\sqrt{A}}{1+2\Delta}\frac{n}{\sqrt{n}+\sqrt{8\pi}}$ bit-meters/sec is achieved.

Discussion on the result $\Theta(W\sqrt{An})$:

- In [Agarwal04], Agarwal and Kumar present improved upper $\sqrt{\frac{8}{\pi}}\frac{W}{\sqrt{(1+\Delta)\sqrt{\Delta(2+\Delta)}}}\cdot\sqrt{An}$ and lower bounds $\sqrt{\frac{1}{\pi}}\frac{W}{\sqrt{(1+\Delta)\sqrt{\Delta(2+\Delta)}}}\cdot\sqrt{An}$. The new bounds bracket the transport capacity to within a factor of $\sqrt{8}$, which means the result is really tight.

- We assume node density $\lambda \equiv \frac{n}{A}$ is a constant, then each node, on average, obtains $\Theta(\frac{W}{\sqrt{\lambda}}) = \Theta(1)$ bit-meters/sec.

1.5 Transport Capacity for Arbitrary Networks under the Physical Model

In this section, we consider a more physical criterion for successful reception specified by the requirement on the signal to interference plus noise ratio (SINR) – called the Physical Model. Since common radio technology for decoding a packet works only if the SINR is sufficiently large, this model is more faithful to physical considerations.

The Protocol Model stipulates local, geometric constraints, while in the Physical Model every transmission influences every reception. They thus seem very different from each other, but, interestingly, as shown by the following result, there is a correspondence between them.

1.5.1 Upper Bounds on Transport Capacity for Arbitrary Networks under the Physical Model

General Cases:

Suppose X_i is transmitting to $X_{R(i)}$ at power level P_i at time instance t, from the physical model we have:

$$\frac{\frac{P_i}{|X_i - X_{R(i)}|^\alpha}}{N + \sum_{k\in\mathcal{T},k\neq i}\frac{P_k}{|X_k - X_{R(i)}|^\alpha}} \geq \beta \Rightarrow \frac{\frac{P_i}{|X_i - X_{R(i)}|^\alpha}}{N + \sum_{k\in\mathcal{T}}\frac{P_k}{|X_k - X_{R(i)}|^\alpha}} \geq \frac{\beta}{\beta+1}.$$

Similar to the proof procedure for the protocol model, we need to get the upper-bound of $\sum_{i\in\mathcal{T}}|X_i - X_{R(i)}|$.

$$\begin{aligned}
|X_i - X_{R(i)}|^\alpha &\leq \frac{\beta+1}{\beta}\frac{P_i}{N + \sum_{k\in\mathcal{T}}\frac{P_k}{|X_k - X_{R(i)}|^\alpha}} \\
&\leq \frac{\beta+1}{\beta}\frac{P_i}{N + \left(\frac{\pi}{4A}\right)^{\alpha/2}\sum_{k\in\mathcal{T}}P_k} \\
&\quad \text{(Since } |X_k - X_{R(i)}| \leq 2\sqrt{\frac{A}{\pi}}\text{, the diameter of the disk with area } A\text{)}
\end{aligned} \tag{9}$$

Summing over all concurrent transmitter-receiver pairs, we have:

$$
\begin{aligned}
\sum_{i\in\mathcal{T}}|X_i - X_{R(i)}|^\alpha &\le \frac{\beta+1}{\beta}\frac{\sum_{i\in\mathcal{T}} P_i}{N + \left(\frac{\pi}{4A}\right)^{\alpha/2}\sum_{k\in\mathcal{T}} P_k} \\
&\le \frac{\beta+1}{\beta}2^\alpha\left(\frac{\pi}{A}\right)^{-\frac{\alpha}{2}} (\text{ Ignoring } N, \sum_{i\in\mathcal{T}} P_i)
\end{aligned}
\tag{10}
$$

Note that function $f(x) = x^\alpha$ is convex. Hence $(\sum_i x_i)^2 \le \sum_i x_i^2$. We have:

We need to use the following generalized Cauchy-Schwartz inequality: If $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{p'} = 1$, then:

$$
\left|\sum_{i=1}^n x_i \cdot y_i\right| \le \left(\sum_{i=1}^n |x_i|^p\right)^{\frac{1}{p}} \cdot \left(\sum_{i=1}^n |y_i|^{p'}\right)^{\frac{1}{p'}}. \tag{11}
$$

Let $p = \alpha$, then $p' = \frac{\alpha}{\alpha-1}$. We have

$$
\begin{aligned}
\sum_{i\in\mathcal{T}}|X_i - X_{R(i)}| &\le \left(\sum_{i\in\mathcal{T}}|X_i - X_{R(i)}|^\alpha\right)^{\frac{1}{\alpha}} \cdot \left(\sum_{i\in\mathcal{T}} 1^{\frac{\alpha}{\alpha-1}}\right)^{\frac{\alpha-1}{\alpha}} \\
&\le \left(\frac{\beta+1}{\beta}2^\alpha\left(\frac{\pi}{A}\right)^{-\frac{\alpha}{2}}\right)^{\frac{1}{\alpha}}\left(\frac{n}{2}\right)^{\frac{\alpha-1}{\alpha}} \\
&= \left(\frac{\beta+1}{\beta}\right)^{\frac{1}{\alpha}}\frac{2^{\frac{2\alpha-1}{\alpha-1}}}{\sqrt{\pi}}\sqrt{A}n^{\frac{\alpha-1}{\alpha}}.
\end{aligned}
\tag{12}
$$

