Correction

A Correction to the Proof of a Lemma in "The Capacity of Wireless Networks"

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***Abstract*—The proof of Lemma 4.8 in [1] is corrected.**

Correction of Proof of Lemma 4.8

In [1, Sec. IV.G], there was an error in the stated sample size bound for the Vapnik–Chervonenkis (VC) theorem. The notation used will be that in [1] with $d(\mathcal{F})$ denoting the VC dimension of the set $\mathcal{F}$. The correct statement is

$$\text{Prob}\left(\sup_{F\in\mathcal{F}}\left|\frac{1}{N}\sum_{i=1}^{N}I(x_i\in F)-P(F)\right|>\epsilon\right) \le 4\left[(2N)^{d(\mathcal{F})+1}+1\right]e^{-\frac{\epsilon^2 N}{8}}, \qquad \text{for } N\ge 2/\epsilon^2. \quad (1)$$

See [2].

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Proof of Lemma 4.8: The probability that a cell V in $\mathcal{V}_n$ contains no nodes is $\le \left(1-\frac{100\log n}{n}\right)^n$. Since there are at most $\frac{n}{100\log n}$ cells

$$\text{Prob}(\text{every cell } V\in\mathcal{V}_n \text{ contains a node}) \ge 1-\frac{n}{100\log n}\left(1-\frac{100\log n}{n}\right)^n.$$

Since

$$\lim_{n\to+\infty} n\left(1-\frac{\log n}{n}\right)^n = 1$$

the right-hand side above converges to one. □

It may be noted that this proof also allows us to use cells of smaller size, dispensing with the factor 100 in [1, eq. (13)]. Another proof of the above lemma is based on the probability of an ϵ-net [3], for which the sample size given in [1] is indeed the appropriate one; however, it does not allow us to dispense with this factor.

It should also be noted that Lemma 4.13 continues to follow from (1) with $\mathcal{F}=\mathcal{D}'$ and $\epsilon=16\sqrt{\frac{\log n}{n}}$, since $d(\mathcal{D}')\le 30$ and c_5 can be chosen as 16 plus the constant in Lemma 4.9. □

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References

[1] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, pp. 388–404, Mar. 2000.

[2] V. N. Vapnik and A. Chervonenkis, "On the uniform convergence of relative frequencies of events to their probabilities," *Theory Probab. its Applic.*, vol. 16, no. 2, pp. 264–280, 1971.

[3] A. Blumer, A. Ehrenfeucht, D. Haussler, and M. Warmuth, "Classifying learnable geometric concepts with the Vapnik-Chervonenkis dimension," in *Proc. 18th ACM Symp. Theory of Computing*, Berkeley, CA, May 1986, pp. 272–282.