Performance of GBN ARQ

In the lecture on data layer control protocols, we present some performance results on ARQ protocols under very restrictive conditions such as the one for GBN. For example, given the slide window size W and packet error probability, we obtain the channel efficiency as follows

$$U_{gbn} = \frac{1-P}{1+(W-1)P}.$$

This is valid only when W is chosen to be the number of frames that can be squeezed into the bit pipe, i.e., the total number of frame transmissions wasted due to an error occurrence. If we assume that we use

$$t_F = t_{frame} + t_{prop} + t_{proc} + t_{ack} + t_{prop} + t_{proc}$$

which is the total time spent for a packet to transmit and to know its status. Assuming that the window size is greater than the total number $m = t_F/t_{frame}$ (the total number of frames that can be transmitted during the round trip time t_F or the number of frames that can be squeezed in the bit pipe), when $W \geq m$, then the frame error will be known after m frames. Thus, under the condition that we allow more frames than the bit pipe could accommodate, the channel efficiency is indeed

$$U_{gbn} = \frac{1-P}{1+(m-1)P}.$$

Thus, we cannot set $m = 1$ to compare with the stop and wait protocol!

The difficult case is when $W < m$. In this case, every W frames can be transmitted and then wait. For each frame, if there is no error, it takes only $t_F - (W-1)t_{frame}$ because $(W-1)$ other frames will not be penalized to that particular frame. If there is an error in the frame, it takes t_F to know its status. After this time period, the frame will be retransmitted. Thus, each error will cause the waste of t_F period. Thus, a packet will have to transmitted exactly k times with probability $P^{k-1}(1-P)$ with spending time $(k-1)t_F + t_F - (W-1)t_{frame}$. Therefore, the average total time spent for this frame is

$$\begin{aligned}
t_{total} &= \sum_{k=1}^{\infty} \left((k-1)t_F + t_F - (W-1)t_{frame}\right) P^{k-1}(1-P) \\
&= \sum_{k=1}^{\infty} k t_F P^{k-1}(1-P) - (W-1)t_{frame} \\
&= t_F/(1-P) - (W-1)t_{frame}
\end{aligned}$$

Thus, the channel efficiency, under $W < T_F/t_{frame}$, is given by

$$U_{gbn} = \frac{t_{frame}}{t_{total}} = \frac{t_{frame}}{t_F/(1-P) - (W-1)t_{frame}} = \frac{1-P}{t_F/t_{frame} - (W-1)(1-P)}.$$

We can see that when $W = 1$, we can obtain the result for stop and wait protocol.

One modification in the above derivation is to let the W multiple packets shared the idle period during the last transmission, i.e., the idle period during t_F is divided by W, which is $[2t_{prop} - Wt_{frame}]/W$ or simply $[t_F - (W-1)t_{frame}]/W$. Use this term to replace $t_F - (W-1)t_{frame}$, we could go through the same procedure to obtain a better approximation.

Another approach is to find the channel efficiency without error first using the following approach under assumption $W < m$. In one transmission cycle (transmit W packets and wait until return), there are W packets delivered, thus the channel efficiency can be approximated by

$$U_{gbn} = \frac{Wt_{frame}}{t_F}.$$

When packet error is P, then the channel efficiency can be approximated by

$$U_{gbn} = \frac{Wt_{frame}}{t_F}(1-P).$$

As you can see, there is no analytically accurate formula. However, the fundamental procedure is more or less the same.