Solutions for Homework 3

3.30 (**Solution**): From Little's law we have that $\Pr(\text{the system is busy}) = \lambda E(X)$, therefore, $\Pr(\text{the system is empty}) = 1 - \lambda E(X)$.

The length of time between busy periods is the length of idle time. The ending point of the busy period is the starting point of the idle period, which is a point between two typical customer arrivals, since the interarrival time is exponentially distributed, according to (strong) memoryless property, the distribution of the residual life of the interarrival time (i.e., the idle period) will be still exponentially distributed with the same rate, thus the idle period is exponentially distributed with parameter λ, thus the average of idle period is $1/\lambda$.

Let B be the average length of busy period and I the average length of idle period, since the busy periods and idle periods alternate, the proportion of time the system is busy is $B/(B+I)$, which is also equal to the probability that the system is busy, i.e., $\lambda E[X]$, thus we have

$$B = \frac{E[X]}{1 - \lambda E[X]}.$$

From this expression, we obtain that the average number of customer served during the busy period is

$$\frac{B}{E[X]} = \frac{1}{1 - \lambda E[X]}.$$

3.31 (**Solution**) The problem with the argument given is that more customers arrive while long-service customers are served, so the average service time of a customer found in service by another customer upon arrival is more than $E[X]$.

3.36 (**Solution**) For each session, the arrival rates, average transmission times and utilization factors for the short packets (class 1), and the long packets (class 2) are

$$\begin{aligned} \lambda_1 &= 0.25, \ \frac{1}{\mu_1} = 0.02, \ \rho_1 = 0.005; \\ \lambda_2 &= 2.25, \ \frac{1}{\mu_1} = 0.3, \ \rho_1 = 0.675; \end{aligned}$$

The corresponding second moments of transmission time are

$$E[X_1^2] = 0.0004, \ E[X_2^2] = 0.09.$$

The total arrival rate for each session is $\lambda = 2.5$ packets per second. The overall 1st and 2nd moments of the transmission time, and the overall utilization factors are given by

$$\begin{aligned} &1/\mu = 0.1 * (1/\mu_1) + 0.9 * (1/\mu_2) = 0.272 \\ &E[X^2] = 0.1 * E[X_1^2] + 0.9 * E[X_2^2] = 0.081 \\ &\rho = \lambda/\mu = 2.5 * 0.272 = 0.68 \end{aligned}$$

We obtain the average time in queue W via the P-K formula

$$W = \frac{\lambda E[X^2]}{2(1-\rho)} = 0.3164.$$

The average time in the system is $T = 1/\mu + W = 0.588$. The average number in the queue and in the system are

$$N_Q = \lambda W = 0.791,\ N = \lambda T = 1.47.$$

The quantities above correspond to each session in the case where the sessions are time-division multiplexed on the line. In the statistical multiplexing case, W, T, N_Q and N are decreased by a factor of 10 (for each session).

In the non-preemptive priority case we obtain the following using the corresponding formulas:

$$W_1 = \frac{\lambda_1 E[X_1^2] + \lambda_2 E[X_2^2]}{2(1-\rho_1)} = 0.108.$$
$$W_2 = \frac{\lambda_1 E[X_1^2] + \lambda_2 E[X_2^2]}{2(1-\rho_1)(1-\rho_1-\rho_2} = 0.38$$
$$T_1 = 1/\mu_1 + W_1 = 0.128$$
$$T_2 = 1/\mu_2 + W_2 = 1.055$$
$$N_Q 1 = \lambda_1 W_1 = 0.027$$
$$N_Q 2 = \lambda_2 W_2 = 0.855$$
$$N_1 = \lambda_1 T_1 = 0.032$$
$$N_2 = \lambda_2 T_2 = 2.273$$

3.37 (**Solution**) (a). $\lambda = 1/60$, probability distribution for number of copies is $p_i = 1/10 = 0.1$ ($i = 1, 2, \ldots, 10$). Thus, $E[X] = \sum_{i=1}^{10}(3*i)*p_i = 16.5$ seconds, and $E[X^2] = \sum_{i=1}^{10}(3*i)^2 * p_i = 346.5$, Thus, we have

$$W = \frac{\lambda E[X^2]}{2(1-\lambda E[X])} = 3.98.$$

(b). Non-preemptive priority. In the following calculation, the superscript 1 will imply the quantities for the priority 1 customers and 2 for priority 2 customers. Underscripted quantities will refer to the overall system. Notice that the problem is equivalent to the following: with probability 0.2 a customer belongs to the class 1 and rest belongs to class 2, the service time will also be classified into two averages. Thus,

$$\lambda = 1/60,\ \lambda_1 = 0.2 * \lambda = 1/300,\ \lambda_2 = 0.8 * \lambda = 1/75$$

$E[X] = 16.5,\ E[X_1 = 0.5 * 3 + 0.5 * 6 = 4.5,\ E[X_2] = 19.5, E[X^2] = 346.5$

$R = \frac{1}{2}\lambda E[X^2] = 2.8875$

$\rho_1 = \lambda_1 E[X_1] = 0.015$

$\rho_2 = \lambda_2 E[X_2] = 0.26$

$W_1 = \frac{R}{1 - \rho_1} = 2.931$

$W_2 = \frac{R}{(1 - \rho_1)(1 - \rho_2)} = 4.043$

$T_1 = 7.4315,\ T - 2 = 23.543$

$T = \frac{\lambda_1 T_1 + \lambda_2 T_2}{\lambda} = 20.217$