Homwork #1

1. The exponential distribution for the random variable ξ has the following density function

$$f(t) = \lambda e^{-\lambda t},\ t \geq 0.$$

   (1). Find the mean, the variance and the coefficient of variation;

   (2). Find the characteristic function, moment generating function and the Laplace transform of the pdf;

   (3). Show that the exponential distribution has the following memoryless property: for any $s \geq 0$ and $t \geq 0$, we have

$$P(\xi \geq s + t | \xi \geq s) = P(\xi \geq t).$$

   In fact, a stronger version can be shown: for any nonnegative random variable ν, we have

$$P(\xi \geq t + \nu | \xi \geq \nu) = P(\xi \geq t).$$

2. Find the pdf for the minimum of K independent random variables, each of which is exponentially distributed with parameter λ. Find the Laplace transform of the pdf for the sum of the K independent random variables, each of which is exponentially distributed?

3. Consider the discrete-time, discrete-state Markov chain with the probability transition matrix

$$\begin{pmatrix} 0.5 & 0.5 \\ 0.75 & 0.25 \end{pmatrix}.$$

   Find the stationary state probability vector π. Suppose that you are an outsider observer and observe the evolution of the Markov chain, how often you find the chain stay in the state 1?