The upper-bound is given by:

$$
W\sum_{i\in\mathcal{T}}|X_i - X_{R(i)}| \le \left(\frac{\beta+1}{\beta}\right)^{\frac{1}{\alpha}}\frac{2^{\frac{2\alpha-1}{\alpha-1}}}{\sqrt{\pi}}W\sqrt{A}n^{\frac{\alpha-1}{\alpha}}.
$$

Special Case where $\frac{P_{max}}{P_{min}} < \beta$:

Suppose X_i is transmitting to $X_{R(i)}$ at power level P_i at time instance t, from the physical model we have:

$$
\frac{\frac{P_i}{|X_i - X_{R(i)}|^\alpha}}{N + \sum_{k\in\mathcal{T},k\ne i}\frac{P_k}{|X_k - X_{R(i)}|^\alpha}} \ge \beta \Rightarrow \frac{\frac{P_i}{|X_i - X_{R(i)}|^\alpha}}{\frac{P_k}{|X_k - X_{R(i)}|^\alpha}} \ge \beta.
$$

Thus

$$
\begin{aligned}
|X_k - X_{R(i)}| &\geq \beta^{\frac{1}{\alpha}} \left(\frac{P_k}{P_i}\right)^{\frac{1}{\alpha}} |X_i - X_{R(i)}| \\
&\geq \left(\frac{\beta P_{min}}{P_{max}}\right)^{\frac{1}{\alpha}} |X_i - X_{R(i)}| \\
&= (1+\Delta)|X_i - X_{R(i)}|
\end{aligned} \tag{13}
$$

where $\Delta = \left(\frac{\beta P_{min}}{P_{max}}\right)^{\frac{1}{\alpha}} - 1$. Thus the same upper bound as for the protocol model carries over with Δ defined as above. Recall that for the protocol model the upper bound is $\sqrt{\frac{8}{\pi}}\frac{W}{\Delta}\sqrt{An}$. Therefore, we have the upper-bound for this special case as: $\sqrt{\frac{8}{\pi}}\frac{W}{\left(\frac{\beta P_{min}}{P_{max}}\right)^{\frac{1}{\alpha}}-1}\sqrt{An}$.

Note that the upper bound for the generalized physical model is discussed in a similar way in Chapter 4.3 in [Xue06] (pp. 177-pp.182). We note that this upper bound, differing by only a constant ratio, for networks under the Physical Model. This is because if a network under the Physical Model can realize a successful transmission between X_i and $X_R(i)$ at rate W bits/sec, the SINR must be no less than β. Then in that case, the Generalized Physical Model also allows a constant rate transmission corresponding to the SINR β in Shannon's formula between nodes X_i and $X_R(i)$.

1.5.2 Lower Bounds on Transport Capacity for Arbitrary Networks under the Physical Model

The original paper do not prove this part. Here we use the argument in Chapter 4.2 in [Xue06]. The following result show that there is a correspondence between the protocol model and the phisical model.

Theorem: Let $\Delta(\beta) = \left(48\frac{2^{\alpha-2}}{\alpha-2}\beta\right)^{\frac{1}{\alpha}}$. Suppose that for $\Delta > \Delta(\beta)$ the Protocol Model allows simultaneous transmissions for all transmitter-receiver pairs in a set $\mathcal{T}(t)$. Then there exists a power assignment $\{P_i, 1 \leq i \leq n\}$ allowing the same set of transmissions under the Physical Model with threshold β.

Proof: See pp. 174-176 in [Xue06].

This shows that any feasible set of active transmitter-receiver pairs in the Protocol Model with sufficiently large Δ, dependent on β, admits a power assignment for the same set of nodes to transmit under the Physical Model. This allows one to get feasibility results for the Physical Model based on those for the Protocol Model. Therefore, we can get the same lower-bound from the result of the Protocol Model.

Note that the above theorem also establishes a similar lower bound, differing by only a constant ratio, for networks under the Generalized Physical Model. This is because a successful transmission in a network under the Physical Model requires that the SINR be larger than a threshold, which enables a constant rate between these two nodes under the Generalized Physical Model.

References

[Penrose03] M. D. Penrose, *Random Geometric Graphs*, volume 5 of *Oxford Studies in Probability*, Oxford University Press, May 2003.

[Gupta98] P. Gupta and P. R. Kumar, "Critical power for asymptotic connectivity in wireless networks," Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W. H. Fleming, 1998.

[Gupta00] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," IEEE Transactions on Information Theory, vol. IT-46, no. 2, pp. 388-404, March 2000.

[Agarwal04] A. Agarwal and P. R. Kumar, "Capacity bounds for ad hoc and hybrid wireless networks," ACM SIGCOMM Computer Communications Reveiew, vol. 34, no. 3, pp. 71C81, July 2004.

[Xue06] F. Xue and P. R. Kumar, *Scaling Laws for Ad Hoc Wireless Networks: An Information Theoretic Approach*, NOW Publishers, Delft, The Netherlands, 2006. (Free online version is available at http://black.csl.uiuc.edu/ prkumar/ps_files/06-07-18-scalinglaws. pdf)